t

1: Consider a consumer who chooses consumption in period t (C_t) for t = 1,2, 3 ... to maximize the Sum from t = 1 to infinity of $(C_t)^{0.5}/(1.10)^t$ subject to the budget constraint that the present discounted value of consumption is equal to initial wealth (K_0) plus the present discounted value of wages (wt)where wages and consumption are discounted at a constant rate r

so K_1 = the sum from 1 to infinity of $(C_t - w_t)/(1+r)^{t-1}$

a) If initial wealth $K_0 = 0$ and wages are 1 what is C_1 as a function of r ?

Euler equation always

$c_{t+1}/c_t = ((1+r)/(1+d))^{1/\theta}$ (in aula S11 I rederived this using a Lagrange multiplier, but you don't have to). In this case 1+d = 1.1 and theta = 0.5

so

$c_{t+1}/c_t = ((1+r)/(1+d))^2$

by induction

$c_t/c_1 = ((1+r)/(1+d))^{2(t-1)}$

Now the budget constraint. It holds with equality for the correct solution

Sum $(ct/(1+r)^t)$ = K_0+ Sum $(W_t/(1+r)^t)$ = Sum$(1/(1+r)^t)$ = $(1/(1-1/(1+r)))/(1+r) = 1/r$

So Sum $C_1((1+r)/(1+d))^{2(t-1)}/(1+r)^t = 1/r =$

Sum $C_1((1+r)/(1+d))^{2(t-1)}/(1+r)^{t-1}$ $/(1+r) = 1/r$

So Sum $C_1((1+r)/(1+d))^{2(t-1)}/(1+r)^{t-1} = (1+r)/r$

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b)Why is there no solution if r>0.21 ?

So Sum $C_1((1+r)/(1.1)^2)^{(t-1)} = (1+r)/r$

So $C_1(1/(1- (1+r)/(1.1^2) = (1+r)/r$

So $C_1 = (1-(1+r)/(1.1^2)(1+r)/r$

b) if $r > 0.21$ this gives negative consumption which is crazy. See that the exponential sum does not converge. The problem is that if $r > 0.21$ the consumer can always get higher utility by saving Until time T earning compound interest and then consuming all the wealth. The utility increases in T up to infinity. The problem is that the problem has no solution. It is impossible to find C_t so that the consumer has the maximum possible utility, because there is no maximum to the possible utility – the consumer can get any utility up to infinity. The problem has no solution if $r>0.21$