

Advanced Microeconomics 2025/2026
Master of Science in Economics
Problem Set 5

05/12/2025

Question 1 Suppose that $f(\cdot)$ is the production function associated with a single-output technology, and let Y be the production set of this technology. Show that if Y satisfies constant returns to scale, $f(\cdot)$ is homogeneous of degree one.

Question 2 Derive the profit function $\pi(p)$ and the supply function $y(p)$ for the single-output technology with production function $f(z) = \sqrt{z_1 + z_2}$.

Question 3 Derive the cost function $C(w, q)$ and the conditional factor demands $z(w, q)$ for the following single-output constant returns to scale technology with production function given by $f(z_1, z_2) = z_1 + z_2$.

Question 4 Consider the single-output technology with production function $f(z) = z_1^\alpha z_2^\beta$ with $\alpha \geq 0$ and $\beta \geq 0$ and answer the following questions.

- Set up and solve the profit maximisation problem, identify the supply correspondence $y(p)$ and the profit function $\pi(p)$ and its dependence on the parameters α and β .
- Evaluate the returns to scale of the production function above for different values of the parameters α and β .
- Derive the conditional factor demands $z(w, q)$ for the same technology, then construct the cost function $C(w, q)$ and explain how it varies in relation to the parameters α and β .

Question 5 Consider the single-output model represented by the production function $f(z_1, z_2) = 4z_1 + 2z_2$. Determine the returns to scale of this productive technology. Explain why the profit function of any firm with such technology can be either $\pi(p) = 0$ or $\pi(p) = +\infty$.