

Microeconomics I

Assignment 1, 2022-2023

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In the course, preferences are build using binary relations that, by assumption, allow consumers to compare any two commodity bundles. In this assignment, we will explore an alternative model in which the consumers are either unable or unwilling to make certain comparisons.

In what follows, we assume that the set of consumption bundles X is finite, and we define two primitive relations on this set, namely *strict preferences* \succ and *positive indifference* \sim . These binary relations will satisfy the following properties:

1. \succ is asymmetric and transitive;
2. \sim is reflexive, symmetric, and transitive;
3. if $x \succ y$ and $y \sim z$, then $x \succ z$; and
4. if $x \sim y$ and $y \succ z$, then $x \succ z$.

Given this basic model, answer the following questions.

- (a) Show that 1-4 imply the following statement: if $x \succ y$, then neither $y \sim x$ nor $x \succ y$.
- (b) Given \succ and \sim with the properties 1-4, construct a *weak preference relation* \succsim such that $x \succsim y$ if $x \succ y$ or $x \sim y$. Is this weak preference relation complete? Is it transitive?
- (c) Suppose we begin with a primitive weak preference relation \succsim and define \succ and \sim in the usual manner: $x \succ y$ if $x \succsim y$ and not $y \succsim x$, and $x \sim y$ if $x \succsim y$ and $y \succsim x$. What properties must \succsim have so that \succ and \sim have the properties 1-4?
- (d) Suppose we have a function $u : X \rightarrow \mathbb{R}$ and we define $x \succ y$ if $u(x) > u(y) + 1$ and $x \sim y$

if $u(x) = u(y)$.¹ Do \succ and \sim constructed in this way from u have any of the properties 1-4?

(e) Suppose we have \succ and \sim satisfying properties 1-4 for a finite set X . Does there exist a function $u : X \rightarrow \mathbb{R}$ such that $u(x) = u(y)$ if and only if $x \sim y$ and $u(x) > u(y) + 1$ if and only if $x \succ y$? If so, construct one such a utility function, otherwise provide a counterexample.

¹In other words, indifference bundles have the same utility, whilst strict preferences require a *large enough* utility difference between the two bundles.