Comparison of risk aversion across individuals

- Let $u_1(.)$ and $u_2(.)$ be two Bernoulli utility functions (i.e. the Bernoulli utility functions of two different individuals)
- We say that individual 2 is more risk averse than individual 1 when any lottery preferred to a riskless outcome $\overline{x}$ by individual 2 is also preferred by individual 1 to the same riskless outcome $\overline{x}$
- Formally,

$$\int u_2(x)\, dF(x) \geq u(\overline{x}) \;\Rightarrow\; \int u_1(x)\, dF(x) \geq u(\overline{x})$$

Risk aversion and its measures

The previous statement is equivalent to

- $u_2(.)$ is 'more concave' than $u_1(.)$ (i.e. there exists an increasing concave function $\psi(.)$ such that $u_2(x) = \psi(u_1(x))$ for all x.)
- the *Arrow Pratt measure of absolute risk aversion* is higher the more risk averse is the individual

$$r_A(x; u_2) \geq r_A(x; u_1) \qquad \text{for every } x$$

- the *certainty equivalent* is lower the more risk averse is the individual

$$c(F, u_2) \leq c(F, u_1) \qquad \text{for every } F(.)$$

- the *probability premium* is higher the more risk averse is the individual

$$\pi(x, \epsilon, u_2) \geq \pi(x, \epsilon, u_1) \qquad \text{for every } x \text{ and } \epsilon$$

Risk aversion and wealth

- Often (observed and) assumed that wealthier individuals are less risk averse than less wealthy individuals.
- This is formalised by the idea of decreasing absolute risk aversion

  decreasing absolute risk aversion $\Leftrightarrow r_A(x, u)$ decreases in x

Risk aversion and wealth (2)

Many alternative and equivalent definitions for
decreasing absolute risk aversion

- probability premium is decreasing in x
- the certainty equivalent of a lottery formed by adding risk z to wealth x, given by the amount c_x (such that $u(c_x) = \int u(x+z)dF(z)$) is such that $(x - c_x)$ decreases with x: the higher is x, the less is the individual willing to pay to avoid the risk
- whenever $x_2 < x_1$, $u_2 = u(x_2 + z)$ is a concave transformation of $u_1 = u(x_1 + z)$
- any lottery preferred to a certain outcome at lower wealth level will be also preferred at a higher wealth

$$\int u(x_2 + z)\, dF(z) \geq u(x_2) \text{ and } x_2 < x_1$$
$$\Rightarrow$$
$$\int u(x_1 + z)\, dF(z) \geq u(x_1)$$

Relative risk aversion

More often, a stronger assumption is used:

nonincreasing relative risk aversion

An individual becomes less adverse with regard to gambles that a are proportional to her wealth as her wealth increases

A measure of *relative* risk aversion is given by the coefficient of relative risk aversion $r_R(x)$,

$$r_R(x) = -x\,\frac{u''(x)}{u'(x)}$$

- Useful to assess the attitude towards risky projects whose outcome are percentage gains or losses of current wealth
- Since $r_R(x) = x r_A(x)$,

  decreasing **relative** RA $\Rightarrow$ decreasing **absolute** RA

  but not viceversa

Comparison of payoff distributions

- Useful to compare distributions of monetary payoffs (rather than utility over them or utility functions)
- Two possible criteria
  - level of returns: look for conditions under which $F(.)$ always yields higher returns than $G(.)$

    $\hookrightarrow$**First-order stochastic dominance**
  - level of risk: look for conditions under which $F(.)$ is always less risky than $G(.)$

    $\hookrightarrow$**Second-order stochastic dominance**

First-order stochastic dominance

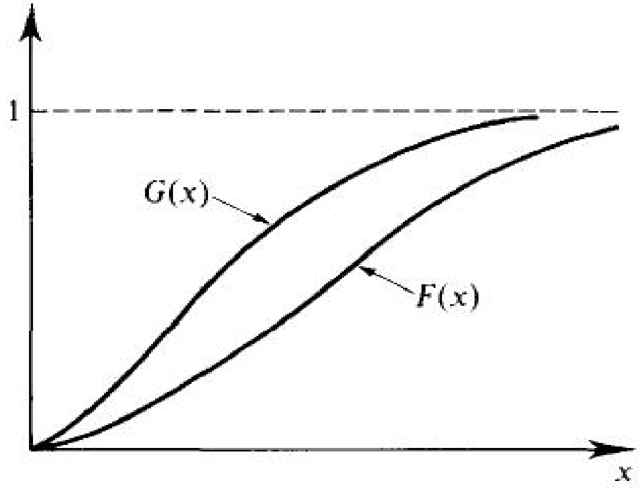
- We say that the distributions of monetary payoffs $F(.)$ *first-order stochastically dominates* $G(.)$ when $F(.)$ always yields higher returns than $G(.)$
- More formally,

  $F(.)$ *first-order stochastically dominates* (FOSD) $G(.)$ when
  $$F(x) \leq G(x) \text{ for all } x$$
  - for every amount of money x, the probabilit of getting at least x is higher under $F(.)$ than under $G(.)$
- Another (equivalent) way to assess returns of distribution of payoffs is by looking at their expected utility

  $$F(.) \text{ FOSD } G(.) \Leftrightarrow \int u(x)dF(x) \geq \int u(x)dG(x) \text{ for every } u(.)$$

  that is, individuals always prefer distribution of monetary payoffs which yields higher returns
- Notice that if $F(.)$ FOSD $G(.)$, then $\int xdF(x) \geq \int xdG(x)$ but not viceversa: a ranking of the means not enough for FOSD, since the entire distribution matters

First-order stochastic dominance



Second-order stochastic dominance

- Focus now on the riskiness or dispersion of a lottery, as opposed to higher/lower returns of lottery (as in FOSD)
- To focus on riskiness, assume that lotteries have the same mean (ie, the same expected return)
- We say that the distributions of monetary payoffs $F(.)$ *second-order stochastically dominates* $G(.)$ when $F(.)$ is less risky than $G(.)$
- Less risky?
  - $F(.)$ *second-order stochastically dominates* $G(.)$ if every risk averse individual prefers $F(.)$ to $G(.)$
  - Alternatively, for every nondecreasing concave function $u(.)$ we have

$$\int u(x)dF(x) \geq \int u(x)dG(x)$$

    that is, the expected utility of $F(.)$ is higher than the expected utility of $G(.)$

Second-order stochastic dominance

Example 1 of SOSD: mean preserving spread

- A mean preserving spread occurs when
  - the probability of some outcomes is spread over the probabilities of other outcomes
  - the mean is left unaltered
- If $G(.)$ is a *mean preserving spread* of $F(.)$, then $F(.)$ SOSD $G(.)$

Second-order stochastic dominance

Example 2 of SOSD: elementary increase in risk

- An elementary increase in risk occurs when
  - all the mass of an interval $[x', x'']$ is transferred to the end points of this interval
  - the mean is left unaltered
- (Notice that an elementary increase in risk is always a mean preserving spread, but not viceversa)
- If $G(.)$ constitute an *elementary increase in risk* from $F(.)$, then $F(.)$ SOSD $G(.)$

Second-order stochastic dominance

More generally,

$$F(.) \text{ SOSD } G(.) \text{ when } \int_0^x G(t)dt \geq \int_0^x F(t)dt \text{ for all } x$$

that is, for any x, the area below $G(.)$ is always larger than the area below $F(.)$