Exercizes Wed April

1) Consumer livess 2 periods and maximizes

$\ln(C1) + (1/1.1)\ln(C2)$

Initial wealth 0 , r = 0.1. W1= 0

W2 =

1.1+(2/3) probability 0.5
3.1 probability 0.5

What is consumption in period 1 (C1) ?

Hint it is an integer

$u'(C1) = E_1 (((1+r)/(1+d)) u'(C2))= ((1+r)/(1+d)) E_1 (u'(C2)) = E_1 (u'(C2))$

st $C1(1.1)+C2 = W2$

$C2 = W2 – C1(1.1)$

max E $u(C1) + (1/1.1) u(W2-(1.1)C1)$

$0 = u'(C1) - E_1 ((1+r)/1.1) u'(W2 -(1+r)C1)$

$u'(C1) = E_1 (u'(W2-(1.1)C1) = 0.5 u'(1.1+(2/3) -1.1C1) + 0.5 u'(3.1-1.1C1)$

$1/C1 = 0.5/(1.1+(2/3) -1.1C1) + 0.5/(3.1-1.1C1)$

infinity = $0.5/(1.1+(2/3)) + 0.5/(3.1)$

check C1=1

? $1 = 0.5/(2/3) = 0.5/2 = 0.5(3/2) + 0.25 = 0.75+0.25 =1$

b) What if W2 = 2.1+(2/3)

$0 = u'(C1) - u'(2.1+(2/3) -(1+r)C1)$

$u'(C1) = u'(2.1+(2/3) -(1+r)C1)$

$1/C1 = 1/(2.1+(2/3) -(1+r)C1)$

$(2.1+(2/3) -(1+r)C1) = C1$

$2.1+(2/3) -1.1C1= C1$

$2.1 + 2/3 = 2.1C1$

$C1 = 1 + 2/(6.3)$

2) Consumption CAPM

max $2(C2)^{0.5}$

2 assets
asset 1 pays 0.25 prob 0.5
pays 2.25 prob 0.5

Consumers start with 1 unit of asset 1

They can also borrow and save at the safe interest rate r
That is there is also a safe asset in zero net supply

1) equilibrium borrowing = 0 for everyone (by symmetry)

r has to be such that each chooses to borrow 0

risky asset has price 1
safe asset which pays 1 for sure in period 2 has price $1/(1+r)$ in period 1.

if agent "buys" x units of safe asset then agent has 1-x units of risky asset

so has $C2 = x(1+r) + (1-x)0.25$ prob 0.5
$x(1+r) + (1-x)2.25$ prob 0.5

r must be so $x = 0$

$E\ 2C2^{0.5}$

$0 = E\ dU/dx = 0.5\ (1+r - 0.25)u'(C2|\text{state } 1) + 0.5(1+r-2.25)u'(C2|\text{state}2)$

state1 $C2 = 0.25$
state 2 $C2 = 2.25 = 9/4$

$u'(C2) = 1/C2^{0.5}$
state 1 $u'(C2) = 2$
state 2 $u'(C2) = 1/(9/4)^{0.5} = 1/(3/2) = 2/3$

$0=0.5\ (1+r - 0.25)2+ 0.5(1+r-9/4)2/3$

$0 = 1+r + (1+r)(1/3) - 0.5(0.25)2 - 0.5(9/4)2/3 = (1+r)4/3 - 0.25-0.75 =(1+r)4/3 - 1 =0$
$1+r = ¾$ $r = -0.25$

2 unsolvable problems

3) max sum $1/(1.1)^{t-1} C_t^{0.5}$

st K = sum $1/(1+r)^{t-1} C_t$

yesterday should have said r < 0.21

today show there is no solution if r = 0.44

There is no solution because there is n o maximum because any u can be achieved

hmm imagine the consumer consumes 0 until t then consumes all she has then consumers zero after t

$Ct = K(1.44)^{t-1}$

$u = (1/(1.1)^{t-1})C_t^{0.5} = (1/(1.1)^{t-1})(K(1.44)^{t-1})^{0.5} = (1/(1.1)^{t-1})(K^{0.5}(1.44)^{(t-1)\,0.5}) =$

$(1/(1.1)^{t-1})(K^{0.5}(1.44)^{0.5(t-1)}) = K^{0.5}(1/(1.1)^{t-1})(1.2^{t-1}) = K^{0.5}(1.2/(1.1))^{t-1}$

4) Ramsey Cass Koopmans theta = 0.5 g = 0.6 rho = 0.2 n = 0, delta =0
production function = whatever K0 is whatever

There is no solution because there is no maximum to U. Any amount can be obtained.

Imagine consumer chooses so c is constant

c = f(k0) – gk0

$C_p = C_{po} e^{gt}$

$u(C_p) = 2 C_p^{0.5} = 2C_{p0}^{0.5} e^{gt/2}$

$e^{-rho\ t} u(C_p) = e^{-rho\ t} 2C_{p0}^{0.5} e^{gt/2} = 2C_{p0}^{0.5} e^{(g/2-rho)t} = 2C_{p0}^{0.5} e^{(g/2-rho)t} = 2C_{p0}^{0.5} e^{0.1t}$

look need rho > (theta)g

Oh see need r steady state = rho +g(theta) > r for golden rule = g