Behavioral Lecture 1

There are good reasons to try to relax the assumption that people have rational expectations, but there is also a big risk. Without the discipline of that assumption it is easy to fit anything and explain nothing. It is very easy to see how strange macroeconomic events and patterns **could** have been caused by irrational hopes and fears, and very hard to know if they were.

For that reason, I think it is very important that any alternative model of expectations be based on micro data, preferably experimental data. Only if there is strong evidence that people form expectations in a particular way does it make sense to consider the implications for the aggregate economy.

Fortunately, there is now a huge psychological literature which has reached some tentative conclusions. For macroeconomists, the most interesting work is based on asking people to forecast time series & rewarding accuracy. I know of work with univariate time series, so subjects only know past values of the variable to be forecast.

In particular there is a fair amount of research on what happens if people are shown a random walk and asked to forecast the next observation – this has the advantage that the optimal formula forecast = most recent value is very simple. It also appeals to financial economists, because forecasting changes in asset prices would be very profitable, and it is very hard, because they are approximately random walks (the series used in the oldest experiments were closing prices of individual stocks).

The rather robust result of these studies are that subjects typically forecast as if the series were **mean reverting**, so if it has recently increased they forecast it will decline again. However, if the series increases for a few periods in a row, subjects **extrapolate** the recent trend forecasting continued increase. Vice versa if the series has decreased.

This mini-course will be based entirely on treating this tentative stylized fact as a known psychological law and considering its implications for macroeconomics. This lecture will just discuss the pattern and consider the oldest simplest economic model in which it is assumed – a model of a purely speculative asset (which I will call bitcoin for some reason).

The first point is that the non-optimal forecasts of random walks seem in some way related to the fact that we can't generate random numbers. If asked for a list of independent random digits from 0 to 9, people give slightly negatively correlated series – there are too few runs of numbers greater than 4 and of numbers less than

5. There are especially too few cases in which the numbers increase for, say, 4 in a row (this is called a “run”). Decades ago, Kahneman and Twersky wrote that it is as if people think the law of large numbers applies to small numbers so the variance of the mean of a small sample of numbers is low.

There seems to be a link between this behaviour and the other than optimal forecasts. The mean reversion corresponds to the thought that changes in the level of the series are random and, to humans, that means slightly negatively correlated. The impression created by a few increases in a row is that the change is not random, even though such patterns aren’t really rare in a random walk.

So one way to summarize the results is that a walk which seems random to us is a mean reverting process, and an actual random walk sometimes looks like an increasing or decreasing trend.

There are two other ways of describing the psychological result.

One of them is that, when faced with a random walk, agents make forecasts which would be optimal if they faced a stationary process around a broken trend. Time series econometricians know that it is very very hard to distinguish a random walk from a stationary process around a broken trend.

Lets say the time series just happens to be a stationary process around 0

So say $X_t = 0.9X_{t-1} + \text{epsilon}_t$

With epsilon an iid disturbance and each period there is a 1% chance that the trend becomes alpha and 1% that it becomes -alpha (here alpha is a known constant). So with probability 1% something happens at time t so after t

$X_{t+s} = (s)\text{alpha} + Y_{t+s}$

And $Y_{t+s} = 0.9Y_{t+s-1} + \text{epsilon}_{t+s}$ is not observed.

The probability that there are n increases in a row if the trend is still zero (or is negative) declines exponentially in n, so pretty soon 0.98 times that probability is less than 0.01 times the fairly high probability that there are n increases in a row if the trend has become positive.

So the observed forecasting rule would make sense if people were faced with a stationary process around a broken trend.

Now in some exxperiments, subjects are definitely definitely faced with a random walk, because changes in the series are generated with a pseudo random number generator.

Another way of describing (or motivating or justifying) the observed behaviour is as follows

First (and this won't work) imagine an experiment in which the experimentor flips a coin (once) and if it comes up heads then makes

$X_{t+1} = 0.9X_t + 0.1X_{t-1} + \text{epsilon}_t$

And if it comes up tails makes

$X_{t+1} = 1.1X_t - 0.1X_{t-1} + \text{epsilon}_t$

The experimental subject knows this and knows that it is either best to predict an increase is followed by another increase or that an increase is followed by a decrease. However, the subject doesn't know if the coin came up heads or tails.

The fully 100% rational agent will update the 50-50 probability with Bayes's formula. This means that, for a while, a series of changes of the same sign (say increases) will convince the agent that the coin probably came up tails and cause they agent to extrapolate.

If epsilon is normally distributed with mean zero and variance 1, the posterior probability that the coin came up heads is

$$\frac{e^{-0.5\sum_{t=3}^{T}(X_t\ -0.9X_{t-1}\ \ -0.1X_{t-2})^2}}{e^{-0.5\sum_{3}^{T}(X_t\ -0.9X_{t-1}\ \ -0.1X_{t-2})^2} + e^{-0.5\sum_{3}^{T}(X_t\ -1.1X_{t-1}\ \ +0.1X_{t-2})^2}}$$

100% fully rational agents will put weights on the two forecasting rules based on past performance.

The problem with this simple model, is that the agent will end up knowing if the coin came up heads or tails with almost complete certainty and so their forecasting rule won't change given new information.

A more interesting model (with very messy math) involves a more mischievous experimentor who rolls a pair of dice every period a if they comes up 1s (or snake eyes) re-flips the coin. Bayes's formula becomes ugly (and I won't even try to type it) but it is clear that lower weight is placed on the rules performance in the distant past (because it is more likely that the rolls of the dice have made it irrelevant).

A simple formula which has something to do with rationality would be

$$\frac{e^{-0.5\sum_{t=3}^{T}(\rho^{T-t}(X_t\ -0.9X_{t-1}\ \ -0.1X_{t-2})^2}}{e^{-0.5\sum_{3}^{T}\rho^{T-t}(X_t\ -0.9X_{t-1}\ \ -0.1X_{t-2})^2} + e^{-0.5\sum_{3}^{T}\rho^{T-t}(X_t\ -1.1X_{t-1}\ \ +0.1X_{t-2})^2}}$$

With $\boldsymbol{\rho}$= 11/12 .  This isn't really the optimal weight on the first (mean reverting) forecasting rule, given the story about the experiment.  I am implicitly making all sorts of barbarous approximations setting logs of sums approximately to sums of logs and stuff.

I hope you think it's OK as motivation for a simple formula which implies behaviour roughly corresponding to the behaviour observed in experiments.