Microeconomics I

Alberto Iozzi

Housekeeping: about your teacher

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Housekeeping: timetable & contacts

- Lectures on Monday, Tuesday and Wednesday, h11-13
- From 2/11 to 11/12
- + tutorials with Gabriele Beccari
- Contact: `alberto.iozzi@uniroma2.it`
- Office hours: TUE, 15-16; or, better, after the lectures or by appointment

Housekeeping: syllabus & texbooks

- Syllabus
  - (Neoclassical) demand theory
  - (Neoclassical) production theory
  - Choice under uncertainty
  - General equilibrium
- Main textbooks:
  - Mas-Colell, A, M D Whiston & J R Green, *Microeconomic Theory* (chaps 1-6)
  - Jehle, J A and P J Reny, *Advanced Microeconomic Theory* (3rd ed., chaps 1-5)
- Useful references are:
  - Deaton, A & J Muellbauer, 1980. *Economics and consumer behaviour*
  - Rubinstein, A, 2009. *Lecture notes in microeconomic theory: the economic agent*
  - Varian H, 2010. *Microeconomic analysis*
  - Miller, N H, 2006. *Notes on Microeconomic Theory* (available at http://www.business.illinois.edu/nmiller/notes.html)
  - Nicholson, W and C M Snyder. *Microeconomic Theory: Basic Principles and Extensions* (11th or 10th edition)

Housekeeping: exam

- Final exam made out of 4 questions
  - 2 exercises
  - 2 essay-type questions
  - students to answer 3 out of 4 questions
- Problem sets to be worked out on your own (alone or with your classmates) within the given deadline.
- Problem sets will be graded and marks will count for your final grade
- The weight of the problem sets in the final grade will be disclosed at the end of the course (it will be a rather small weight)
- Grades in the problem set valid in the exam winter session ONLY

Consumption Theory

Main assumptions 1

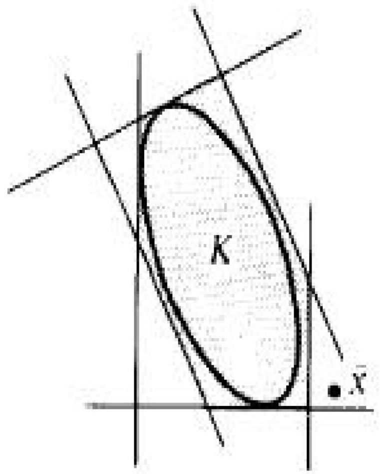
- 1 representative consumer
- L commodities, available for certain, physically different from each other
- commodities are available in *commodities bundles*.
- a commodity bundle is normally denoted by x's, where

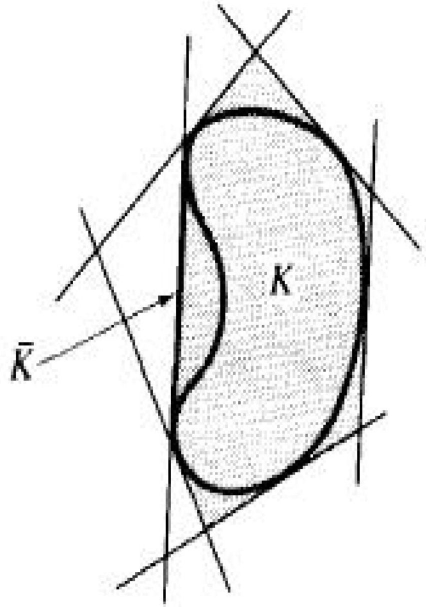
$$x = \begin{pmatrix} x_1 \\ \dots \\ x_\ell \\ \dots \\ x_L \end{pmatrix}$$

Main assumptions 2

- commodity bundles belong to the *consumption set* X, the set of consumption bundles which are physically feasible. We will normally use $X = \Re^L_+$
- consumption set X is *convex*: any linear combination of any two bundles belonging to the consumption set also belongs to the consumption set

To fix ideas: convex/not convex set





A convex set: a set with *no notches and no holes*

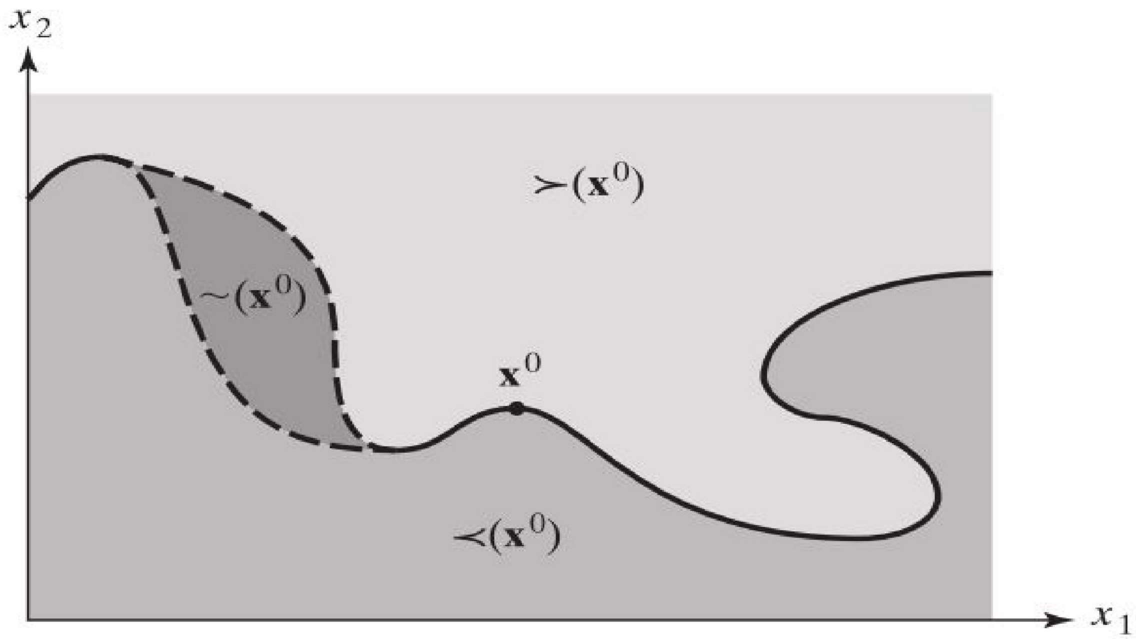
The *preference relation* $\succeq$

- Consumers have a *preference relation* $\succeq$ over commodity bundles
- Preference relation $\succeq$ means *is at least as well liked as* (or, in short, *is at least as good as*).
- We'll also use $\sim$ which means *is liked as* (or, in short, *is as good as*)

Properties of the *preference relation* $\succeq$

For all x, y and $z \in X$, the *preference relation* $\succeq$ satisfies:

1. Rationality:
   - *completeness*: either $x \succeq y$ or $y \succeq x$
   - *transitiveness*: if $x \succeq y$ and $y \succeq z$, then $x \succeq z$



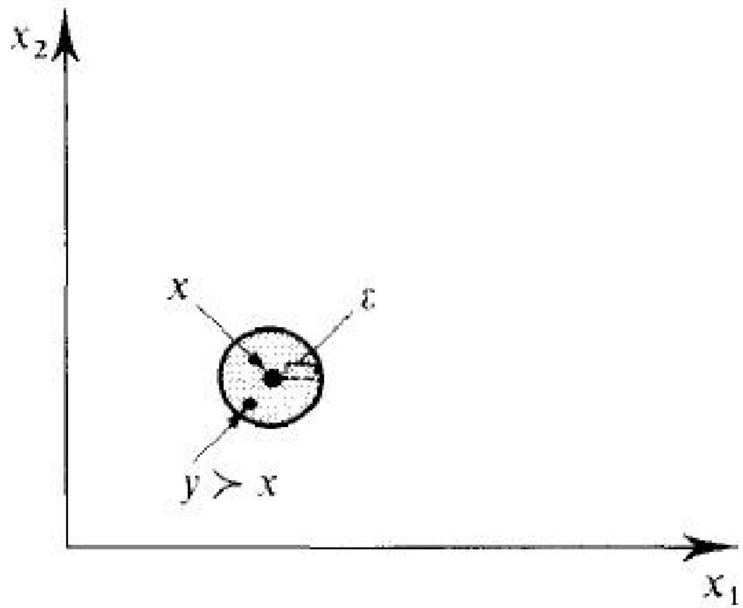
Properties of the *preference relation* $\succeq$

For all x, y and $z \in X$, the *preference relation* $\succeq$ satisfies:

2. Desiderability - Main idea: more is better!!
   - *monotonicity*: $x \gg y$ implies $x \succ y$ (where $x \gg y$ means $x_\ell > y_\ell$ for all ℓ)
   - *strong monotonicity*: $x_\ell \geq y_\ell$ for all ℓ and $x_\ell > y_\ell$ for at least one ℓ implies $x \succ y$.

   Often sufficient to appeal to

   - *local non-satiation*: for every $\varepsilon > 0$, there is y such that $\|x - y\| \leq \varepsilon$ and $y \succ x$

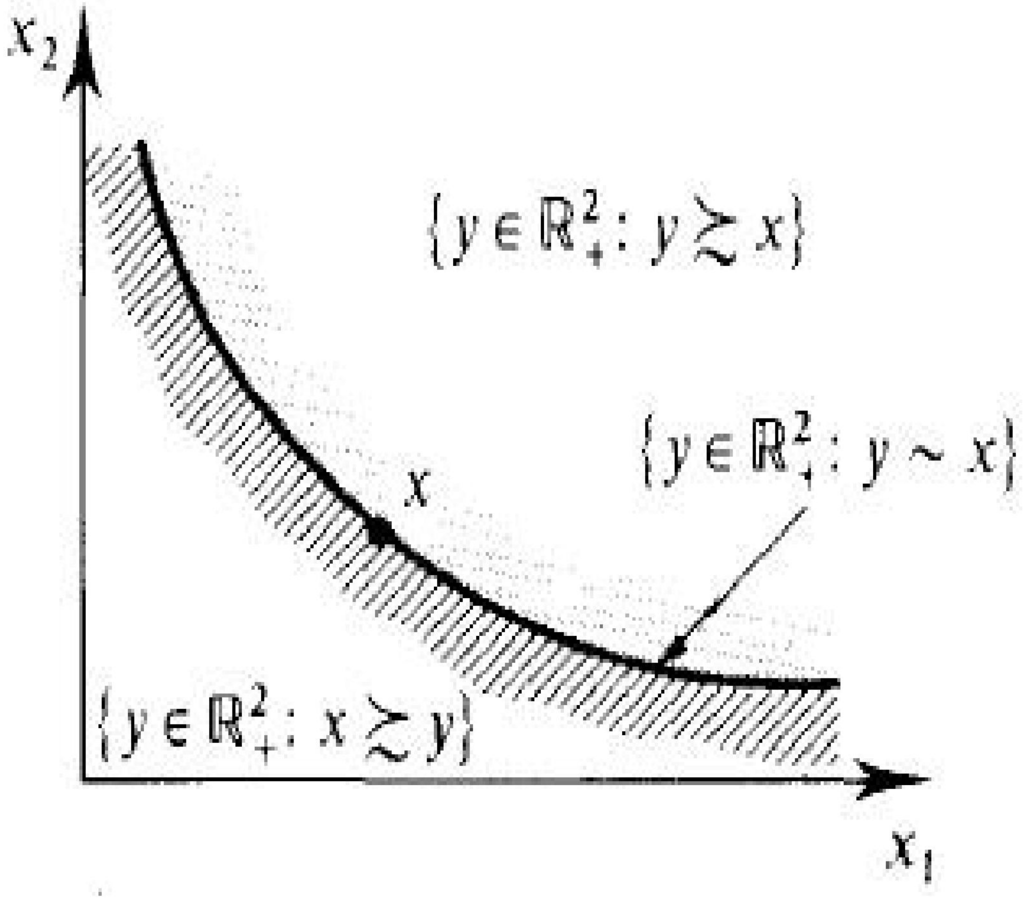


Consumption set

From these assumptions, for any consumption bundle x, can define 3 sets of consumption bundles

- *indifference set*. All consumption bundles that are indifferent to x: $\{y \in X : y \sim x\}$
- *upper contour set*. All consumption bundles that are at least as good as x: $\{y \in X : y \succeq x\}$
- *lower contour set*. All consumption bundles that x is at least as good as: $\{y \in X : x \succeq y\}$

Indifference sets



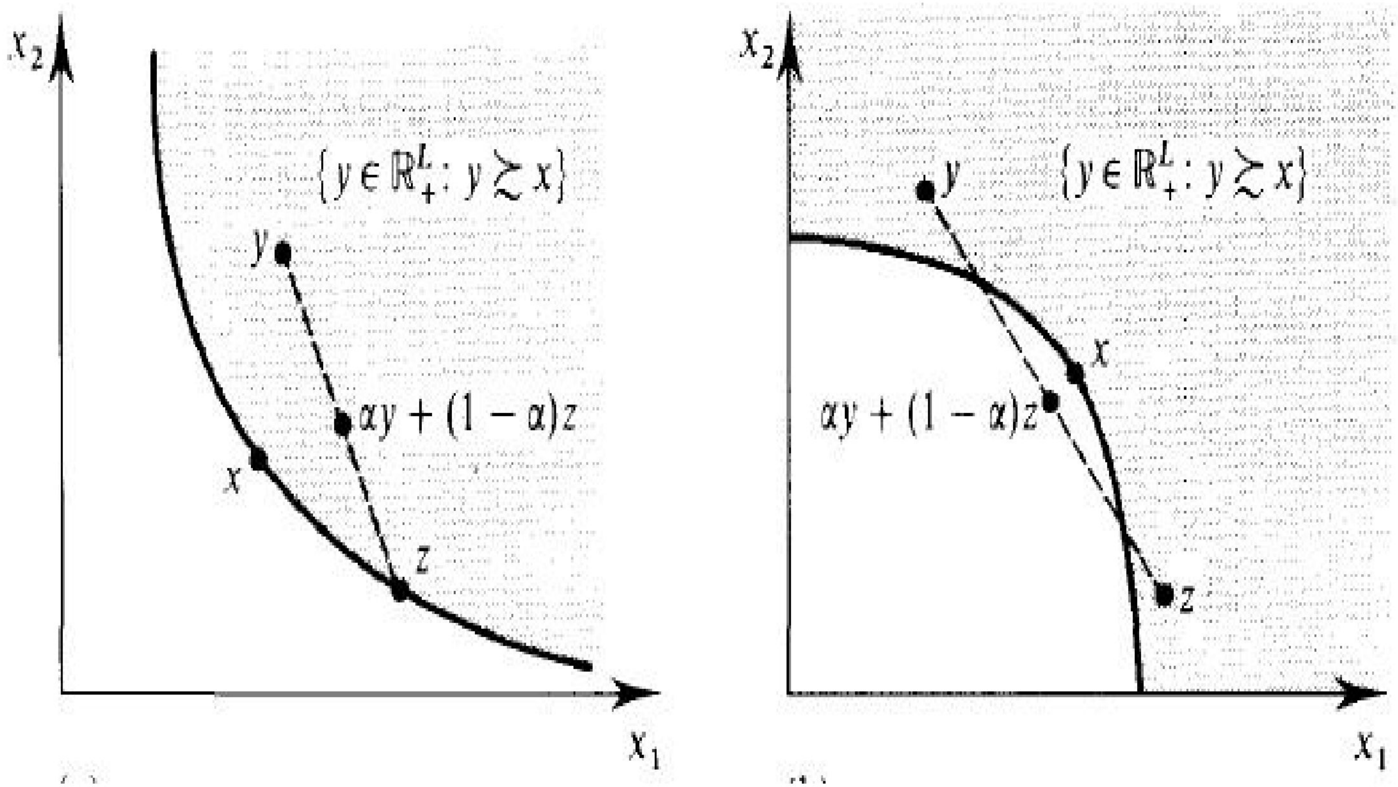
Properties of the preference relation $\succeq$ (more)

For all x, y and $z \in X$

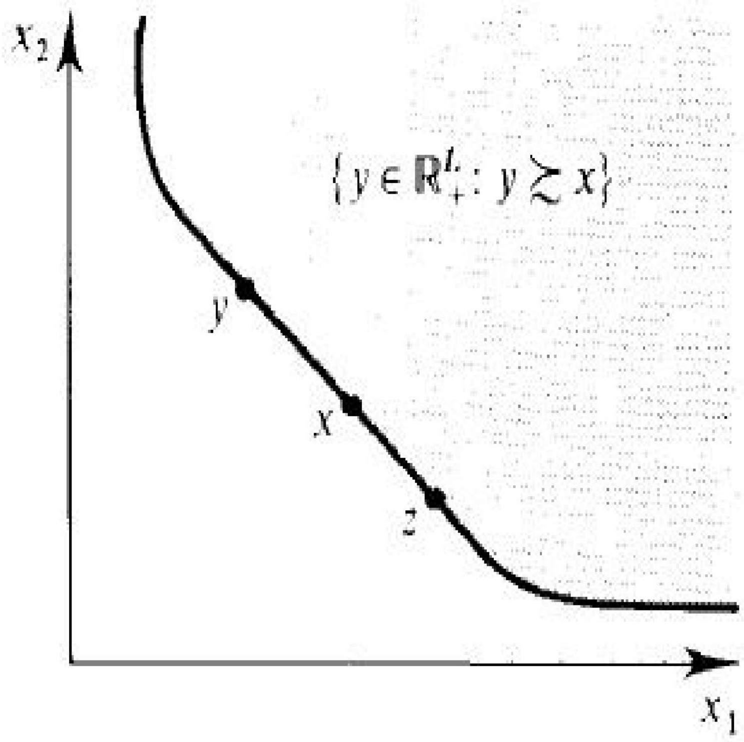
3. Convexity:
   - *convexity*: A preference relation $\succeq$ is convex whenever its upper contour set is convex.
     For any $\alpha \in [0, 1]$ and for any y and z such that $y \succeq x$ and $z \succeq x$, then $\alpha y + (1 - \alpha)z \succeq x$ [$\alpha y + (1 - \alpha)z \succ x$ for *strict* convexity]

   Main idea: preference for diversification (or, in jargon, diminishing marginal rate of substitution, see below)

Strictly convex/not convex preference relations



Convex/not convex preference relations



From preference relation to utility function 1

- Useful to represent the preference relation $\succeq$ by means of a *utility function*.
- A further assumption on the preference relation $\succeq$ is needed. Then, for all x and $y \in X$
  4. Continuity:
     - *continuity*: A preference relation $\succeq$ is continuous if
       - the upper contour set $\{y \in X : y \succeq x\}$ and
       - the lower contour set $\{y \in X : x \succeq y\}$
       are both closed, that is they contain their boundaries.

       Purely technical assumption, needed to avoid sudden jumps in preferences

From preference relation to utility function 2

- Rationality and continuity assumptions are sufficient for the existence of a continuous utility function $u(x)$ that represents $\succeq$
- $u(x)$ needs not to be unique: any $z(x) = f(u(x))$ where $f(.)$ is a strictly increasing function, also represents $\succeq$.
  - utility function is **ordinal**
- Notice that, of course, continuity does not imply differentiability.
- Normally, it is assumed that $u(x)$ is twice differentiable

From restrictions on $\succeq$ to properties of $u(x)$

Properties of the preference relation $\succeq$ translate into properties of the utility function $u(x)$.

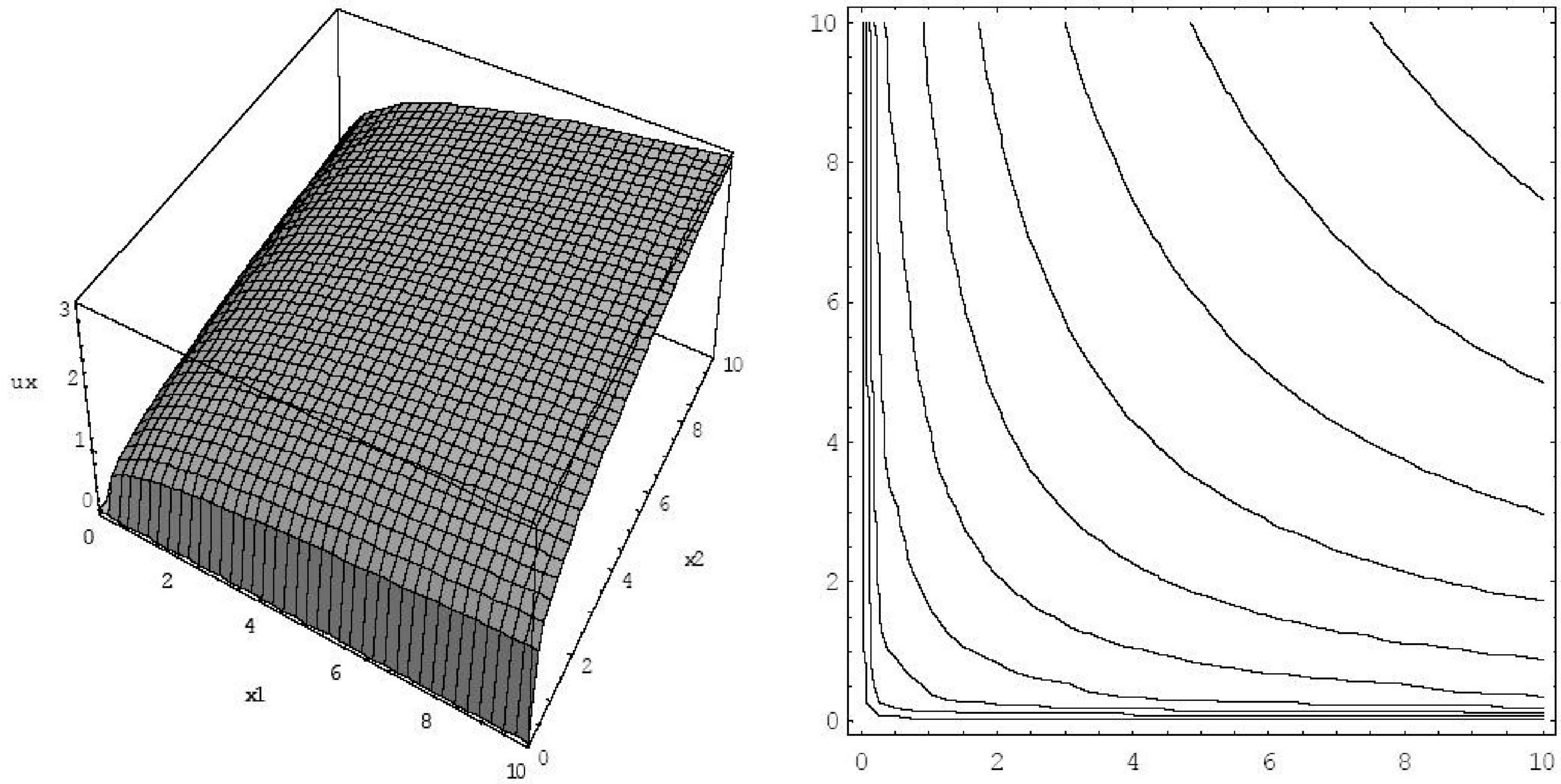
- monotonicity of $\succeq \Rightarrow u(.)$ is increasing:

$$x \gg y \Rightarrow x \succ y \Rightarrow u(x) > u(y)$$

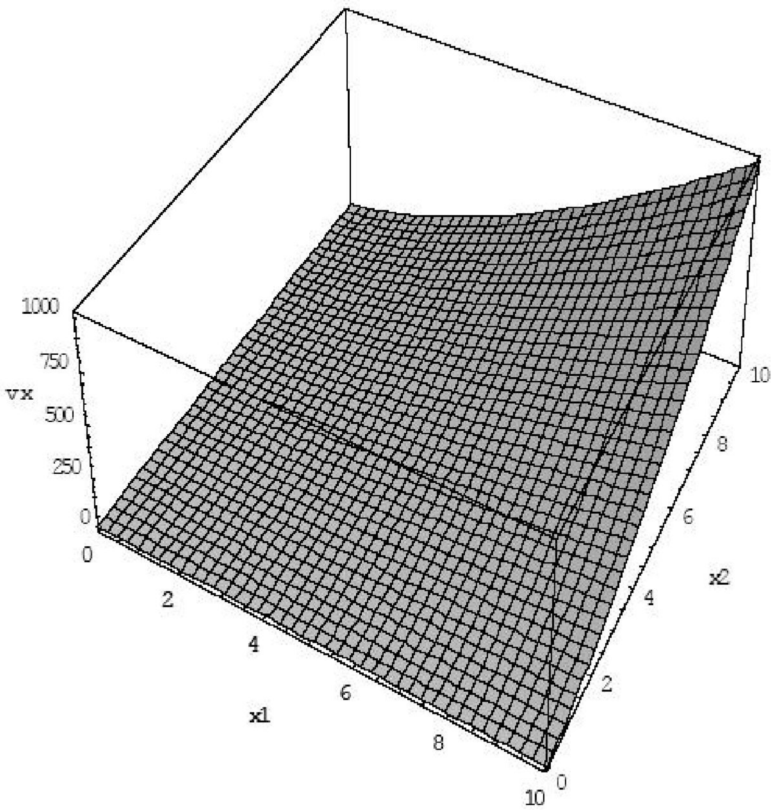
- convexity of $\succeq \Rightarrow$ quasi-concavity of $u(.)$:

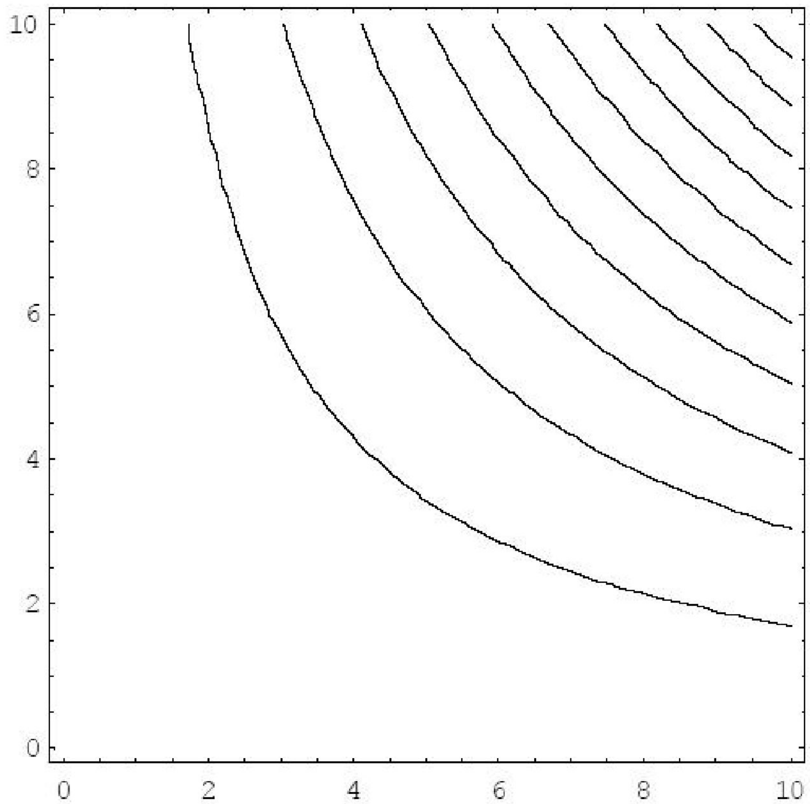
almost tautological:
quasi-concavity of $u(x)$ means that,
for all x,
the set $\{y \in \Re^L_+ : u(y) \geq u(x)\}$ is convex.

To fix ideas: concave function



To fix ideas: quasi-concave function



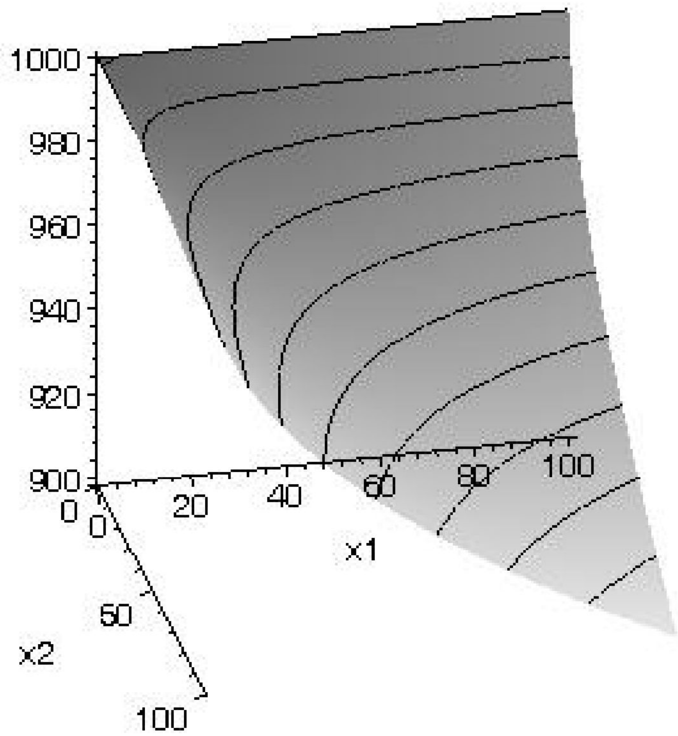


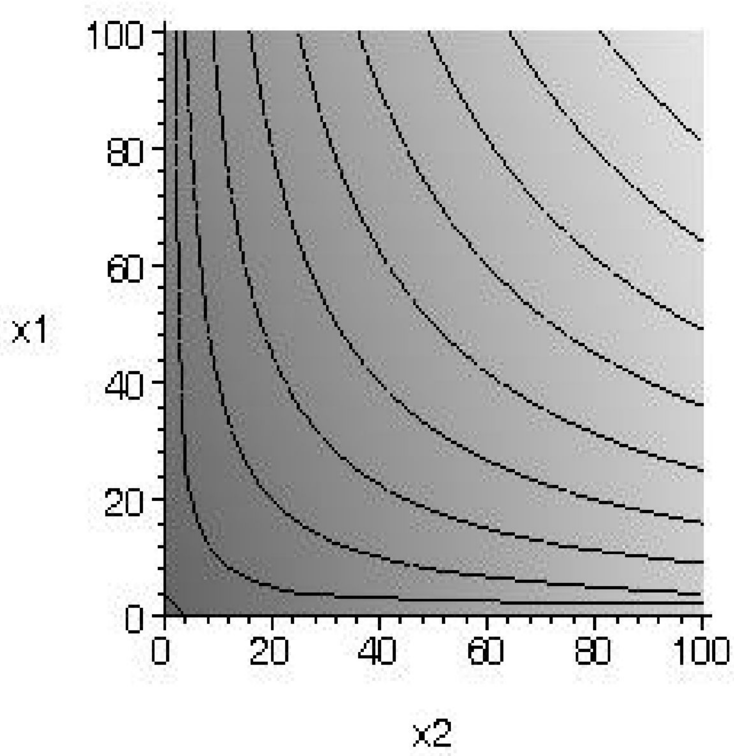
quasi-concave functions

have

convex upper contours

To fix ideas: quasi-convex function





quasi-convex functions
have
convex lower contours

Marginal rate of substitution (two-goods case)

- Useful property of indifference curves and contours is the marginal rate of substitution (MRS):

  local measure of how much our consumer substitutes one good with another, mantaining his/her utility constant

- Technically, it is the slope of the indifference curve or, equivalently, the slope of the utility function contour
- To compute the local measure of the slope of the utility function contour in the case of two goods
  - totally differentiate the utility function, keeping $u(x_1, x_2)$ constant

$$\frac{\partial u(x_1, x_2)}{\partial x_1} dx_1 + \frac{\partial u(x_1, x_2)}{\partial x_2} dx_2 = du(.) = 0$$

  - ... and rearrange

$$MRS(x_1, x_2) = \frac{dx_2}{dx_1} = -\frac{\frac{\partial u(x_1, x_2)}{\partial x_1}}{\frac{\partial u(x_1, x_2)}{\partial x_2}} = -\frac{MU_{x_2}}{MU_{x_1}}$$

- $MRS(x_1, x_2)$ is the ratio between the marginal utilities MU_{x_1} and MU_{x_2}

Types of preferences

Widely used types of preference relations / utility functions are

- Homothetic preferences
- Quasi-linear preferences
- Leontief preferences

Homothetic preferences

- Preferences are homothetic if

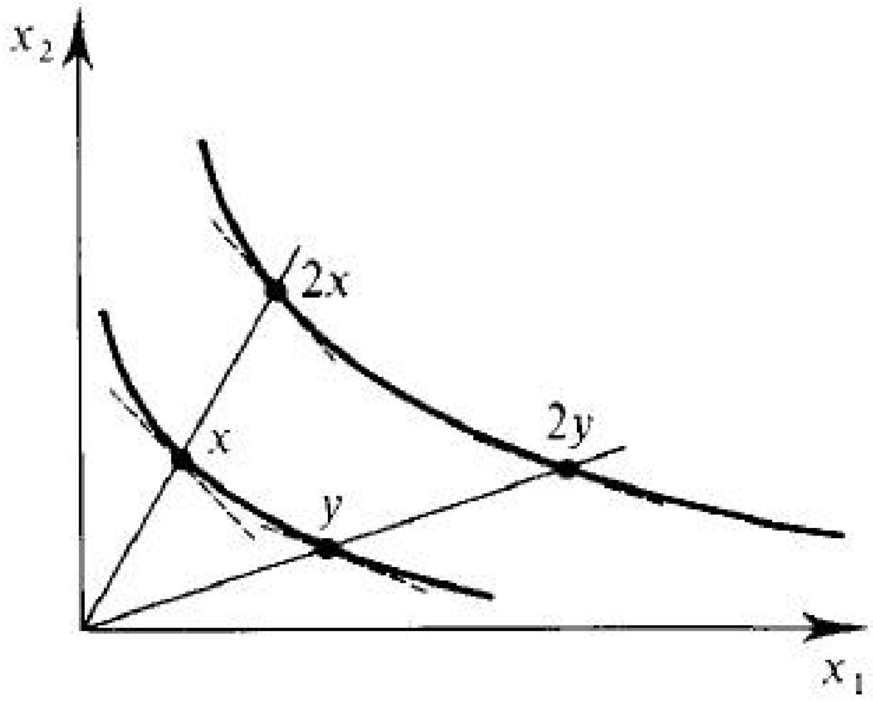
  all indifference curves are related
  by a proportional expansion along rays

- That is, for any x, y such that $x \sim y$, then $\alpha x \sim \alpha y$ for any $\alpha \geq 0$.
- Homothetic preference relations are representable by a utility function that is homogenous of degree one

$$u(\alpha x) = \alpha u(x) \text{ for all } \alpha > 0.$$

- Notice however that not every utility function that represents homothetic preferences must be homogenous of degree one (see example below)

Homothetic preferences



Cobb-Douglas preferences

- Cobb-Douglas preferences are a very widely used type of preferences
- In the case of two goods

$$u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$$

- Often, it is assumed that coefficients sum up to 1

$$u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$$

Cobb-Douglas utility function and homotheticy

- Are Cobb-Douglas preferences homothetic?
- REMEMBER
  a function $f(x)$ is homogenous of degree k if $f(tx) = t^k f(x)$
- A Cobb-Douglas utility function is homogenous of degree $\alpha + \beta$

$$u(tx_1, tx_2) = (tx_1)^\alpha (tx_2)^\beta = t^{\alpha+\beta} x_1^\alpha x_2^\beta = t^{\alpha+\beta} u(x_1, x_2)$$

- Notice that Cobb-Douglas utility functions are such that, for any $\frac{x_1}{x_2}$, MRS is constant along the ray

$$\begin{aligned} MRS(tx_1, tx_2) &= -\frac{\partial u(tx_1, tx_2)/\partial x_1}{\partial u(tx_1, tx_2)/\partial x_2} = -\frac{\partial t^k u(x_1, x_2)/\partial x_1}{\partial t^k u(x_1, x_2)/\partial x_2} \\ &= -\frac{t^k \partial u(x_1, x_2)/\partial x_1}{t^k \partial u(x_1, x_2)/\partial x_2} = -\frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2} \\ &= MRS(x_1, x_2) \end{aligned}$$

Cobb-Douglas utility function and homotheticity (ctd)

- However, take the following transformation

$$v(x_1, x_2) = [u(x_1, x_2)]^{\frac{1}{\alpha+\beta}} \qquad \alpha, \beta > 0, \alpha + \beta \geq 1$$

then

$$\begin{aligned} v(tx_1, tx_2) &= [u(tx_1, tx_2)]^{\frac{1}{\alpha+\beta}} = [(tx_1)^{\alpha}(tx_2)^{\beta}]^{\frac{1}{\alpha+\beta}} \\ &= (t^{\alpha+\beta})^{\frac{1}{\alpha+\beta}}(x_1^{\alpha}x_2^{\beta})^{\frac{1}{\alpha+\beta}} = t[u(x_1, x_2)]^{\frac{1}{\alpha+\beta}} \\ &= tv(x_1, x_2) \end{aligned}$$

Lesson: Not every utility function that represents homothetic preferences must be homogenous of degree 1 !!

- While homothetic preferences can always be represented by a utility function homogenous of degree 1, they can also be represented by many other utility functions that are not homogenous of degree 1

Quasi-linear preferences

Let

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ \dots \\ 0 \end{pmatrix}$$

Preference relation on $X = (-\infty, \infty) \times \Re_+^{L-1}$ is quasi-linear w. r. to commodity 1 (the numeraire) if

- commodity 1 is desirable:

  $$x + \alpha e_1 \succ x \text{ for all } x \text{ and } \alpha > 0$$

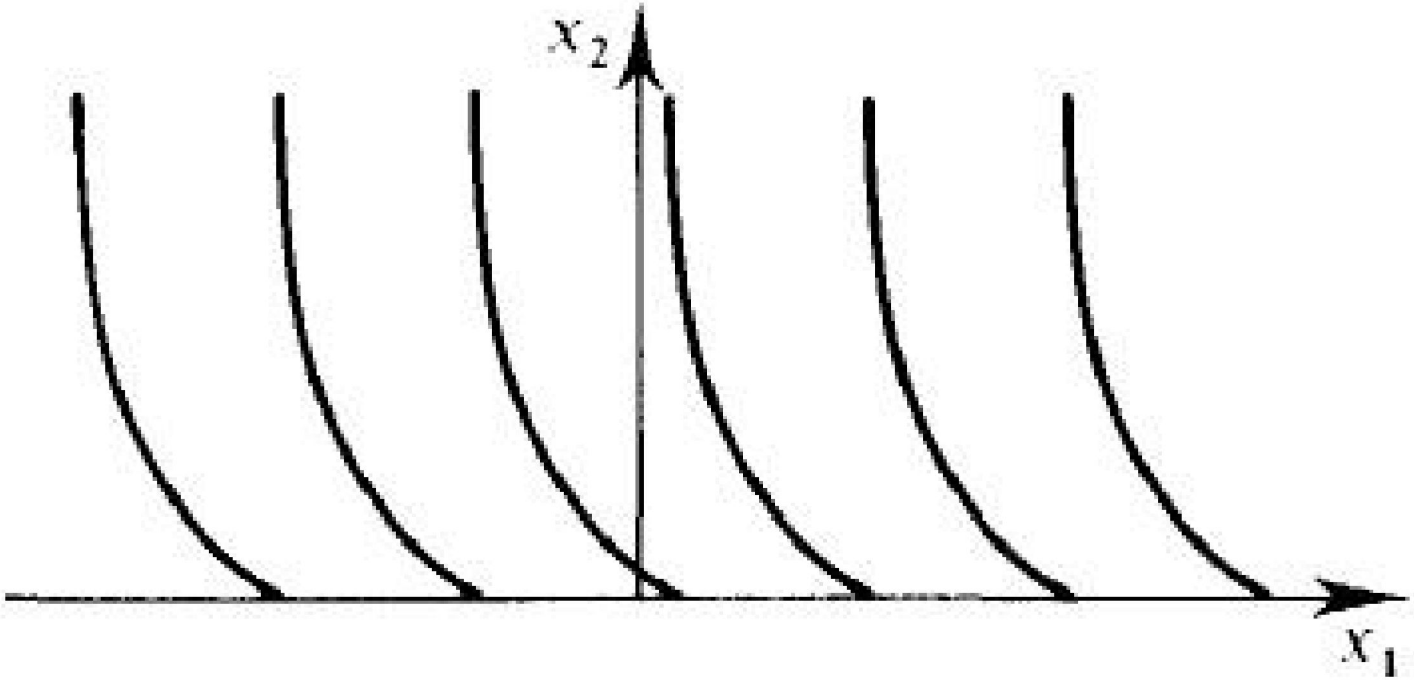
  Int.: if $\alpha = 1$, adding 1 unit of good 1 makes the consumpion bundle more desirable
- indifference set are parallel displacements of each other along the axis of commodity 1

  if $x \sim y$, then $(x + \alpha e_1) \sim (y + \alpha e_1)$ for all x, y, and $\alpha > 0$

  Int.: if $\alpha = 1$ and two bundles are equally liked, adding 1 unit of good 1 to both keeps them equally liked

Quasi-linear preference relations are representable by an utility function $u(x) = \beta x_1 + \phi(x_2, \dots, x_L)$.

Quasi-linear preferences



Quasi-linear preferences (ctd)

- For quasi-linear preferences, MRS between the numeraire and any not-numeraire good is constant for any level of the numeraire good
- That is, an increase in the availability of the numeraire does not affect the slope of the indifference curve
- Example:
  - Let $u(x_1, x_2) = \beta x_1 + \phi(x_2)$
  - Then,

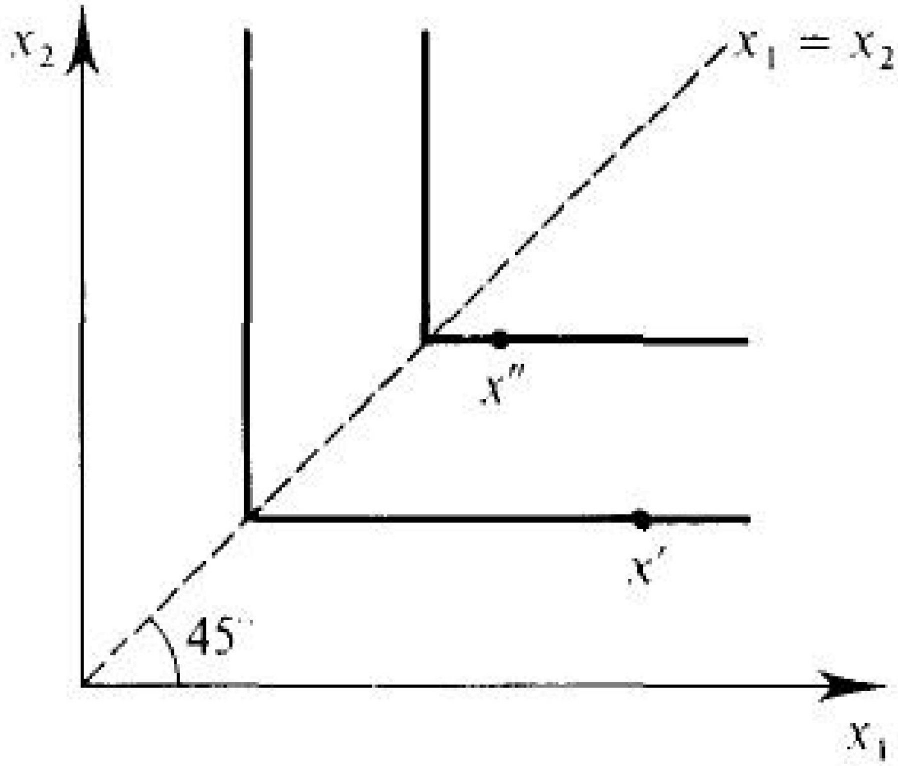
$$MRS = -\frac{\phi'(x_2)}{\beta}$$

    which is independent on x_1

Leontief preferences

- A Leontief preference relation is such that $x' \succeq x''$ iff $\min\{\alpha_1 x_1', \ldots, \alpha_n x_n'\} \geq \min\{\alpha_1 x_1'', \ldots, \alpha_n x_n''\}$ for $\alpha_\ell > 0$
- Notice: $u(x)$ is continuous but not differentiable

Leontief preferences



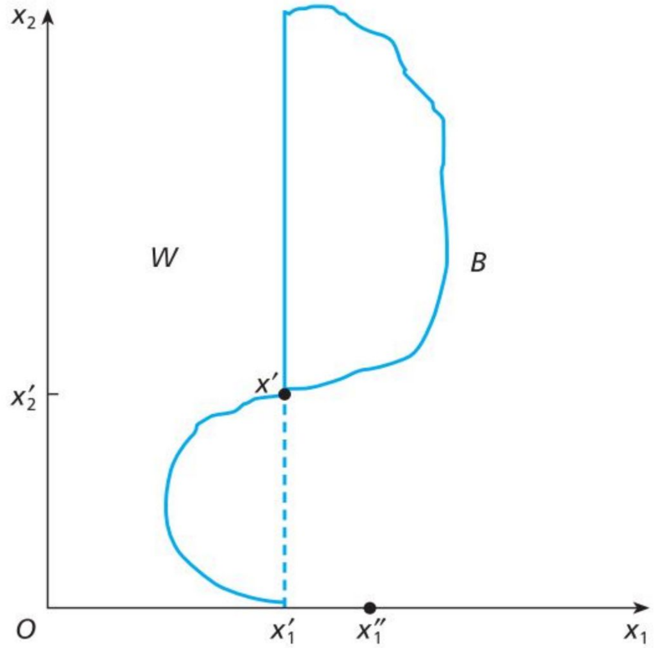
Lexicographic preferences

- In the two-good case

$$(x_1, x_2) \succeq (y_1, y_2) \text{ iff } \begin{cases} x_1 > y_1 \text{ or} \\ x_1 = y_1 \text{ and } x_2 \geq y_2 \end{cases}$$

- Like a dictionary, first the consumer compares the first component of the consumption bundle; if they are the same, then he/she jumps into the second component, and so on...
- Indifference sets are single point: preferences are not continuous, and then not representable by a utility function

Lexicographic preferences



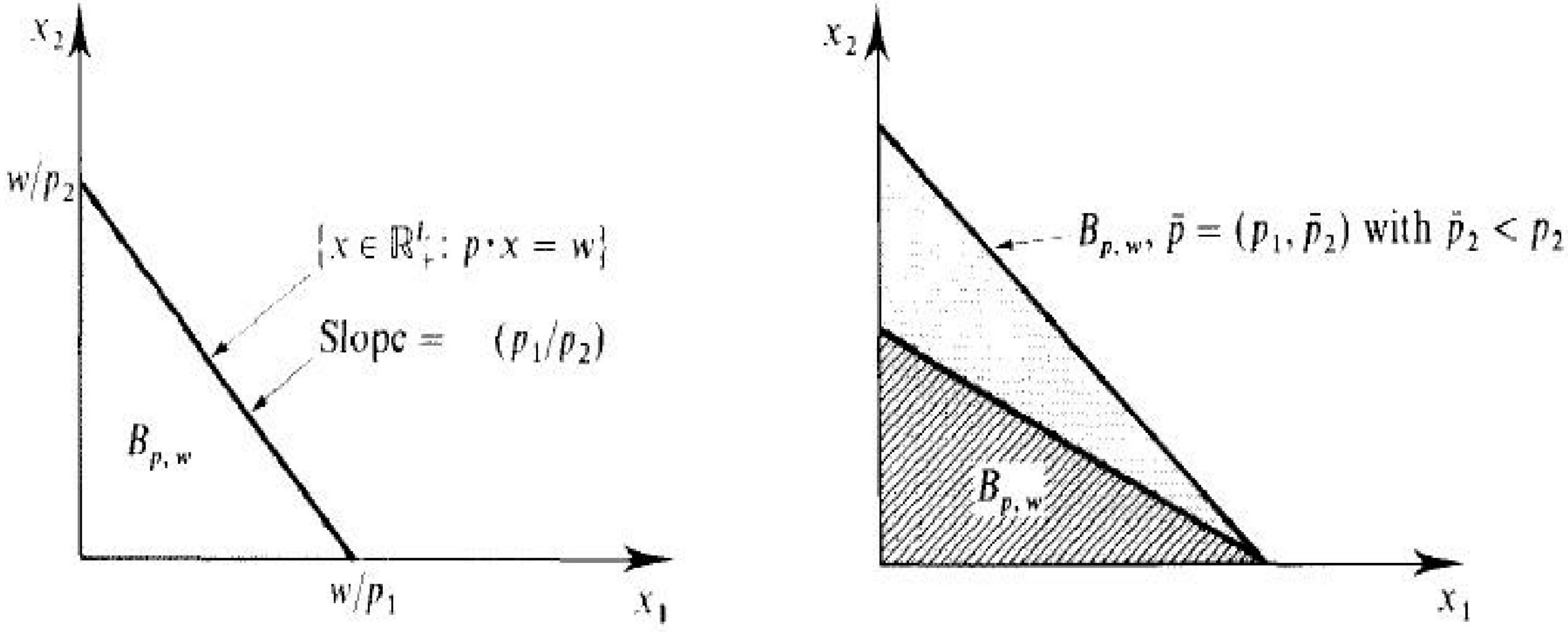
- Take the bundle x': what points are preferred to it and to what points is it preferred?
- area B and points on the solid line above x' must all be preferred to x', since contains weakly more of x_1
- area W and points on the broken line below x', must all be such that x' is preferred to them since contains less of x_1
- $\Rightarrow$ no other points are indifferent to x'
- each point in the space lies in an indifference set consisting only of itself.

Budget set 1

- Consumer's choice restricted to admissible bundles
- Admissible bundles are those within the budget set $B_{p,w}$

$$B_{p,w} = \{x \in \Re^L_+ : p \cdot x \leq w\}$$

where p are commodities' prices and w consumer's wealth/income
("$\cdot$" to indicate product operation between two vectors)



Budget set 2

Important assumptions on prices

- prices p are all positive: $p \gg 0$
- prices p are known
- prices are given to the consumer
- prices are linear

Important assumptions on income/wealth

- income/wealth w is given

$\Rightarrow$ budget set $B_{p,w}$ is convex.

A budget set which is not convex