More on Consumption

Recall for a fixed known interest rate r

old14) $E_t[u'(c_{t+1})/u'(c_t)] = (1+d)/(1+r)$

which is equivalent to

1) $u'(c_t) = E_t[(1+r)u'(c_{t+1})]/(1+d)$

On ~~Monday~~ Tuesday, I made a bold approximation for logarithms and noted that equation 14 and the assumption of a constant elasticity of substitution utility (CES) function

$u(c) = [c^{(1-\theta)}/(1-\theta)$

approximately imply

2) $E_t[\log(c_{t+1})] = \log(c_t) + ((r - d)/\theta$

this is a standard equation, but it is not exact. A rational consumer will consider the fact that next period's consumption isn't exactly predictable. Unexpectedly good news implies higher than expected consumption and vice versa.

The expected value of the marginal utility of consumption in period t+1 is not equal to the marginal utility that one would get with the expected level of consumption.

$u'(c_{t+1})$ is sometimes higher than
$u'(E_t(c_{t+1}))$ and sometimes lower. It is higher on average if the third derivative of the utility function is positive $u'''()$ is positive. For the CES utility function the third derivative is $[c^{-\theta-2}]\,\theta(\theta+1)) > 0$.

All standard utility functions have a positive third derivative so economists suspect that the growth rate of consumption is higher than that implied by equation old14. This is an issue on the order of the difference between the rate of growth $(c_{t+1}-c_t)/c_t$ and the change in log consumption $\log(c_{t+1}) - \log(c_t)$. I ignored both issues when making the bold assumption ~~Monday~~ Tuesday.

The extra growth of consumption (extra savings) increases the more uncertain the consumer is. This increase is saving is called precautionary saving. Economists don't agree if this is an important issue.

If the real interest rate varies and is stochastic (casuale) and not completely predictable, it is standard to call the interest rate paid in period t

r_t so the first order condition becomes

3) $u'(c_t) = E_t[(1+r_{t+1})u'(c_{t+1})]/(1+d)$

With the assumption that the utility function is constant elasticity of substitution AND the bold approximation for logarithms this becomes

4) $E_t[\log(c_{t+1})] = \log(c_t) + (E_t(1+r_{t+1}) - d)/\theta$

This is the form in which the equation is usually tested. To obtain $E_t(1+r_{t+1})$ one regresses the real interest rate on lagged information and uses the fitted values from the regression.

Now what if there is more than one asset, more than one way to save (say Treasury bills (BOTs) and stocks (Azioni) ? Consider two assets one of which pays 1+r and one of which pays $1+\pi_t$

Let's say in period t, the agent invests amount A in the asset which pays $1+\pi_{t+1}$ and the rest of her wealth in the asset which pays a safe certain return 1+r The first order condition for optimal A is

4) $E_t[(1+r)u'(c_{t+1})] = E_t[(1+\pi_{t+1})u'(c_{t+1})]$

Equation 4 usually will **not** hold if

$1+r = E_t(1+\pi_{t+1})$

For example if the second asset is a balanced portfolio of stock (azioni) then it will have high returns when the economy does well and one would expect high consumption then too.

To figure out what the expected returns on assets should be, economists tend to assume that there is a safe asset and expand equation 4 as follows. first note that if r is a constant then

5) $E_t[(1+r)u'(c_{t+1})] = (1+r)E_t[u'(c_{t+1})]$

then note

6) $E_t[(1+\pi_{t+1})u'(c_{t+1})] =$

$E_t[(1+E_t(\pi_{t+1})+\pi_{t+1} - E_t(\pi_{t+1}))\ u'(c_{t+1})] =$

$(1+E_t(\pi_{t+1}))E_t(u'(c_{t+1})) +$

$E_t[(\pi_{t+1} - E_t(\pi_{t+1}))u'(c_{t+1})]$

so

7) $(1+r)E_t[u'(c_{t+1})] =$

$(1+E_t(\pi_{t+1}))E_t(u'(c_{t+1})) + E_t[(\pi_{t+1} - E_t(\pi_{t+1}))u'(c_{t+1})]$

$= (1+E_t(\pi_{t+1}))E_t(u'(c_{t+1})) + E_t[(\pi_{t+1} - E_t(\pi_{t+1}$

$))(u'(c_{t+1})-E_t(u'(c_{t+1})) +$

$E_t[(\pi_{t+1} - E_t(\pi_{t+1}))(E_t(u'(c_{t+1}))]$

$= (1+E_t(\pi_{t+1}))E_t(u'(c_{t+1})) + E_t[(\pi_{t+1} - E_t(\pi_{t+1}$

$))(u'(c_{t+1})-E_t(u'(c_{t+1}))]$

so

8) $1+r = 1+E_t(\pi_{t+1}) +$

$E_t[(\pi_{t+1} - E_t(\pi_{t+1}))u'(c_{t+1})]/E_t(u'(c_{t+1}))$

8 implies

9) $1+E_t(\pi_{t+1}) = 1+r -$

$E_t[(\pi_{t+1}-E_t(\pi_{t+1}))(u'(c_{t+1})-E_t(u'(c_{t+1}))]/E_t(u'(c_{t+1}))$

I have done a bit of algebra but also noted that, since $E_t(c_{t+1})]$ is a constant,

$E_t[(\pi_{t+1}-E_t(\pi_{t+1}))E_t(u'(c_{t+1}))]=0$.

This says that if the return on an asset has a negative correlation with the marginal utility of consumption then the expected return on the asset should be higher than the safe real interest rate. Other things equal, that means that if the correlation with the return on an asset and consumption is positive, the asset should have a higher expected return than the safe real interest rate.

Oddly the return on stock has a low correlation with aggregate consumption. Equation 8 and the idea that consumption can be modelled by consumption of a rational agent don't work well together.

There are assets which have high returns when the marginal utility of consumption is high. They are called insurance.