Problem Set - General Equilibrium

Problem 1

Let $\mathcal{E} = \{u^i, e^i\}_{i=1}^I$ be an exchange economy. Show that if the allocation $\overline{x}$ is a solution for

$$\begin{array}{lll} \max_x & \lambda_1 u^1(x^1) + \ldots + \lambda_I u^I(x^I) & \\ s.t. & \sum_{i=1}^I x^i = \sum_{i=1}^I e^i & \quad (PC), \\ & x^i \geq 0, i = 1, \ldots, I & \end{array}$$

where $\lambda_i > 0$ for all $i \in \{1, \ldots, I\}$, then $\overline{x}$ is Pareto efficient.

Obs.: Assume that u^i is concave for all i. In this case, you can show that if $\overline{x}$ is Pareto efficient, then there exists a vector $(\lambda_1^*, \ldots, \lambda_I^*) > 0$ such that $\overline{x}$ is a solution to (PC) when $\lambda_i = \lambda_i^*$ for $i = 1, \ldots, I$.

Problem 2

Let $\mathcal{E} = \{u^i, e^i\}_{i=1}^I$ be an exchange economy.

(a) Show that if the allocation $\overline{x}$ is Pareto efficient, then $\overline{x}$ is a solution for

$$\begin{array}{ll} \max_x & u^1(x^1) \\ s.t. & u^i(x^i) \geq u^i(\overline{x}^i), i = 2, \ldots, I \\ & \sum_{i=1}^I x^i = \sum_{i=1}^I e^i \\ & x^i \geq 0, i = 1, \ldots, I \end{array}$$

(b) Suppose now that the utility functions u^i and continuous and strongly increasing. Show that if the allocation $\overline{x}$ is a solution for

$$\begin{array}{ll} \max_x & u^1(x^1) \\ s.t. & u^i(x^i) \geq u^i(\overline{x}^i), i = 2, \ldots, I \\ & \sum_{i=1}^I x^i = \sum_{i=1}^I e^i \\ & x^i \geq 0, i = 1, \ldots, I \end{array}$$

then $\overline{x}$ is Pareto efficient.

Problem 3

Consider the 2×2 exchange economy where

$$\begin{aligned} u^1(x_1, x_2) &= x_1 x_2 & \text{and } e^1 = (18, 4) \\ u^2(x_1, x_2) &= \ln x_1 + 2 \ln x_2 & \text{and } e^2 = (3, 6) \end{aligned}$$

(a) Find a Walrasian equilibrium of this economy.

(b) Find the contract curve of this economy, i.e. determine the set of all Pareto efficient allocations.

Problem 4

Consider the production economy

$$\mathcal{E} = \left(\{u^i, e^i\}_{i=1}^I, \{Y^j\}_{j=1}^J, \{\theta^{i1}, \dots, \theta^{iJ}\}_{i=1}^I \right).$$

We say the list $(\overline{x}, \overline{y}, p^*)$, where $(\overline{x}, \overline{y})$ is an allocation and $p^* > 0$ is a price vector, is an equilibrium with transfers if there exists $w = (w^1, \dots, w^I) > 0$ with

$$\sum_{i=1}^I w^i = p^* \sum_{i=1}^I e^i + p^* \sum_{j=1}^J \overline{y}^j$$

such that:

(i) for all $j \in \{1, \dots, J\}$, $p^* \cdot y^j \leq p^* \cdot \overline{y}^j$ for all $y^j \in Y^j$;

(ii) for all $i \in \{1, \dots, I\}$, $\overline{x}^i$ is a solution for

$$\begin{aligned} \max \quad & u^i(x) \\ s.t. \quad & p^* \cdot x \leq w^i \\ & x \geq 0 \end{aligned}$$

(iii) $\sum_{i=1}^I \overline{x}^i = \sum_{i=1}^I e_i + \sum_{j=1}^J \overline{y}^j$.

Show that:

(a) Every Walrasian equilibrium of $\mathcal{E}$ is an equilibrium with transfers.

(b) Suppose the utility functions u^i are locally non–satiated. If $(\overline{x}, \overline{y}, p^*)$ is an equilibrium with transfers, then $(\overline{x}, \overline{y})$ is Pareto efficient.

Problem 5

Consider a two-consumer, two-commodity pure exchange economy where the consumers' preferences are continuous, strictly convex and strongly monotone. The consumers' initial endowments are $\omega_1 = (\omega_1^1, \omega_1^2)$ and $\omega_2 = (\omega_2^1, \omega_2^2)$. For each of the statements below, determine whether they are True of False, and justify your answers.

(a) If the preferences of the two consumers are quasilinear with respect to the same numeraire, and the endowments of consumer 1 are increased to $\omega_1' >> \omega_1$ whilst ω_2 remains the same, then at equilibrium the utility of consumer 1 increases.

(b) Suppose that preferences are quasilinear, but an increase in 1's endowments comes from a transfer from consumer 2, i.e. $\omega_1' = \omega_1 + z$ and $\omega_2' = \omega_2 - z$ for some $z \geq 0$. Then, in equilibrium, consumer 1's utility cannot decrease.

(c) Consider again a transfer as in the previous item, but preferences are no longer quasilinear. If the transfer z is small and the change in equilibrium relative price is restricted to be small, than the equilibrium utility of consumer 1 must not decrease.

Problem 6

Consider a pure exchange economy with two goods (X and Y) and two consumers (1 and 2) where free disposal is impossible. A consumption bundle is denoted by (x, y), meaning x units of good X and Y unites of good Y. Everyone's consumption set is $\mathbb{R}^2_+$. Consumer 1's endowment is $\omega_1 = (5, 0)$, and consumer 2's is $\omega_2 = (0, 5)$. Consumer 1's preferences are represented by the utility function

$$u_1(x, y) = 1 - (x - 3)^2 - (y - 4)^2,$$

while consumer 2's preferences is the lexicographic ordering that ranks X before Y.

(a) Find the set of all Pareto efficient allocations.

(b) Find the set of all allocations that can be supported as a Walrasian equilibrium with transfers.

Problem 7

Let $\mathcal{E} = \{u^i, e^i\}_{i=1}^2$ be an exchange economy where $X^i = \mathbb{R}^2_+$ for $i = 1, 2$ and $u^i : \mathbb{R}^2_+ \to \mathbb{R}$ are continuous, strictly quasi-concave, and strongly increasing. We call $u^i(x_1^i, x_2^i)$ agent i's *material payoff function.* Agent 2 displays *purely selfish* preferences summarized by the utility function

$$\widetilde{u}^2(x_1, x_2) = u^2(x_1^2, x_2^2)$$

while agent 1 is characterized by *altruistic preferences*

$$\widetilde{u}^1(x_1, x_2) = u^1(x_1^1, x_2^1) + \alpha u^2(x_1^2, x_2^2),$$

where $\alpha \in (0, 1]$ denote agent 1's degree of altruism.

(a) Show that if the allocation $\overline{x}$ is a solution for

$$\begin{array}{lll} \max_x & \widetilde{u}^1(x) & \\ s.t. & \sum_{i=1}^2 x^i = \sum_{i=1}^2 e^i & \\ & x^i \geq 0 & i = 1, 2 \end{array}$$

then $\overline{x}$ is Pareto efficient.

(b) Suppose that $u^i(x_1, x_2) = x_1^{1/2} x_2^{1/2}$, $e^1 = (10, 5)$, and $e^2 = (5, 10)$. Find a Walrasian equilibrium of this economy. How does it change when α goes to zero?

Exercises

Solve exercises 15.B.9, 16.C.3 and 16.D.1 from Mas-Colell, Whinston and Green.