Assignment No. 1
Microeconomics I (Prof. Alberto Iozzi)

Gabriele Beccari

PhD student *in Economics and Finance*
Department of Economics and Finance
"Tor Vergata" University of Rome
gabriele.beccari@uniroma2.it

Academic year 2019-20

Please return your answers by the beginning of the first practice (15/11/2019).

Exercise 1

Let $u(x)$ be a *constant elasticity of substitution* utility function of the form

$$u(x) = \left(x_1^\rho + x_2^\rho\right)^{1/\rho} \tag{1}$$

1. Choose a consumption bundle in the Walrasian budget set

   $$B_{p,w} = \{x \in \mathbb{R}^L_+ : p \cdot x \leq w\}$$

   to maximise the utility level;
2. sketch a graph and list three properties that the solution bundle possesses;
3. compute the *Indirect utility function* $v(p, w)$, that is, find $u(x^*)$ for any $x^* \in x(p, w)$;
4. list four properties that $v(p, w)$ possesses;
5. derive the corresponding *Expenditure function* $e(p, u)$ from the relation $v(p, w) = e\big(p, v(p, w)\big)$;
6. from point *5.* derive the *Hicksian demand function* $(h(p, u) \subset \mathbb{R}^L_+)$[1]. $\square$

Exercise 2

A consumer lives for two periods $T = \{1, 2\}$ only. Understandably enough, she wants to maximise her utility function

$$u(c_t) = \alpha \cdot ln(c_1) + \beta \cdot \frac{ln(c_2)}{1+\delta} \qquad \delta > 0 \tag{2}$$

[1] This point requires a lot of patience, as it is very computationally intensive. Proceed with method.

subject to the inter-temporal budget constraint

$$c_1 + \frac{c_2}{1+r} = e_1 + \frac{e_2}{1+r} \qquad\qquad r > 0$$

where e_t is the endowment she receives at every period $t \in T$.

1. Give an economic interpretation of both δ and r;
2. choose c_t in order to maximise $u(c_t), \forall t \in T$. □

Exercise 3

Let $u(x)$ be a *Cobb-Douglas* utility function of the form

$$u(x) = x_1^{\mu} \cdot x_2^{1-\mu} \tag{3}$$

1. Compute the minimal level of wealth required to reach utility level u, that is, find the point in the set

   $$\{x \in \mathbb{R}_+^L : u(x) \geq u > u(0)\}$$

   that lies on the lowest possible budget line associated with a price vector $p > 0$;
2. compute the *Expenditure function* $e(p, u)$ and list its four properties;
   (a) One of the properties will be the degree of homogeneity that $e(p, u)$ exhibits. In this case, prove your answer;
   (b) *for extra points*, feel free to provide a proof for all the four properties. □

References

- Mas-Colell, A., Whinston, M. D., & Green, J. R. (1995). *Microeconomic theory.* New York: Oxford University Press.
  - Chapter 3: **Classical Demand Theory**.