Microconomics I, 2024/2025
Master of Science in Economics
Problem Set 5

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Question 1 Solve Exercise 15.B.1 of Mas Colell-Whinston-Green.

Question 2 Solve Exercise 16.C.2 of Mas Colell-Whinston-Green.

Question 3 Let $\mathcal{E} = \{u_i, \omega_i\}_{i=1}^I$ be an exchange economy with L commodities. Show that if the allocation $\overline{x}$ is a solution for

$$\begin{array}{lll} \max_x & \lambda_1 u_1(x_1) + \ldots + \lambda_I u_I(x_I) & \\ s.t. & \sum_{i=1}^I x_i = \sum_{i=1}^I \omega_i & \qquad (PC), \\ & x_i \geq 0, i = 1, \ldots, I & \end{array}$$

where $\lambda_i > 0$ for all $i \in \{1, \ldots, I\}$, then $\overline{x}$ is Pareto efficient.

Obs.: Assume that u_i is concave for all i. In this case, you can show that if $\overline{x}$ is Pareto efficient, then there exists a vector $(\lambda_1^*, \ldots, \lambda_I^*) > 0$ such that $\overline{x}$ is a solution to (PC) when $\lambda_i = \lambda_i^*$ for $i = 1, \ldots, I$.

Question 4 Consider the $I = L = 2$ exchange economy where

$$\begin{array}{ll} u_1(x_{11}, x_{21}) = x_{11}x_{21} & \text{and } \omega_1 = (18, 4) \\ u_2(x_{12}, x_{22}) = \ln x_{12} + 2\ln x_{22} & \text{and } \omega_2 = (3, 6). \end{array}$$

1. Find a Walrasian equilibrium of this economy.
2. Find the set of all Pareto efficient allocations.

Question 5 Consider the production economy $\left(\{X_i, \succsim_i\}_{i=1}^I, \{Y_j\}_{j=1}^J, \{\theta_{i1}, \ldots, \theta_{iJ}\}_{i=1}^I, (\overline{\omega}_1, \ldots, \overline{\omega}_L)\right)$. We say the list $(\overline{x}, \overline{y}, p^*)$, where $(\overline{x}, \overline{y})$ is an allocation and $p^* > 0$ is a price vector, is an equilibrium with transfers if there exists $(w_1, \ldots, w_I) \gg 0$ with

$$\sum_{i=1}^I w_i = p^* \sum_{i=1}^I \omega_i + p^* \sum_{j=1}^J \overline{y}_j$$

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such that:
(i) for all $j \in \{1, \ldots, J\}$, $p^* \cdot y_j \leq p^* \cdot \overline{y}_j$ for all $y_j \in Y_j$;
(ii) for all $i \in \{1, \ldots, I\}$, $\overline{x}_i$ is a solution for

$$\begin{array}{ll} \max & u_i(x_i) \\ s.t. & p^* \cdot x_i \leq w_i \\ & x_i \geq 0 \end{array}$$

(iii) $\sum_{i=1}^{I} \overline{x}_i = \sum_{i=1}^{I} \omega_i + \sum_{j=1}^{J} \overline{y}_j$.

Show that every Walrasian equilibrium of $\mathcal{E}$ is an equilibrium with transfers.

Question 6 Let $\mathcal{E} = \{u_i, \omega_i\}_{i=1}^{I}$ be an exchange economy with L commodities. Show that if the allocation $\overline{x}$ is a solution for

$$\begin{array}{ll} \max_x & \lambda_1 u_1(x_1) + \ldots + \lambda_I u_I(x_I) \\ s.t. & \sum_{i=1}^{I} x_i = \sum_{i=1}^{I} \omega_i \qquad\qquad (PC), \\ & x_i \geq 0, i = 1, \ldots, I \end{array}$$

where $\lambda_i > 0$ for all $i \in \{1, \ldots, I\}$, then $\overline{x}$ is Pareto efficient.

Obs.: Assume that u_i is concave for all i. In this case, you can show that if $\overline{x}$ is Pareto efficient, then there exists a vector $(\lambda_1^*, \ldots, \lambda_I^*) > 0$ such that $\overline{x}$ is a solution to (PC) when $\lambda_i = \lambda_i^*$ for $i = 1, \ldots, I$.