Economics of Growth and Development

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Classical Growth Theory

All great classical economists were concerned with economic growth and development: Adam Smith; David Ricardo; Thomas Malthus; John Stuart Mill; Karl Marx

Smith optimistic about growth based on division of labour (specialisation) and increasing returns in industry, but division of labour limited by extent of the market(anticipates endogenous growth theory).

Malthus, Ricardo, Mill pessimistic because diminishing returns in agriculture would cause famine, a rising price of food and decline in the profit rate on capital.

Marx predicted breakdown of capitalism itself because of falling rate of profit and 'realization' crises (lack of demand: anticipates Keynes).

However neoclassical economists not interested in growth.

Harrod-Domar Growth Model I

Modern growth theory started with Roy Harrod in 1939. Domar derived same results (independently) in 1946

Harrod specifies three growth rates which may equal each other only by chance: actual growth rate (g); warranted equilibrium growth rate (g_w); natural growth rate (g_n) determined by labour force growth and technical progress –exogenously determined

If $g \neq g_w$, cyclical fluctuations occur

If $g_w \neq g_n$, secular stagnation if $g_w > g_n$; structural unemployment and inflation if $g_n > g_w$

Most developing countries have labour and technical progress growing faster than capital accumulation i.e. g $g_n > g_w$ –hence growing unemployment of structural variety

Nothing in Harrod model to equalize g, g_n and g_w

Harrod-Domar Growth Model II

g is calculated as follows: S=sY and K=cY with s (savings ratio) and c (capital-output ratio, assuming a Leontief production function so that Y=min $K/c, L/b$).

In equilibrium I=dK=S then g= dY/Y=dK/K=sY/(Yk)=s/c. So growth will be higher the higher is the savings ratio and the lower the capital-output ratio.

However investment will be as follows (Acceleration Principle):

$\frac{I}{Y}=c_r\frac{dEY}{Y}$,where dEY/Y is the increase in GDP that firms expect.

To realise these expectations dEY/Y=$\frac{s}{c_r}$ =g_w must equal dY/Y=s/c=g.

If $g< g_w$,i.e.if $c>c_r$, i.e. if investment is too low, aggregate demand (Y) wil be too low to use all the machines, so subsequently investment will go further down as machines will be idle. Fall in I will cause income to go down as well, through the multiplier 1/s. The opposite will be true if $g>g_w$.

Equilibrium is unstable.

Moreover even if $g=g_w$ there could be unemployment if the labor force

Households and Firms I

At the center of the Solow growth model is the *neoclassical* aggregate production function and in particular capital accumulation: the elasticity of substitution between labour and capital is positive.

- Closed economy, with a unique final good.
- Discrete time running to an infinite horizon, time is indexed by $t = 0, 1, 2, ...$
- To fix ideas, assume all households are identical.

Households and Firms II

- As in HD households save a constant exogenous fraction s of their disposable income
- Assume all firms have access to the same production function (as in HD):
- Aggregate production function for the unique final good $Y(t)$is

$$Y(t) = F[K(t), L(t), A(t)] \tag{1}$$

- Assume capital $K(t)$ is the same as the final good of the economy..$L(t)$ denotes the demand for labor.
- $A(t)$ is technology.
- Major assumption: technology is **free**; it is publicly available as a non-excludable, non-rival good.

Key Assumption

Assumption 1 **(Differentiability, Positive and Diminishing Marginal Products, and Constant Returns to Scale)** The production function $F : \mathbb{R}^3_+ \to \mathbb{R}_+$ is twice continuously differentiable in K and L, and satisfies

$$F_K(K, L, A) \equiv \frac{\partial F(\cdot)}{\partial K} > 0, \quad F_L(K, L, A) \equiv \frac{\partial F(\cdot)}{\partial L} > 0,$$
$$F_{KK}(K, L, A) \equiv \frac{\partial^2 F(\cdot)}{\partial K^2} < 0, \quad F_{LL}(K, L, A) \equiv \frac{\partial^2 F(\cdot)}{\partial L^2} < 0.$$

Moreover, F exhibits constant returns to scale in K and L.

- Assume F exhibits *constant returns to scale* in K and L. I.e., it is *linearly homogeneous* (homogeneous of degree 1) in these two variables.

Definition The function $g : \mathbb{R}^2 \to \mathbb{R}$ is homogeneous of degree m in $x \in \mathbb{R}$ and $y \in \mathbb{R}$ if and only if

$$g(\lambda x, \lambda y) = \lambda^m g(x, y) \text{ for all } \lambda \in \mathbb{R}_+, x \in \mathbb{R}, y \in \mathbb{R}$$

Notice: the above is an identity!

Theorem **(Euler's Theorem)** Suppose that $g : \mathbb{R}^2 \to \mathbb{R}$ is continuously differentiable in $x \in \mathbb{R}$ and $y \in \mathbb{R}$, with partial derivatives denoted by g_x and g_y and is homogeneous of degree m in x and y. Then:

1) $mg(x, y) = g_x(x, y)\, x + g_y(x, y)\, y$,for all $x \in \mathbb{R}$, $y \in \mathbb{R}$
2)$g_x(x, y)$ and $g_y(x, y)$ are themselves homogeneous of degree $m - 1$ in x and y.

Proof:
$\mathrm{d}g(\lambda x, \lambda y)/d\lambda = xg_x(\lambda x, \lambda y) + yg_y(\lambda x, \lambda y) = m\lambda^{m-1} g(x, y)$,for $\lambda = 1$,so we have proved 1.
$\partial g(\lambda x, \lambda y)/\partial x = \lambda g_x(\lambda x, \lambda y) = \lambda^m g_x(x, y), \Leftrightarrow g_x(\lambda x, \lambda y) = \lambda^{m-1} g_x(x, y)$for $\lambda = 1$,so we have proved 2.

Market Structure and Endowments I

- We will assume that markets are competitive.
- Households own all of the labor $\bar{L}(t)$, which they supply inelastically.
- Households own the capital stock of the economy and rent it to firms.
- Denote the *rental price of capital* at time t be $R(t)$.

Market Structure and Endowments II

- The price of the final good is normalized to 1 *in all periods*
- Assume capital depreciates, with "exponential form," at the rate δ: out of 1 unit of capital this period, only $1-\delta$ is left for next period.
- This affects the interest rate (rate of return to savings) faced by the household.
- *Interest rate* faced by the household will be $r(t) = R(t) - \delta$.

Firm Optimization I

- Only need to consider the problem of a *representative firm*:

$$\max_{L(t)\geq 0, K(t)\geq 0} F[K(t), L(t), A(t)] - w(t)\, L(t) - R(t)\, K(t)\,. \qquad (2)$$

- This is a static maximization problem.
  1. Firm is taking as given $w(t)$ and $R(t)$.
  2. Concave problem, since F is concave.

Firm Optimization II

- Since F is differentiable, first-order necessary conditions imply:

$$w(t) = F_L[K(t), L(t), A(t)], \tag{3}$$

and

$$R(t) = F_K[K(t), L(t), A(t)]. \tag{4}$$

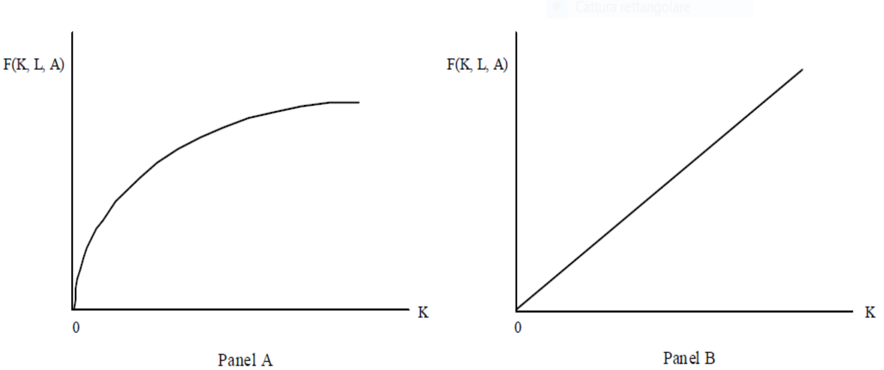
Firm Optimization III

Given RCS firms make no profits, and in particular,

$$Y(t) = w(t) L(t) + R(t) K(t).$$

- This follows immediately from Euler Theorem.

Production Functions



Fundamental Law of Motion of the Solow Model I

- K depreciates exponentially at the rate δ, so

$$K(t+1) = (1-\delta) K(t) + I(t), \tag{5}$$

  where $I(t)$ is investment at time t.
- From national income accounting for a closed economy,

$$Y(t) = C(t) + I(t), \tag{6}$$

- Using (1), (5) and (6), any allocation in this economy must satisfy

$$K(t+1) \leq F[K(t), L(t), A(t)] + (1-\delta) K(t) - C(t)$$

  for $t = 0, 1, ...$
- *Behavioral rule* of the constant saving rate simplifies the structure of equilibrium considerably.

Fundamental Law of Motion of the Solow Model II

- Since the economy is closed (and there is no government spending),

$$S(t) = I(t) = Y(t) - C(t).$$

- Individuals are assumed to save a constant fraction s of their income,

$$S(t) = sY(t), \tag{7}$$

$$C(t) = (1 - s) Y(t) \tag{8}$$

- Implies that the supply of capital resulting from households' behavior can be expressed as

$$K^s(t+1) = (1-\delta)K(t) + sY(t).$$

Fundamental Law of Motion of the Solow Model III

- Setting supply and demand equal to each other, this implies $K^s(t+1) = K(t+1)$.
- The *labor market clearing* for labour can be expressed as:

$$L(t) = \bar{L}(t) \tag{9}$$

for all t.

- Combining these market clearing conditions with (1) and (5), we obtain *the fundamental law of motion* the Solow growth model:

$$K(t+1) = sF\left[K(t), L(t), A(t)\right] + (1-\delta) K(t). \tag{10}$$

- Nonlinear *difference equation*.
- Equilibrium of the Solow growth model is described by this equation together with laws of motion for $L(t)$ (or $\bar{L}(t)$) and $A(t)$.

Equilibrium Without Population Growth and Technological Progress I

- Make some further assumptions, which will be relaxed later:
  1. There is no population growth; total population is constant at some level $L > 0$. Since individuals supply labor inelastically, $L(t) = L$.
  2. No technological progress, so that $A(t) = A$.
- Define the capital-labor ratio of the economy as

$$k(t) \equiv \frac{K(t)}{L}, \tag{11}$$

- Using the constant returns to scale assumption, we can express output (income) per capita, $y(t) \equiv Y(t)/L$, as

$$\begin{aligned} y(t) &= F\left[\frac{K(t)}{L}, 1, A\right] \\ &\equiv f(k(t)). \end{aligned} \tag{12}$$

Equilibrium Without Population Growth and Technological Progress II

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- From Profit maximization and Euler Theorem ,

$$\begin{aligned} R(t) &= f'(k(t)) > 0 \text{ and} \\ w(t) &= f(k(t)) - k(t) f'(k(t)) > 0. \end{aligned} \quad (13)$$

- 1)Since F is homogeneous of order 1, its partial derivatives are homogenous of order 0, ie they will depend on the ratio between arguments not their levels. This means $\mathrm{F}_K(K, L) = f'(k)$. 2) Again from Euler
$\mathrm{Y}{=}\mathrm{F}_K K + F_L L = \mathrm{F}_K K + wL$, $Y/L = f(k) = k(t) f'(k(t)) + w$.

Example: The Cobb-Douglas Production Function I

- $$Y(t) = AK(t)^{\alpha} L(t)^{1-\alpha}, \ 0 < \alpha < 1. \tag{14}$$
- Satisfies Assumptions 1 and 2.
- Dividing both sides by $L(t)$,

$$y(t) = Ak(t)^{\alpha},$$

- From equation (22),

$$\begin{aligned} R(t) &= \frac{\partial Ak(t)^{\alpha}}{\partial k(t)}, \\ &= \alpha Ak(t)^{-(1-\alpha)}. \end{aligned}$$

Example:The Cobb-Douglas Production Function II

- Alternatively, in terms of the original production function (14),

$$\begin{aligned} R(t) &= \alpha A K(t)^{\alpha-1} L(t)^{1-\alpha} \\ &= \alpha A k(t)^{-(1-\alpha)}, \end{aligned}$$

- Similarly, from (22),

$$\begin{aligned} w(t) &= A k(t)^{\alpha} - \alpha A k(t)^{-(1-\alpha)} \times k(t) \\ &= (1-\alpha) A K(t)^{\alpha} L(t)^{-\alpha}, \end{aligned}$$

Equilibrium Without Population Growth and Technological Progress I

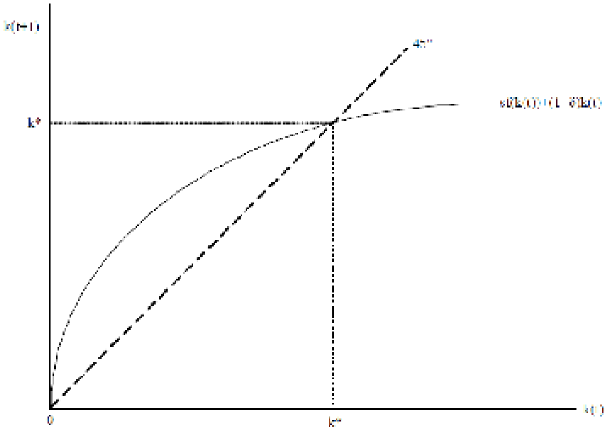
- The per capita representation of the aggregate production function enables us to divide both sides of (10) by L to obtain:

$$k(t+1) = sf(k(t)) + (1-\delta)k(t). \tag{15}$$

- The other equilibrium quantities can be obtained from the capital-labor ratio $k(t)$.

Definition A steady-state equilibrium without technological progress and population growth is an equilibrium path in which $k(t) = k^*$ for all t.

- The economy will tend to this steady state equilibrium over time.


k*
0
k*

Equilibrium Without Population Growth and Technological Progress II

- Thick curve represents (19) and the dashed line corresponds to the 45° line.
- Their (positive) intersection gives the steady-state value of the capital-labor ratio k^*,

$$\frac{f\left(k^*\right)}{k^*} = \frac{\delta}{s}. \tag{16}$$

- There is another intersection at $k = 0$, because the figure assumes that $f\left(0\right) = 0$.

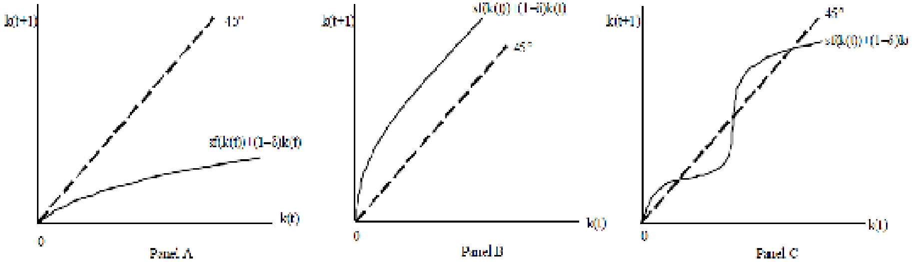
Equilibrium Without Population Growth and Technological Progress V

Proposition In the basic Solow growth model there exists a unique steady state equilibrium where the capital-labor ratio $k^* \in (0, \infty)$ is given by (20)., per capita output is given by

$$y^* = f(k^*) \tag{17}$$

and per capita consumption is given by

$$c^* = (1 - s) f(k^*) . \tag{18}$$


k(t+1)
45°
k(t)
0
Panel A
k(t+1)
45°
k(t)
0
Panel B
k(t+1)
45°
k(t)
0
Panel C

Second Useful Assumption

Assumption 2 **(Inada conditions)** F satisfies the Inada conditions

$$\begin{aligned}\lim_{K\to 0} F_K(\cdot) &= \infty \text{ and } \lim_{K\to\infty} F_K(\cdot) = 0 \text{ for all } L > 0 \text{ all } A \\ \lim_{L\to 0} F_L(\cdot) &= \infty \text{ and } \lim_{L\to\infty} F_L(\cdot) = 0 \text{ for all } K > 0 \text{ all } A.\end{aligned}$$

- Important in ensuring the existence of *interior equilibria*.$\lim_{K\to 0} F_K(\cdot) = \infty$ avoids case A, $\lim_{K\to\infty} F_K(\cdot) = 0$avoids case B. Concavity avoids case C.

Equilibrium Without Population Growth and Technological Progress VI

- Figure shows through a series of examples why Assumptions 1 and 2 cannot be dispensed with for the existence and uniqueness results.
- Assume that

$$f(k) = a\tilde{f}(k),$$

- $a > 0$, with greater values corresponding to greater productivity of factors..
- Since $f(k)$ satisfies the regularity conditions imposed above, so does $\tilde{f}(k)$.

Equilibrium Without Population Growth and Technological Progress VII

Denote the steady-state level of the capital-labor ratio by $k^* (a, s, \delta)$ and the steady-state level of output by $y^* (a, s, \delta)$ when the underlying parameters are a, s and δ. Then we have

$$\frac{\partial k^* (\cdot)}{\partial a} > 0, \; \frac{\partial k^* (\cdot)}{\partial s} > 0 \text{ and } \frac{\partial k^* (\cdot)}{\partial \delta} < 0$$

Equilibrium Without Population Growth and Technological Progress VIII

- Follow immediately by writing

$$\frac{\tilde{f}\left(k^*\right)}{k^*} = \frac{\delta}{as},$$

and noticing that $\frac{\tilde{f}(k^*)}{k^*}$ decreases in k^*.

Equilibrium Without Population Growth and Technological Progress IX

- First important prediction of Solow Model, broadly(far from perfectly) confirmed by data: countries with higher investment rates will have higher income per capita.
- With a Cobb Douglas $y_i^* = A^{\frac{1}{1-\alpha}} \left(\frac{s_i}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$,so $\frac{y_i^*}{y_j^*} = \left(\frac{s_i}{s_j}\right)^{\frac{\alpha}{1-\alpha}}$, for any two countries i, j, so it is possible to make quantitative predictions. These leads however to a severe underestimation of US income per capita with respect to other countries.

- These are ss predictions: countries may be away from ss: we'll talk about convergence to the ss later.
- However question remains: why do some countries invest less than others?
- If the saving rate increases with income the possibility of multiple equilibria arises.If $s=s_1$ for $y \leq y_1$ and $s=s_2$ otherwise then we will have two possible steady states, and if initial income is less or equal than y_1,the economy will converge to the one with lower saving and income.

Review of the Discrete-Time Solow Model

- Per capita capital stock evolves according to

$$k(t+1) = sf(k(t)) + (1-\delta)k(t). \tag{19}$$

- The steady-state value of the capital-labor ratio k^* is given by

$$\frac{f(k^*)}{k^*} = \frac{\delta}{s}. \tag{20}$$

- Consumption is given by

$$C(t) = (1-s)Y(t) \tag{21}$$

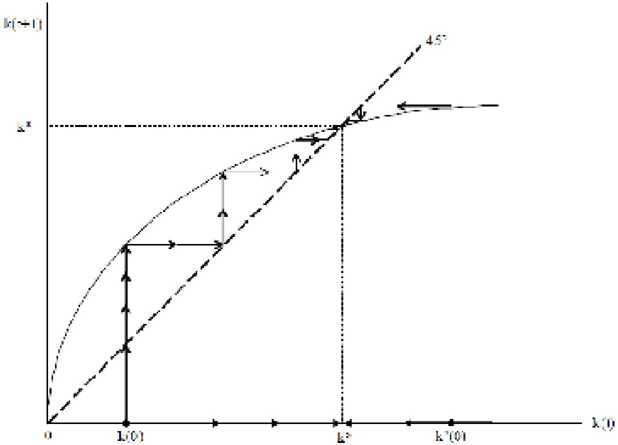
- And factor prices are given by

$$\begin{aligned} R(t) &= f'(k(t)) > 0 \text{ and} \\ w(t) &= f(k(t)) - k(t) f'(k(t)) > 0. \end{aligned} \tag{22}$$

Transitional Dynamics in the Discrete Time Solow Model III

- Stability result can be seen diagrammatically in the Figure:
  - Starting from initial capital stock $k(0) < k^*$, economy grows towards k^*, *capital deepening* and growth of per capita income.
  - If economy were to start with $k(0) > k^*$, reach the steady state by decumulating capital and contracting.

Transitional Dynamics in Figure



A First Look at Sustained Growth I

- Simplest model of sustained growth essentially takes $\alpha = 1$ in terms of the Cobb-Douglas production function above.
- Suppose:
$$F\left[K(t), L(t), A(t)\right] = AK(t), \tag{23}$$
where $A > 0$ is a constant.
- So-called "AK" model, and in its simplest form output does not even depend on labor (even if can be introduced by admitting imperfect competiton or externalities to capital).

A First Look at Sustained Growth II

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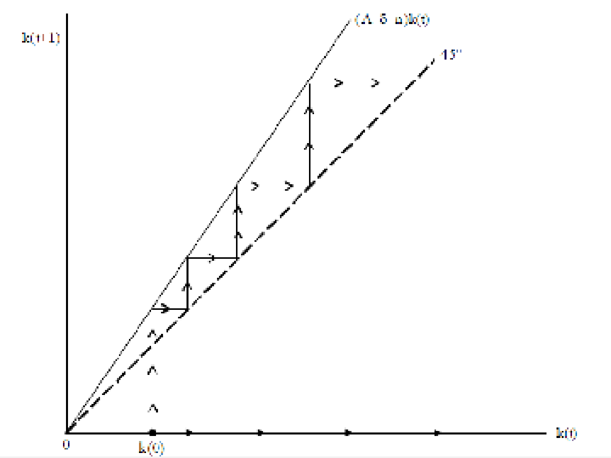
- Combining with the production function (23),

$$\frac{k(t+1)}{k(t)} = sA + 1 - \delta.$$

- Therefore, if $sA - \delta > 0$, there will be sustained growth in the capital-labor ratio.
- From (23), this implies that there will be sustained growth in output per capita as well. $\frac{k(t+1)-k(t)}{k(t)} = sA - \delta$

Sustained Growth in Figure



Growth Accounting I

- Aggregate production function :

$$Y(t) = K(t)^{\alpha} L(t)^{1-\alpha} A(t).$$

- Combined with competitive factor markets, gives Solow (1957) *growth accounting framework.*
- Continuous-time economy: take logs and differentiate with respect to time.
- Dropping time dependence,

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \alpha \frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L}. \tag{24}$$

- Witth perfect competition $\alpha \equiv RK/Y$ Denote growth rates of output, capital stock and labor by $g \equiv \dot{Y}/Y$, $g_K \equiv \dot{K}/K$ and $g_L \equiv \dot{L}/L$,.x=$\frac{\dot{A}}{A}$

$$x = g - \alpha_K g_K - \alpha_L g_L. \tag{25}$$

- Gives estimate of contribution of technological progress, *Total Factor Productivity* (TFP) or Multi Factor Productivity.

- Solow's (1957) applied this framework to US data: a large part of the growth was due to technological progress.
- From early days, however, a number of pitfalls were recognized.(from endogeneity of $\frac{\dot{K}}{K}$ to problematic assumption of perfect competition)
  - Moses Abramovitz (1956): dubbed the $\hat{x}$ term "the measure of our ignorance".
  - If we mismeasure g_L and g_K we will arrive at inflated estimates of $\hat{x}$.
  - Reasons for mismeasurement:
  - what matters is not labor hours, but effective labor hours
    - changes in the *human capital* of workers.
  - measurement of capital inputs:
    - capital and consumption not the same good, need assumptions about how relative prices of machinery change over time.
    - typical assumption was to use capital expenditures but if machines become cheaper would severely underestimate g_K

Solow Model and Regression Analyses I

- Another popular approach of taking the Solow model to data: *growth regressions*, (for those interested see the textbook by Barro and Sala-i-Martin 2004 Economic Growth) . With some algebra it can be shown that:

$$g \simeq x - (1-\alpha)(\delta + x + n)(\log y(t) - \log y^{*}(t)) . \qquad (26)$$

- Two sources of growth in Solow model: g, the rate of technological progress, and “convergence”.

Solow Model and Regression Analyses II

- Latter source, convergence:
  - Negative impact of the gap between current level and steady-state level of output per capita on rate of capital accumulation.
  - The lower is $y(t)$ relative to $y^*(t)$, hence the lower is $k(t)$ relative to k^*, the greater is $f(k^*)/k^*$, and this leads to faster growth in the effective capital-labor ratio.
- Speed of convergence in (26), measured by the term $(1-\alpha)(\delta+x+n)$, depends on:
  - $\delta+x+n$: determines rate at which effective capital-labor ratio needs to be replenished.
  - α when α is high, we are close to a linear—AK—production function, convergence should be slow.
  - Enables us to "calibrate" the speed of convergence in practice
- Focus on advanced economies
  - $g \simeq 0.02$ for approximately 2% per year output per capita growth,
  - $n \simeq 0.01$ for approximately 1% population growth and
  - $\delta \simeq 0.05$ for about 5% per year depreciation.

Solow Model and Regression Analyses III

- Using discrete time approximations, equation (26) yields:

$$g_{i,t,t-1} = b^0 + b^1 \log y_{i,t-1} + \varepsilon_{i,t}, \tag{27}$$

- $\varepsilon_{i,t}$ is a stochastic term capturing all omitted influences.
- If such an equation is estimated in the sample of core OECD countries, b^1 is indeed estimated to be negative.
- But for the whole world, no evidence for a negative b^1. If anything, b^1 would be positive.
- I.e., there is no evidence of world-wide convergence,
- Barro and Sala-i-Martin refer to this as "unconditional convergence."

Solow Model and Regression Analyses IV

- Unconditional convergence may be too demanding:
  - requires income gap between any two countries to decline, irrespective of what types of technological opportunities, investment behavior, policies and institutions these countries have.
  - If countries do differ, Solow model would *not* predict that they should converge in income level.
- If countries differ according to their characteristics, a more appropriate regression equation may be:

$$g_{i,t,t-1} = b_i^0 + b^1 \log y_{i,t-1} + \varepsilon_{i,t}, \tag{28}$$

- Now the constant term, b_i^0, is country specific.
- Slope term, measuring the speed of convergence, b^1, should also be country specific.
- May then model b_i^0 as a function of certain country characteristics.

Solow Model and Regression Analyses VI

- If the true equation is (28), (27) would not be a good fit to the data.
- I.e., there is no guarantee that the estimates of b^1 resulting from this equation will be negative.
- In particular, it is natural to expect that $Cov\left(b_i^0, \log y_{i,t-1}\right) < 0$:
  - economies with certain growth-reducing characteristics will have low levels of output.
  - Implies a negative bias in the estimate of b^1 in equation (27), when the more appropriate equation is (28).
- With this motivation some favor the notion of "conditional convergence:"
  - convergence effects should lead to negative estimates of b^1 once b_i^0 is allowed to vary across countries.

Solow Model and Regression Analyses VII

- Barro and Sala-i-Martin (2004) estimate models where b_i^0 is assumed to be a function of:
  - male schooling rate, female schooling rate, fertility rate, investment rate, government-consumption ratio, inflation rate, changes in terms of trades, openness and institutional variables such as rule of law and democracy.
- In regression form,

$$g_{i,t,t-1} = \mathbf{X}_{i,t}'\beta + b^1 \log y_{i,t-1} + \varepsilon_{i,t}, \tag{29}$$

- $\mathbf{X}_{i,t}$ is a vector including the variables mentioned above (and a constant).
- Imposes that b_i^0 in equation (28) can be approximated by $\mathbf{X}_{i,t}'\beta$.
- Conditional convergence: regressions of (29) tend to show a negative estimate of b^1.
- But the magnitude is much lower than that suggested by the computations in the Cobb-Douglas Example.

Drawbacks of Growth Regressions

- Regressions similar to (29) have not only been used to support "conditional convergence," but also to estimate the "determinants of economic growth".
- Coefficient vector $\boldsymbol{\beta}$: information about *causal effects* of various variables on economic growth.
- Several problematic features with regressions of this form. These include:
- **Many variables in $X_{i,t}$ and $\log y_{i,t-1}$, are econometrically endogenous: jointly determined with $g_{i,t,t-1}$.**
    - May argue b^1 is of interest even without "causal interpretation".
    - But if $\mathbf{X}_{i,t}$ is econometrically endogenous, estimate of b^1 will also be inconsistent (unless $\mathbf{X}_{i,t}$ is independent from $\log y_{i,t-1}$).
    - **Equation for (26) is derived for closed Solow economy.**

Challenges to the Regression Analyses I

- **Technology differences across countries are not orthogonal to all other variables.**
- $\bar{A}_j$ (not observable so it goes in the error term) is correlated with measures of s for two reasons.
    1. *omitted variable bias*: societies with high $\bar{A}_j$ will be those that have invested more in technology for various reasons; same reasons likely to induce greater investment in physical and human capital as well (a variable Mankiw- Romer and Weil (1992) focus on).
    2. *reverse causality*: complementarity between technology and physical or human capital imply that countries with high $\bar{A}_j$ will find it more beneficial to increase their stock of human and physical capital.
- Key right-hand side variables are correlated with the error term, ε_j.

Calibrating Productivity Differences: Development/Levels Accounting

- Given series for H_j and K_j and a value for α, construct "predicted" incomes at a point in time using

$$\hat{Y}_j = K_j^{1/3} \left(A_{US} H_j\right)^{2/3}$$

- where A_{US} is computed so that $Y_{US} = K_{US}^{1/3} \left(A_{US} H_{US}\right)^{2/3}$ (values for US).
- Once a series for $\hat{Y}_j$ has been constructed, it can be compared to the actual output series.
- Gap between the two series represents the contribution of technology.
- Alternatively, could back out country-specific technology terms (relative to the United States) as

$$\frac{A_j}{A_{US}} = \left(\frac{Y_j}{Y_{US}}\right)^{3/2} \left(\frac{K_{US}}{K_j}\right)^{1/2} \left(\frac{H_{US}}{H_j}\right).$$

Calibrating Productivity Differences

Calibrating Productivity Differences

Calibrating Productivity Differences

The following features are noteworthy:

1. Differences in physical and human capital still matter a lot.
2. However, differently from the regression analysis, this exercise also shows significant *technology (productivity) differences*.
3. Also interesting is the pattern that the empirical fit of the neoclassical growth model seems to deteriorate over time.
4. Problem: data on capital and labor shares across countries are not widely available. Almost all exercises use the Cobb-Douglas approach (i.e., a constant value of α_K equal to $1/3$).

Endogenous Growth

In Solow technical progress is exogenous, endogenous growth theory suggests that growth is generated by endogenous technical change resulting from innovation.
Assuming constant technological change, potential for growth can be boundless surmounting even limits to resources.
Technology depends on knowledge, and the more knowledge there is in an economy, the higher the growth rate.
Unlike human capital, human knowledge is not embodied in people but can be transmitted across time and space.
Technology and knowledge develop through investing in research, basic and applied, done by firms or governments.
Knowledge can spillover, grow and accumulate indefinitely. It is only constrained by the number of people producing it.

Knowledge as a Non-Rival Good

Knowledge is a non-rival good and can still be used by the seller even after it is sold.

Knowledge is also often an excludable good. Enforced property rights (such as patents and copyrights), for example, prevent access to knowledge.

Such exclusions can be important because people and firms need to have an incentives (monopoly rents) to invest in research and development.

Suppose the units of a newly invented good were sold at their marginal cost (which would happen in perfect competition if the design for the good were public knowledge so any firm produce the good). This price would not cover the R&D costs that were necessary for developing the prototype.

Imperfect competition is necessary to encourage growth. (Recall that the Solow model assumes perfect competition.)

A balance must be struck between granting temporary monopoly rights and promoting competition over the long run.

Basic Equations of the Romer Model

Since knowledge (A_t) depends on the allocation of labor to research, it is endogenous to the model and can vary over time.
The increment of knowledge is given by the stock of knowledge multiplied by the labor force in the research sector L_A:

$$L = L_Y + L_A$$

$$\Delta A_t = BA_{t-1}L_A$$

Notice however that growth will vanish asymptotically if $\Delta A_t = BA_{t-1}^{\beta}L_A, \beta < 1$.
Hypothesis of CRS to knowledge in knowledge production function crucial for unbounded growth.
Condition for unbounded endogenous growth: CRS to a reproducible factor of production. That factor was simply K in the AK model.

Key Differences with Solow

Innovation is endogenous in the Romer model, depending on stock of knowledge and the R&D needed to build that knowledge. Innovation is exogenous in the Solow model.

In the Romer Model, imperfect competition is necessary for innovation and growth, while the Solow model assumes perfect competition.

Solow predicts that countries with low capital intensity should grow faster (convergence). In Romer there is no convergence mechanism. Rich countries will grow faster than poor countries if they invest more in R&D and if poor countries cannot copy innovations.

The Romer model (as the Solow Model) represents a closed economy. But huge role for trade.1) creates bigger markets for new goods 2) allows international diffusion of technology by imports of capital goods, backward engineering etc.