Growth exercizes 2022

Romer 86 (too easy for exam)

There are three steps

1) Find r (it will be a constant)
2) Find C^{dot}/C (for standard utility functions it will be constant)
3) Find C/K so Cdot/C = Kdot/K

Example

$Y = 0.1K^{0.5}(AL)^{0.5}$

A = K/L

Rho = 0.04

n = 0.01

Max integral $e^{-0.03t} C_{pt}^{0.5}$

Note theta = 0.5

Step 1 r = partial Y/partial K = $0.05\ K^{-0.5}(AL)^{0.5}$

AL= K so r = 0.05

Step 2 C_{pt}^{dot}/C_{pt} = (r-rho)/theta = (r-0.04)/0.5 = 0.02

$C = C_pL$ so

Cdot/C = C_{pt}^{dot}/C_{pt} + n = 0.03

Kdot/K = Y/K –C/K = 0.1 – C/K

So C/K = 0.07

$K_t = K_0e^{0.03t}$

$e^{-rt}K_t = e^{-0.05t}\ K_0e^{0.03t} = K_o\ e^{-0.02t}$

so the present value at time t of K at time t goes to zero as K goes to infinity, so the transversality condition is satisfied, that is the budget constraint holds with equality.

2. Romer 86 again (I can't make a hard problem)

Not all constant returns to scale production functions are Cobb Douglas

What if Y = 0.1/((1/K)+(1/(AL))) ?

This looks odd (especially with the equation written as plain text) but is CRS. It remains easy if

A=K/L

r = partial Y/partial K = $0.2(1/K^2)/((1/K)+(1/(AL)))^2$

now plug in AL = K

r = $0.2(1/K)^2/((1/K)+(1/K))^2 = 0.05$

also Y/K = (0.2/(2/K))/K = 0.1

so $K^{dot}/K = 0.1-C/K$

with r and K^{dot}/K (and some assumptions about consumption decisions) I can solve the model without worrying about the production function anymore.

Say I make the same assumptions as in problem 1

Rho = 0.04

n = 0.01

Max integral $e^{-0.03t}\ C_{pt}^{\ 0.5}$

Then I get the exact same phase diagram as before.

3 Romer 90 (solution too long and messy for exam)

$Y = L^{0.75}\ \Sigma^{M}\ x_j^{0.25}$

$\eta = 5$

L = 1

$P_j = 1/\text{alpha} = 4$

FOC of x_j

$0.25L^{0.75}\ xj^{-0.75} - 4 = 0$

$L^{0.75}\ x_j^{-0.75} = 16$

$x_j^{0.75} = L^{0.75}/16$

$x_j^{0.75} = L^{0.75}/16$

$x_j = 16^{-4/3}\ L$

plug in L=1 and

$x_j = 16^{-4/3}$

profit per unit of time = $(4-1)\ 16^{-4/3} = 3(16^{-4/3})$

a patent is worth $3(16^{-4/3})/r$ and must be worth 5 given free entry

so

$r = 3(16^{-4/3})/5$

ugly but constant.

$Y = L^{0.75}M(16^{4/3})^{0.25}$

Plug in L = 1 and get

$Y = M16^{-1/3}$

Good is used to consume, make intermediate goods or make inventions so

$M^{dot} = (Y - Mx_j - C)/5 = (M16^{-1/3} - M16^{-4/3} - C)/5$

$M^{dot}/M = (16^{-1/3} - 16^{-4/3})/5 \ - C/(5M)$

That's all I need to solve given assumptions about C

Let's say

Rho = 0.02

n = 0.01

Max integral $e^{-0.01t}\ C_{pt}^{0.5}$

So $C^{dot}/C = (r-0.04)/0.5 \ + 0.1 = 2(3(16^{-4/3})/5 - 0.4)+0.1$

For the budget constraint to hold with equality we must have

$M^{dot}/M = C^{dot}/C = 2(3(16^{-4/3})/5 - 0.4)+0.1$

SO $(16^{-1/3} - 16^{-4/3})/5 \ - C/(5M) = 2(3(16^{-4/3})/5 - 0.4)+0.1$

And

$C/(5M) = (16^{-1/3} - 16^{-4/3})/5 \ - (2(3(16^{-4/3})/5 - 0.4)+0.1)$

That's ugly but give C_0 as a function of M_0 then C and M grow at rate

$2(3(16^{-4/3})/5 - 0.4)+0.1$

4) Human capital

$Y = 0.2K^{0.3}H^{0.2}(AL)^{0.5}$

$n = 0.01$

$g = 0.01$

$S_K = 0.2$

$S_H = 0.1$

Find the balanced growth path (which will look like a steady state of ratios over AL

Solution

$k = K/(AL)$

$h = H/(AL)$

$y = Y/(AL)$

$y = 0.2k^{0.3}h^{0.2}$

$K^{dot} = 0.04K^{0.3}H^{0.2}(AL)^{0.5}$

$k^{dot} = 0.04k^{0.3}h^{0.2} – (n+g)k = 0.04k^{0.3}h^{0.2} – 0.02k$

$h^{dot} = 0.02k^{0.3}h^{0.2} – 0.02h$

steady state $k^{dot} = 0$ so

$0.02k = 0.04k^{0.3}h^{0.2}$

$k = 2k^{0.3}h^{0.2}$

$h^{dot} = 0$

$0.02h = 0.02k^{0.3}h^{0.2}$

so

$0.02h = 0.02k^{0.3}h^{0.2}$

$h = k^{0.3}h^{0.2} = 0.5k$

$k = 2k^{0.3}(0.5k)^{0.2} = (2/2^{0.2})k^{0.5}$ so

$k^{0.5} = 2^{0.8}$

$k = 2^{1.6}$

$h = 2^{1.6}/2 = 2^{0.6}$

$y = 0.2(2^{1.6})^{0.3}(2^{0.6})^{0.2} = 0.2(2^{0.48+1.2}) = 0.2(2^{0.6})$ all in steady state (really balanced growth path)