1 Solow Problem

1. $Y = 0.1K^{0.5}(AL)^{0.5}$

$n = 0.02$
$g = 0.03$
delta = 0
$k = K/(AL)$

1. What steady state k maximizes c ?
2. If investment = SY what is steady state k as a function of S ?
3. What savings rate S leads to maximum steady state c ?

$y = Y/(AL) = F(k,1) = f(k) = 0.1K^{0.5}(AL)^{0.5} /(AL) = S0.1k^{0.5}$
$c = C/(AL)$

$k^{dot} = S\ 0.1K^{0.5}(AL)^{0.5} /(AL) - (0.05)k = S0.1k^{0.5} - (0.05)k$

$k^{dot} = 0.1K^{0.5}(AL)^{0.5} /(AL) - C/(AL)\ (0.05)k = 0.1k^{0.5} - (0.05)k -c$
$k^{dot} = 0.$
$0= 0.1k^{0.5} - (0.05)k -c$

$c = 0.1k^{0.5} - (0.05)k$

FOC

$0 = 0.05k^{-0.5} - (0.05)$

$0.05k^{-0.5} = (0.05)$

$k^{0.5} = 0.05/0.05 = 1$

$k = 1.$

b) $k^{dot} = S0.1k^{0.5} - (0.05)k$

$k^{dot} = 0.$

$0 = S0.1k^{0.5} - (0.05)k$

$S0.1k^{0.5} = (0.05)k$
$(S)0.1 = (0.05)k^{0.5}$

$S = (0.5)k^{0.5}$

$S^2 = 0.25\ k$
$k = 4S^2$

c) $S = 0.5$

so $SY = rK$

$Sy = rk$

$k^{dot} = rk - (0.05)k$

$r = (n+g+delta)$

$r = 0.05k^{-0.5}$

$rk = 0.05k^{0.5} = 0.5y$

$Sy = rk = 0.5y$

$S = 0.5$

Stupid way

$k = 4S^2$

$y = 0.1k^{0.5} = (0.1)2S = 0.2S$

$c = (1-S)y = 0.2(1-S)S = 0.2(S-S^2)$

FOC

$0 = 0.2 - 0.4S$

$0.2 = 0.4S$

$S=0.5$

$c = Sy$

$C=SY$

Ramsey Cass Koopmans

2: Consider a Solow growth model with no depreciation or population growth. The rate of technological progress $g = 0.02$.
1) $Y = 0.08K^{0.6}(AL)^{0.4}$
a) Ramsey Cass Koopmans Find the steady state capital to effective labour ratio if consumers act to maximize the presented discounted value of the square root of consumption the square root of consumption (so theta = 0,5) with a discount rate rho of 0.04
b) Draw a phase diagram of c and k illustrating the convergence to the steady state you found in a.
c)If rho = 0.01, the problem has no solution. Why can't it be solved ?

$c^{dot}/c = (r\text{-rho})/\text{theta} - g$

$(r\text{-rho})/\text{theta} - g = 0$

$(r-0.04)/(0.5) - 0.02 = 0$

$r- 0.04 = (0.5)(0.02) = 0.01$ thanks (vittoria & Amin)
$r = 0.05$

$r = (0.08)(0.6)K^{-0.4}(AL)^{0.4} = 0.048\ k^{-0.4} = 0.05$

$y = f(k) = 0.08k^{0.6}$

$r = f'(k) = 0.048\ k^{-0.4} = 0.05$

$0.048 = 0.05\ k^{0.4}$

$0.96 = k^{0.4}$
$0.96^{2.5} = (k^{0.4})^{2.5} = k$ (thanks Eduardo)

b) see whiteboard or spreadsheet

c There is no maximum to V, they can have any V up to infinity.