Welfare evaluation

- May want to evaluate effect on utility of changes in the economic environment, typically in prices.
- Utility and indirect utility functions are useful instrument.
- However, measure (but not sign) of the effect
  - depends on functional forms.
  - not comparable across individuals

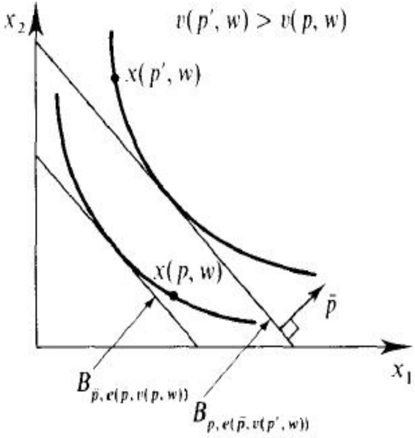
Money metric indirect utility fct

- Money metric utility fct gives in money terms the effect on utility of changes in p
- Take any two utility levels u^0, u^1 and a reference price vector $\overline{p}$.

$$\underline{e(\overline{p}, u^0) \text{ and } e(\overline{p}, u^1) \text{ are money measures of}}$$
$$\underline{\text{utility } u^0 \text{ and } u^1 \text{ at prices } \overline{p}}.$$

- Assume now that u^0 and u^1 are optimal utility levels for price p^0 and p^1, so that $u^0 = v(p^0, w)$ and $u^1 = v(p^1, w)$. Then
  - $e(\overline{p}, v(p^0, w))$ and $e(\overline{p}, v(p^1, w))$ are money measures of indirect utility
  - $e(\overline{p}, v(p^0, w)) - e(\overline{p}, v(p^1, w))$ is a money measure of the welfare effect of the price change

Money metric indirect utility fct



Equivalent and compensating variations

Natural choices for reference price vector $\overline{p}$ are either p^0 or p^1.

Using $e(p^0, u^0) = e(p^1, u^1) = w$, measures of welfare changes

- EQUIVALENT VARIATION: the welfare effect of the price change measured at initial prices

  $\Rightarrow$ how much money the consumer would pay/accept instead of the price change

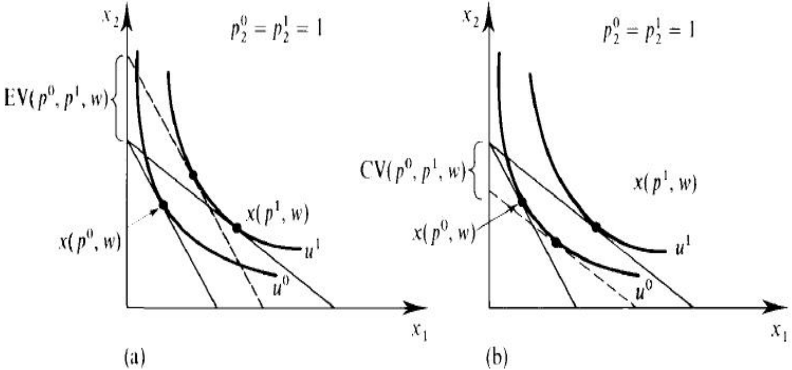
$$\begin{aligned} EV(p^0, p^1, w) &= e(p^0, u^1) - e(p^0, u^0) \\ &= e(p^0, u^1) - w \end{aligned}$$

- COMPENSATING VARIATION: the welfare effect of the price change measured at final prices

  $\Rightarrow$ how much money the consumer would pay/accept after the price change to bring him/her back to the original utility level

$$\begin{aligned} CV(p^0, p^1, w) &= e(p^1, u^1) - e(p^1, u^0) \\ &= w - e(p^1, u^0) \end{aligned}$$

Equivalent and compensating variations



Equivalent variation and hicksian demand

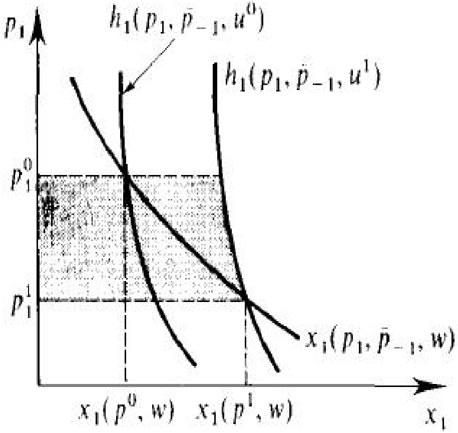
The Equivalent Variation has an immediate intepretation in terms of Hicksian demand

Suppose only price of good 1 is varying.

$$
\begin{aligned}
EV(p^0, p^1, w) &= e(p^0, u^1) - w \\
&= e(p^0, u^1) - e(p^1, u^1) \\
&= \int_{p_1^1}^{p_1^0} h_1(\tilde{p}_1, p_2, \ldots, p_L, u^1) d\tilde{p}_1
\end{aligned}
$$

EV measured by the area between the prices and to the left of the Hicksian demand curve at u^1.

Equivalent variation and hicksian demand



Compensating variation and hicksian demand

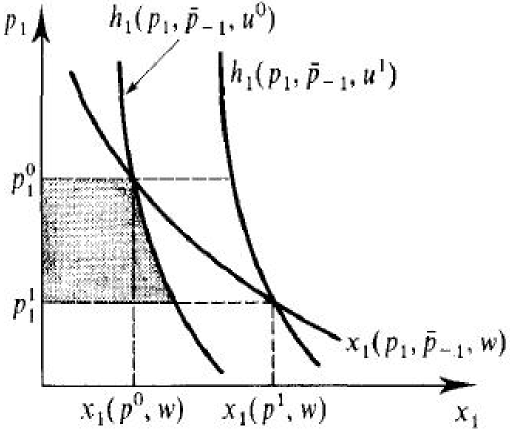
The Compensating Variation too has an immediate intepretation in terms of Hicksian demand

Suppose only price of good 1 is varying.

$$
\begin{aligned}
CV(p^0, p^1, w) &= w - e(p^1, u^0) \\
&= e(p^0, u^0) - e(p^1, u^0) \\
&= \int_{p_1^1}^{p_1^0} h_1(\tilde{p}_1, p_2, \ldots, p_L, u^0) d\tilde{p}_1
\end{aligned}
$$

CV measured by the area between the prices and to the left of the Hicksian demand curve at u^0.

Compensating variation and hicksian demand



Equivalent vs compensating variations

- Clearly, EV and CV give different measures of the welfare change.
- For normal goods, and for a price reduction
$$EV(p^0, p^1, w) > CV(p^0, p^1, w)$$
(recall the relative slope of the demand functions??)
- Not surprisingly, the two measures are identical when there are not wealth effects, i.e. preferences are quasi-linear. In this case,
$$h_1(p^1, u^0) = x_1(p^1, w) = h_1(p^1, u^1)$$

A choice-based theory of preferences

- So far, we started from assuming our representative consumer is endowed with (rational) preferences, and analysed how he/she would make his/her choices
- Now, we start from looking at actual choices: we want to derive predictions on the consumer's choices from the consumer's observable choices!!

Basic idea: actual choices reveal preferences!!

- if a bundle $\tilde{x}$ is chosen when another bundle $\tilde{y}$ is also available, then bundle $\tilde{x}$ is **revealed preferred** to $\tilde{y}$

Note: variables with tildes are actual choices!!

To make sure that consumer's choices are consistent, we need to make sure they satisfy some (minimal) rationality requirements

- WARP: weak axiom of revealed preference

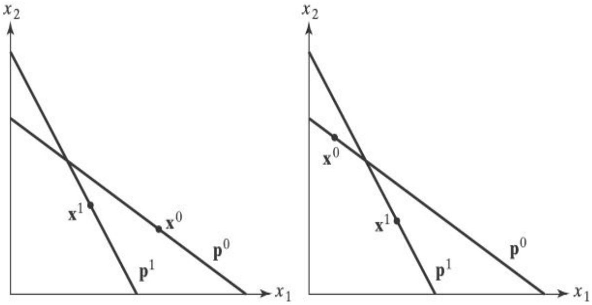
WARP

- Start with some definitions
  - let (p^0, w^0) and (p^1, w^1) be any two price-wealth combinations,
  - let $\tilde{x}^0$ be the actual choice made by the consumer at price-wealth combination (p^0, w^0)
    - $\tilde{x}^0$ is revealed preferred to any other available bundle
  - let $\tilde{x}^1$ be the actual choice made the consumer at price-wealth combination (p^1, w^1)
    - $\tilde{x}^1$ is revealed preferred to any other available bundle
- WARP is satisfied if $\tilde{x}^0$ and $\tilde{x}^1$ are such that:

$$p^0 \cdot \tilde{x}^1 \leq p^0 \cdot \tilde{x}^0 \Rightarrow p^1 \cdot \tilde{x}^0 > p^1 \cdot \tilde{x}^1$$

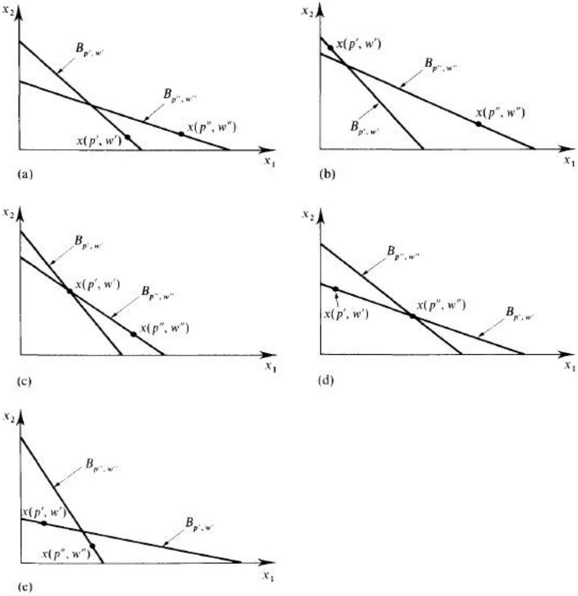
  - In words, WARP is satisfied if, whenever $\tilde{x}^0$ is revealed preferred to $\tilde{x}^1$, $\tilde{x}^1$ is NEVER revealed preferred to $\tilde{x}^0$

Weak axiom of revealed preference



(a) (b)

Weak axiom of revealed preference



On an alternative way of looking at consumer's choices

- Denote with $\tilde{x}(p, w)$ the consumer's choice at price-wealth combination (p, w)
  - this is not a demand function, since it does not derive from an utility function!!
- Want to check what properties this choice function satisfies and, most importantly, if this choice function could be generated by utility maximisation
- Adding the assumption that the consumer always uses his/her entire budget to buy his/her consumption bundle (i.e. $p \cdot \tilde{x} = w$), it is easy to prove that choice function $\tilde{x}(p, w)$ is
  - homogenous of degree 0
  - its Slutsky matrix is negative semidefinite
  - its Slutsky matrix is symmetric (but only for two goods, strongest assumption is required with more goods)

On the meaning of WARP

- If choices satisfy WARP (and budget balancedness),

  there exist a utility function that would yield the observed choices as the outcome of the utility maximisation process
- This utility function is said to 'rationalise' the choice function
- In other words, if observed choices satisfy WARP, our consumer's theory is confirmed by actual choices
- However, not assuming the existence of an utility fct, WARP may be at odds with the evaluation of the consumer's level of well-being

Production Theory

Technology 1

- Consider an economy with L goods.
- A firm is seen as a "black box" which uses these goods to serve as inputs and/or outputs.
- The **production plan** is a vector

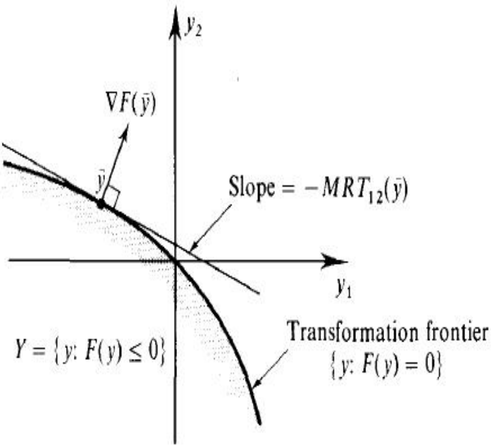
$$\mathbf{y} = (y_1, ..., y_L) \in \Re^L$$

  that describes the (net) output of the L goods of the production process of the firm.
- By convention,
  - a positive y_i indicates a good which is a net output and
  - a negative y_i indicates a good which is a net input.

Technology 2

- The existing technology, taken as primitive datum, defines the **production set** $Y \subset \Re^L$, which is the set of the feasible production plans
- Can describe the production set using the **transformation function** $F(.)$, which is such that:
  - $Y = \{\mathbf{y} \in \Re^L : F(\mathbf{y}) \leq 0\}$;
  - $F(\mathbf{y}) = 0$ iff there is no $\mathbf{y}' \in Y$ such that $\mathbf{y}' \geq \mathbf{y}$.
    $\hookrightarrow$ $F(\mathbf{y}) = 0$ iff $\mathbf{y}$ is (technically) efficient, that is there is no way to produce more of a least one output with the same inputs or the same output with less of at least one input.
- The set $\{\mathbf{y} \in \Re^L : F(\mathbf{y}) = 0\}$ is defined as the **transformation frontier**.

Production set and transformation frontier



Marginal rate of transformation

- Provided that $F(.)$ is differentiable and $F(\bar{\mathbf{y}}) = 0$, then the **marginal rate of transformation** (at $\bar{\mathbf{y}}$) is given by:

$$MRT_{lk}(\bar{\mathbf{y}}) = \frac{\frac{\partial F(\bar{\mathbf{y}})}{\partial y_l}}{\frac{\partial F(\bar{\mathbf{y}})}{\partial y_k}}$$

obtained simply by totally differentiating $F(.)$ and evaluating it at $\bar{\mathbf{y}}$.

<u>how much the (net) output of good k can</u>
<u>increase if the firm decreases marginally</u>
<u>the (net) output of good ℓ</u>

- Graphically, this is simply the slope of the **transformation frontier**.

Technology 3

Sometimes we'll use two simplifications

- separation between inputs and outputs.
  With $L - M$ inputs (always negative) and M outputs (always positive),

$$\begin{aligned} Y &= \{(-z_1, ..., -z_{L-M}, q_1, ..., q_M) : \\ &\quad (z_1, ..., z_{L-M}), (q_1, ..., q_M) \geq 0 \\ &\quad \text{and } F(.) \leq 0\}; \end{aligned}$$

- single-output technology. With $L - 1$ inputs and 1 output, we can make use of the *production function* $q = f(z_1, ..., z_{L-1})$ defined as

$$\begin{aligned} Y &= \{(-z_1, ..., -z_{L-1}, q) : \\ &\quad q - f(z_1, ..., z_{L-1}) \leq 0 \\ &\quad \text{and } (z_1, ..., z_{L-1}) \geq 0\}. \end{aligned}$$

Single output technology

Useful concepts in the single-output case:

- input requirement set
- isoquant
- marginal rate of technical substitution
- elasticity of substitution

Single output technology

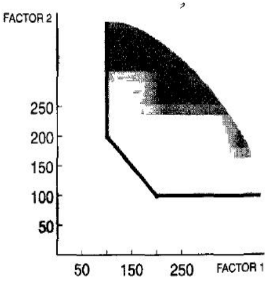
Useful concepts in the single-output case:

- $\hookrightarrow$ input requirement set
- isoquant
- marginal rate of technical substitution
- elasticity of substitution

Input requirement set

- the **input requirement set** $V(q)$ the set of all input bundles that produce *at least* q, given by:

$$\begin{aligned} V(q) &= \{(z_1, ..., z_{L-1}) : (z_1, ..., z_{L-1}) \geq 0 \text{ and} \\ &\quad (-z_1, ..., -z_{L-1}, q') \in Y \text{ with } q' \geq q\}. \end{aligned}$$



Single output technology

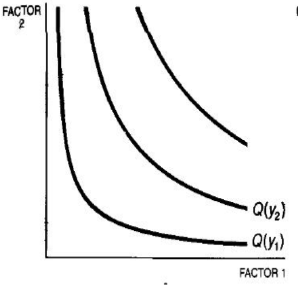
Useful concepts in the single-output case:

- input requirement set
- $\hookrightarrow$ isoquant
- marginal rate of technical substitution
- elasticity of substitution

Isoquant

- the **isoquant** $Q(q)$ is the set of all input bundles that produce *exactly* q, given by:

$$\begin{aligned} Q(q) \quad = \quad & \{(z_1, ..., z_{L-1}) : (z_1, ..., z_{L-1}) \geq 0\ , \\ & (z_1, ..., z_{L-1}) \in V(q) \text{ and} \\ & (z_1, ..., z_{L-1}) \notin V(q') \text{ if } q' > q\}. \end{aligned}$$



Single output technology

Useful concepts in the single-output case:

- input requirement set
- isoquant
- $\hookrightarrow$ marginal rate of technical substitution
- elasticity of substitution

Marginal rate of technical substitution

- if $f(.)$ is differentiable, the **marginal rate of technical substitution** of input k for input l ($MRTS_{kl}$), (holding output fixed at $\overline{q} = f(\overline{\mathbf{z}})$) is the given by:

$$MRTS_{kl} = -\frac{\frac{\partial f(\overline{\mathbf{z}})}{\partial z_l}}{\frac{\partial f(\overline{\mathbf{z}})}{\partial z_k}}$$

  simply obtained by totally differentiating $f(.)$.
  The $MRTS_{kl}$ is simply the slope of the isoquant $Q(q)$ and it is the analogue of the MRT_{kl} (when k and l are inputs).

Single output technology

Useful concepts in the single-output case:

- input requirement set
- isoquant
- marginal rate of technical substitution
- $\hookrightarrow$ elasticity of substitution

Elasticity of substitution

- if the **marginal rate of technical substitution** gives the *slope* of an isoquant, the **elasticity of substitution** measures the *curvature* of an isoquant.
  More technically, the **elasticity of substitution** of input k for input l (with output fixed at $\overline{q}$) is given by :

$$\sigma_{kl} = \frac{\frac{\Delta(z_l/z_k)}{(z_l/z_k)}}{\frac{\Delta MRTS_{kl}}{MRTS_{kl}}}$$

  or, for infinitesimal variations

$$\sigma_{kl} = \frac{MRTS_{kl}}{(z_l/z_k)} \frac{d(z_l/z_k)}{dMRTS_{kl}} = \frac{d\ln(z_l/z_k)}{d\ln|MRTS_{kl}|}$$

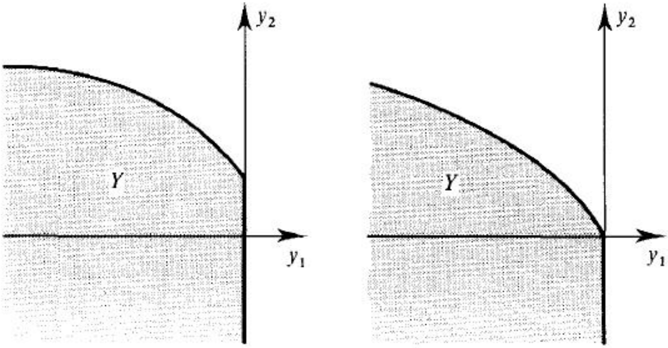
  Intuitively, the more the factor input ratio changes for a given change in the slope of the isoquant, the larger the elasticity of substitution.

Properties of ALL production sets 1

- Y is *closed*.
  If $y^n \to y$ and $y^n \in Y$, then $y \in Y$: the production set contains its own boundary;
- Y is *no empty*:
  At least one productions plan is always possible;

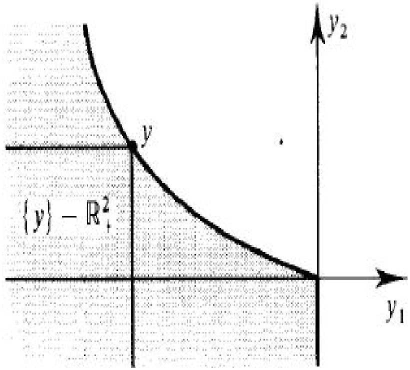
Properties of ALL production sets 2

- Y satisfies *no free lunch*:
  not possible to produce something from nothing (i.e. if $\mathbf{y} \in Y$ and $\mathbf{y} \geq 0$, then $\mathbf{y} = 0$)



Properties of ALL production sets 4

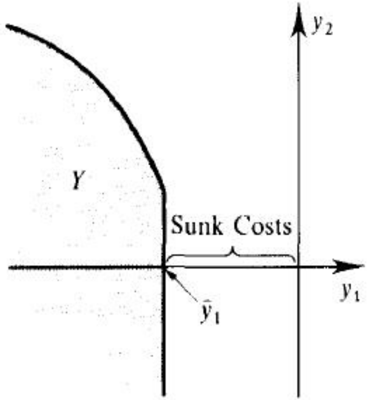
- Y satisfies *free disposal*:
  If $\mathbf{y} \in Y$ and $\mathbf{y}' \leq \mathbf{y}$, then $\mathbf{y}' \in Y$: always possible to throw away (at no cost) some inputs or outputs



(Possible) properties of production sets 1

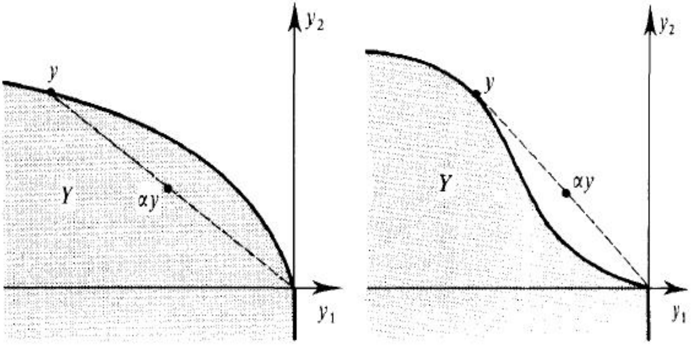
Not always satisfied, sometimes mutually exclusive

- Y satisfies *possibility of inaction*:
  $0 \in Y$: firm can shut down production.
  It holds before any production decision is made. Otherwise, sunk costs or fixed factors of production may make it invalid.



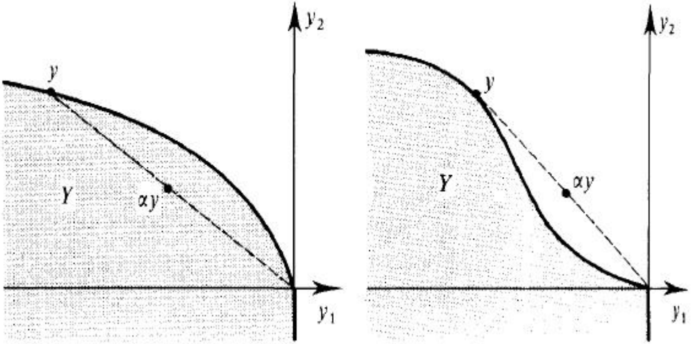
(Possible) properties of production sets 2

- Y is convex.
  If $\mathbf{y}, \mathbf{y}' \in Y$ and $\alpha \in [0, 1]$, then $\alpha\mathbf{y} + (1 - \alpha)\mathbf{y}' \in Y$;
  - nonincreasing returns to scale
  - balanced input combinations are more productive than unbalanced ones.



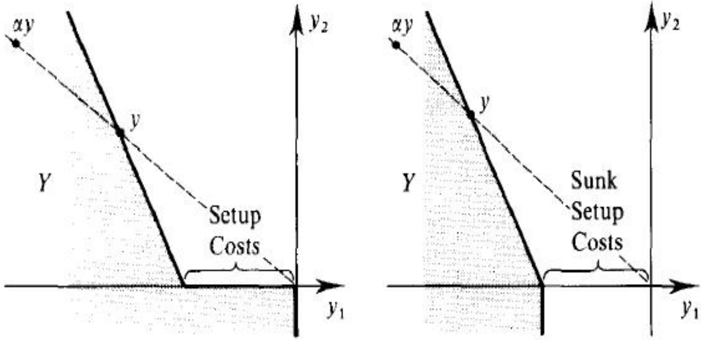
(Possible) properties of production sets 3

- **nonincreasing returns to scale**:
  $\mathbf{y} \in Y \Rightarrow \alpha\mathbf{y} \in Y, \ \forall \alpha \in [0, 1]$. Any feasible production plan can be scaled down.



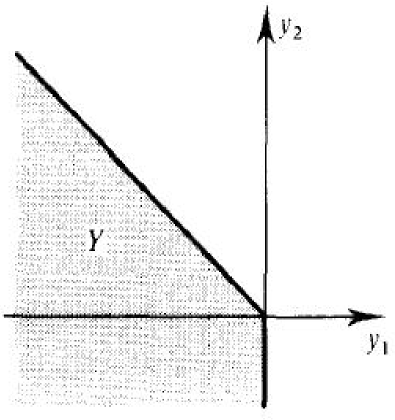
(Possible) properties of production sets 4

- **nondecreasing returns to scale**:
  $\mathbf{y} \in Y \Rightarrow \alpha \mathbf{y} \in Y$, $\forall \alpha \in [1, \infty]$. Any feasible production plan can be scaled up.



(Possible) properties of production sets 5

- **constant returns to scale**: $\mathbf{y} \in Y \Rightarrow \alpha\mathbf{y} \in Y,\ \forall\alpha \in [0, \infty]$.



Scale in the single-output case

In the single-output case, $\forall\ t > 1$, then

- $f(t\mathbf{z}) < tf(\mathbf{z}) \Rightarrow$ **nonincreasing** RS;
- $f(t\mathbf{z}) = tf(\mathbf{z}) \Rightarrow$ **constant** RS;
- $f(t\mathbf{z}) > tf(\mathbf{z}) \Rightarrow$ **nondecreasing** RS;