Problem Set 1

Problem 1

Consider the strict preference relation $\succ$ and the indifference relation $\sim$ studied in class. Show that $\succ$ and $\sim$ (a) are transitive; and (b) are NOT complete.

Problem 2

Suppose that the consumption set X is finite and that $\succsim$ is complete and transitive. Show that there exists an utility function $u : X \to \mathbb{R}$ that represents $\succsim$.

(*Hint:* use the no-better-than-x set $\precsim(x)$ to construct such an utility function.)

Problem 3

Show an example of a utility function that is locally non–satiated, but is not monotonic.

Problem 4

Suppose preferences are represented by the Cobb-Douglas utility function $u(x_1, x_2) = Ax_1^{\alpha}x_2^{1-\alpha}$, for $A > 0$ and $\alpha \in (0, 1)$.

(a) Assuming an interior solution, solve for the Marshallian demand functions.

(b) Consider the logarithmic transformation $v(x) = \ln u(x)$ of the Cobb-Douglas utility function above, and verify that the Marshallian demand functions are identical to those derived in the previous item.

(c) Suppose preferences are represented by the utility function $u(x)$, and consider the transformation $f(u(x))$, where $f' > 0$. Show that the consumer's demand behavior is invariant to positive monotonic transformations of the utility function.

(d) Using the results in *(a)*, derive the consumer's indirect utility function and use it to compute the Hicksian demand functions.

(e) Compute the Hicksian demand functions for the logarithmic transformation of item *(b)*. Is it true that they are invariant to monotonic transformations as well?

Problem 5

Suppose the consumer's preferences can be represented by a utility function $u : X \to \mathbb{R}_+$

that is continuous, strongly monotonic and strictly quasi-concave. Let $\mathcal{U} \subseteq \mathbb{R}_+$ be the image of u, and $v : X \to \mathbb{R}_+$ be such that $v(x) = \tau(u(x))$ where, for $a, b \in \mathbb{R}_+$ and $\overline{u} \in \mathcal{U}$, $\tau : \mathcal{U} \to \mathbb{R}$ is given by

$$\tau(u) = \begin{cases} au & \text{if } u \geq \overline{u} \\ bu & \text{if } u < \overline{u} \end{cases}$$

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(a) Suppose that $a > b > 0$. Do u and v represent the same preference relation? Why?

(b) Suppose now that $a > b = 0$. Do u and v represent the same preference relation? Why?

Problem 6

Suppose that an individual's choice set is $X = \mathbb{R}^2_+$, and that his preferences can be represented by the utility function

$$u(x_1, x_2) = -(x - \overline{x}_1)^2 - (x_2 - \overline{x}_2)^2,$$

for some given $(\overline{x}_1, \overline{x}_2) \gg 0$. Let $B(p, w)$ denote the agent's budget set for prices $p = (p_1, p_2) \gg 0$ and wealth $w > 0$. For each statement below, determine if it is True and False, and justify your answer.

(a) The Marshallian demand function is homogeneous of degree zero.

(b) Walras' law holds, i.e. $\langle p, x \rangle = w$ for all $x \in x(p, w)$.

(c) The indirect utility function $v(p, w)$ is strictly increasing in w.

Problem 7

Consider an economy with two goods, 1 and 2, and a consumer whose demand function $x(p, w)$ satisfies Walras' law. If the demand for the first commodity is given by $x_1(p, w) = \frac{\alpha w}{p_1}$, what is the demand for the second commodity? Does this demand for the second commodity satisfy homogeneity of degree zero?

Exercises

Solve exercises 3.B.2, 3.C.1 and 3.C.4 from Mas-Colell, Whinston and Green.