Microeconomics I
Assignment on Choice under Uncertainty, 2022-2023

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There is a consumer who will do all her consuming one year hence. She has w dollars to invest and no other sources of income except for the returns on her investments with which to finance her future consumption. We denote the proceeds from her investments by y, and assume that her preferences over lotteries with prizes y possess a von Neumann-Morgenstern utility function V, which is strictly increasing, concave, and differentiable.

This consumer can invest her money in one of two assets. The first of these assets is a riskless asset: for every euro invested, she will receive $r > 1$ euros next year. The second asset is risky: its gross return, θ, has a simple probability distribution π.[1]

The consumer's problem, then, is to decide how much money to invest in each of the two assets. Since every euro invested in the second asset is precisely one euro less invested in the first asset, we write her decision problem as a problem in one variable a, the amount of money she invests in the second, risky, asset. Thus, if θ is the gross return on the risky asset, investing a in this asset yields the consumer

$$y = \theta a + r(w - a) = a(\theta - r) + rw$$

euros to spend on consumption next period. Her problem is then

$$\begin{array}{ll} \max_a & \sum_{\theta \in supp(\pi)} \pi(\theta) V(a(\theta - r) + rw) \\ s.t. & a \geq 0. \end{array}$$

Show that

(a) If $\mathbb{E}[\theta] < r$, then $a = 0$ is the unique solution to the consumer's problem.

[1] By gross return, we mean that every euro invested today yields θ euros next year. Also, $\pi(\theta)$ represents the probability that the risky asset has a gross return of θ euros.

(b) If $\mathbb{E}[\theta] = r$, then $a = 0$ is a solution to the consumer's problem.

(c) If $\mathbb{E}[\theta] > r$, then, if there is a solution to the problem, that solution must be such that $a > 0$.

(d) If the consumer is risk-neutral, i.e., if $V(y) = Ay + B$ for constants $A > 0$ and B, then there is no solution if $\mathbb{E}[\theta] > r$, every $a \geq 0$ is a solution if $\mathbb{E}[\theta] = r$, and $a = 0$ is the only solution if $\mathbb{E}[\theta] < r$.

In what follows, suppose that

1. $V(y) = -e^{-\lambda y}$ for some $\lambda > 0$;

2. $\mathbb{E}[\theta] > r$; and

3. There exists at least one value in the support of π that is strictly less than r.

Show that

(e) The solution to the consumer's problem, $a(w, \lambda)$ is independent of w and nonincreasing in λ. Interpret the results.