Three Exercises
Please solve two (not 1 not 3, 2)

1: Consider a consumer who chooses consumption in period t (C_t) for t = 1,2, 3 ... to maximize the Sum from t = 1 to infinity of $-1/(C_t)/(1.10)^t$ subject to the budget constraint that the present discounted value of consumption is equal to initial wealth (K_1) where consumption is discounted at a constant rate r (note this implies that the consumer has no labour income ever)

so K_1 = the sum from 1 to infinity of $C_t/(1+r)^{t-1}$

If initial wealth $K_1 = 100000$ what is C_1 as a function of r ?

a) Euler equation $u'(C_t) = ((1+r_{t+1})/(1+d))\, u'(C_{t+1})$
that is
$u'(C_{t+1}) = ((1+d)/(1+r_{t+1}))\, u'(C_t)$

$u'(C) = C^{-2}$

$C_t^{-2} = ((1+r)/1.1)\, C_{t+1}^{-2}$

$C_{t+1}^2 = ((1+r)/1.1)\, C_t^2$

$C_{t+1} = ((1+r)/1.1)^{0.5}\, C_t$

mathematical induction

$C_t = ((1+r)/1.1)^{0.5(t-1)} C_1$

$C_t/(1+r)^{(t-1)} = ((1+r)/1.1)^{0.5(t-1)} /(1+r)^{(t-1)}\, C_1$

$(1/((1+r)1.1))^{0.5(t-1)}\, C_1$

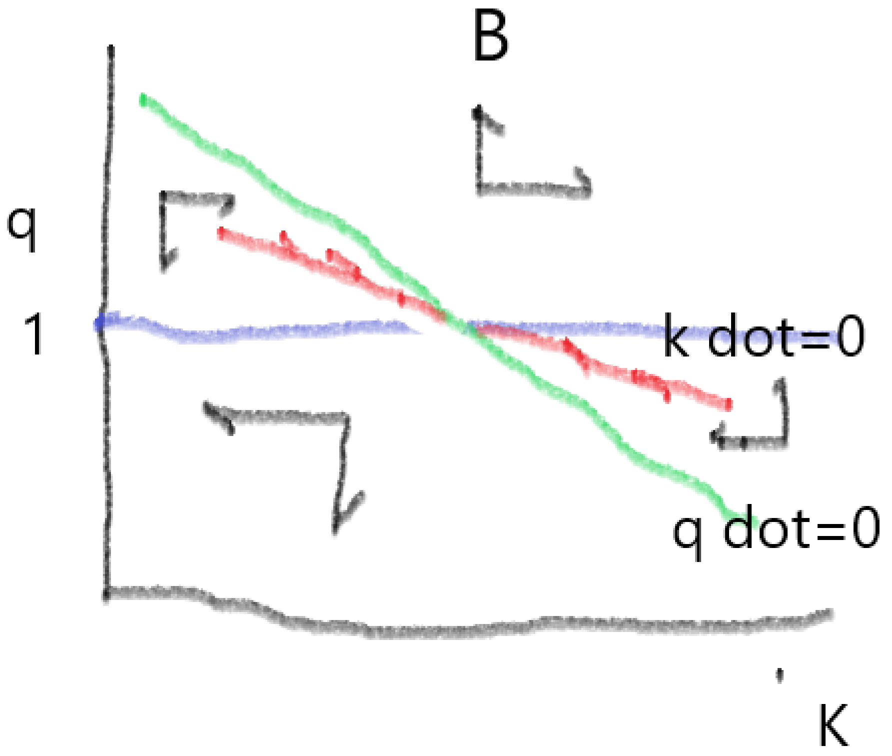
K = Sum_t $(1/((1+r)1.1))^{0.5(t-1)}\, C_1$
$= C_1$ Sum_t $((1/(1+r)1.1))^{0.5(t-1)}$
$= C_1/ (1- (1/((1+r)1.1))^{0.5})$

$C_1 = 100000(1- (1/((1+r)1.1))^{0.5})$

2) Consider the model of investment with no financial market imperfections presented by Romer.
a)Write down the equations for Kdot and Qdot -- the time derivatives of K and Q (you are not obliged to re-derive them)

K dot = $I = C'^{-1}(q-1)$
q dot = $rq - \pi(K)$

b) draw a graph of Q on K showing the Qdot = 0 curve and the Kdot=0 curve (that is draw the phase diagram)



c) K* is the steady state level of capital. Write down the equation for K* (that is find the point where Kdot=0 and Qdot= 0)

q=1

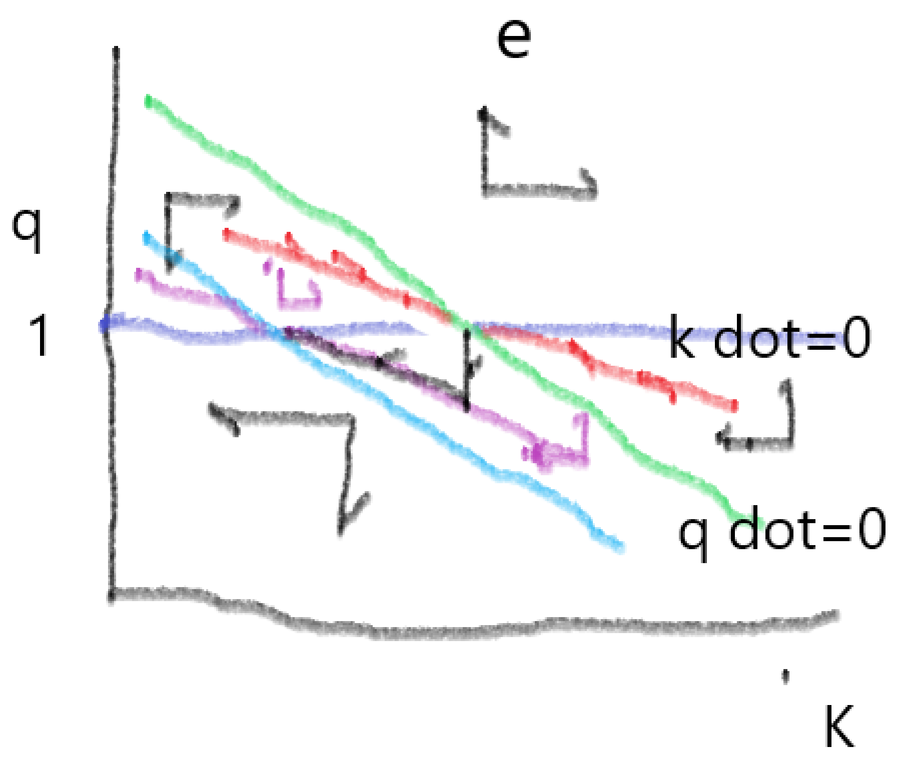
qdot = 0 = rq – π(K) = r – π(K)

π(K) = r

d) Imagine that initial K = K* . Assume there are no taxes and at first agents all believe there will never be taxes, but then shockingly at t1 the state introduces a tax τ on profits so from the point of view of the firm π(K) is replaced by (1- τ)π(K). All agents assume that τ will then remain the same forever . Look at the equations for Qdot and Kdot. Which one is changed by the tax ?

A rq= π(K) (1-τ)

e) Illustrate what happens after t1 .



3: Consider a Solow growth model with no depreciation or population growth. The rate of technological progress g = 0.02..

1) $Y = 0.1K^{0.5}(AL)^{0.5}$

a)Ramsey Cass Koopmans Find the steady state capital to effective labour ratio if consumers act to maximize the presented discounted value of the square root of consumption (so theta = 0,5) with a discount rate rho of 0.05

b)Draw a phase diagram of c and k illustrating the convergence to the steady state you found in c.

C_p dot/Cp = (r-rho)/theta = (r-0.05)/0.5

c dot/c = (r-0.05)/0.5 - g= (r-0.05)/0.5 - 0.02

$y = 0.1\ k^{0.5}$

$r = 0.05k^{-0.5}$

c dot/c = (r-0.05)/0.5 - g= $(0.05k^{-0.5}-0.05)/0.5$ - 0.02

a) c dot/c = 0 = $(0.05k^{-0.5}-0.05)/0.5$ - 0.02

$0.02 = (0.05k^{-0.5}-0.05)/0.5$

$0.01 = (0.05k^{-0.5}-0.05)$

$0.06 = 0.05k^{-0.5}$

$0.06k^{0.5}$ 0.05

$k = (5/6)^2 = 25/36$