1 Cournot model

The inverse demand function in a market in which n firms compete is

$$p = p(Q)$$

where:

$$Q = \sum_{i=1}^{n} q_i.$$

Firms have constant marginal costs but may differ in their level of efficiency. So firm i's cost function is

$$C_i = c_i q_i$$

and their profit functions are

$$\pi_i = (p(Q) - c_i)\, q_i.$$

In a Nash equilibrium all firms maximize their profit, given the quantity produced by rivals. The FOC for the profit maximization problem of firm i is

$$\frac{\partial \pi_i}{\partial q_i} = \frac{\partial p}{\partial Q}\frac{\partial Q}{\partial q_i} q_i + (p(Q) - c_i) = 0.$$

We know that:

$$\frac{\partial Q}{\partial q_i} = 1$$

(i.e. if firm i increases its output - q_i - by x units, total output - Q - increases by x units as well), so the FOC becomes

$$\frac{\partial p}{\partial Q} q_i + (p(Q) - c_i) = 0;$$

this FOC can be rewritten as

$$\frac{p - c_i}{p} = \frac{\partial p}{\partial Q}\frac{q_i}{p};$$

now we can multiply and divide the right-hand side of this equation by Q

$$\frac{p - c_i}{p} = \frac{\partial p}{\partial Q}\frac{q_i}{p}\frac{Q}{Q},$$

so that it becomes

$$\frac{p - c_i}{p} = \frac{\partial p}{\partial Q}\frac{Q}{p}\frac{q_i}{Q}.$$

We know that $\frac{\partial p}{\partial Q}\frac{Q}{p}$ is the inverse of the market demand price elasticity and that $\frac{q_i}{Q}$ is firm i's market share; moreover we know that $\frac{p-c_i}{p}$ represents firm

i's market power (Lerner Index, L_i). thus we can say that in equilibium the following condition will hold

$$L_i = \frac{s_i}{\varepsilon},$$

where s_i is firm i's market share and ε is the market demand price elasticity. We can conclude that in a Cournot game firm i's marjet power is negatively related to the demand price elasticity and positively related to its market share.

If we want to have a measure of the degree of market power held by the n firms in the market, we can compute the weighted average level of market power using as weight firms' market shares. We get

$$L = \sum_{i=1}^{n} s_i L_i = \sum_{i=1}^{n} \frac{(s_i)^2}{\varepsilon} = \frac{1}{\varepsilon} \sum_{i=1}^{n} (s_i)^2 .$$

The expression $\sum_{i=1}^{n} (s_i)^2$ is a measure of the degree of concentration in the market that is called the Herfindahl-Hirschman Index (HHI). By convention the HHI is computed as

$$HHI = 10000 \times \sum_{i=1}^{n} (s_i)^2 .$$

HHI equals 10,000 in case of monopoly and approaches zero as the market becomes less and less concentrated. It is an index that is widely used by competition authorites to assess the risk that a merger may lead to the creation of excessive market power.