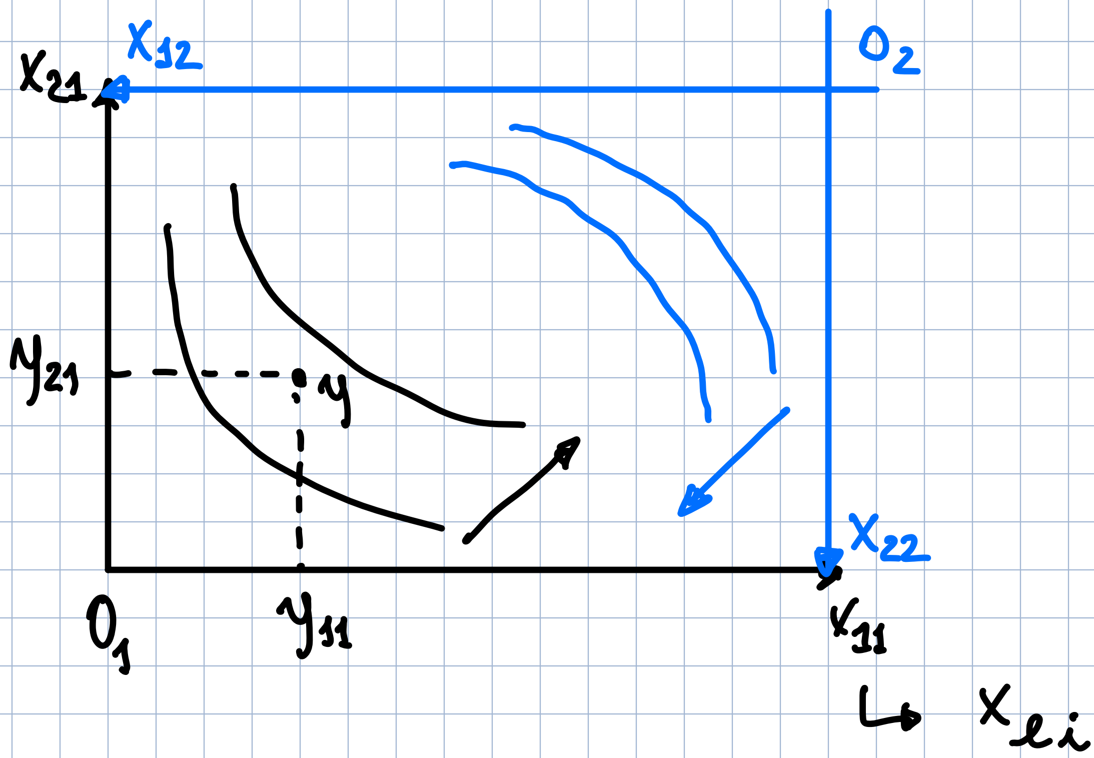
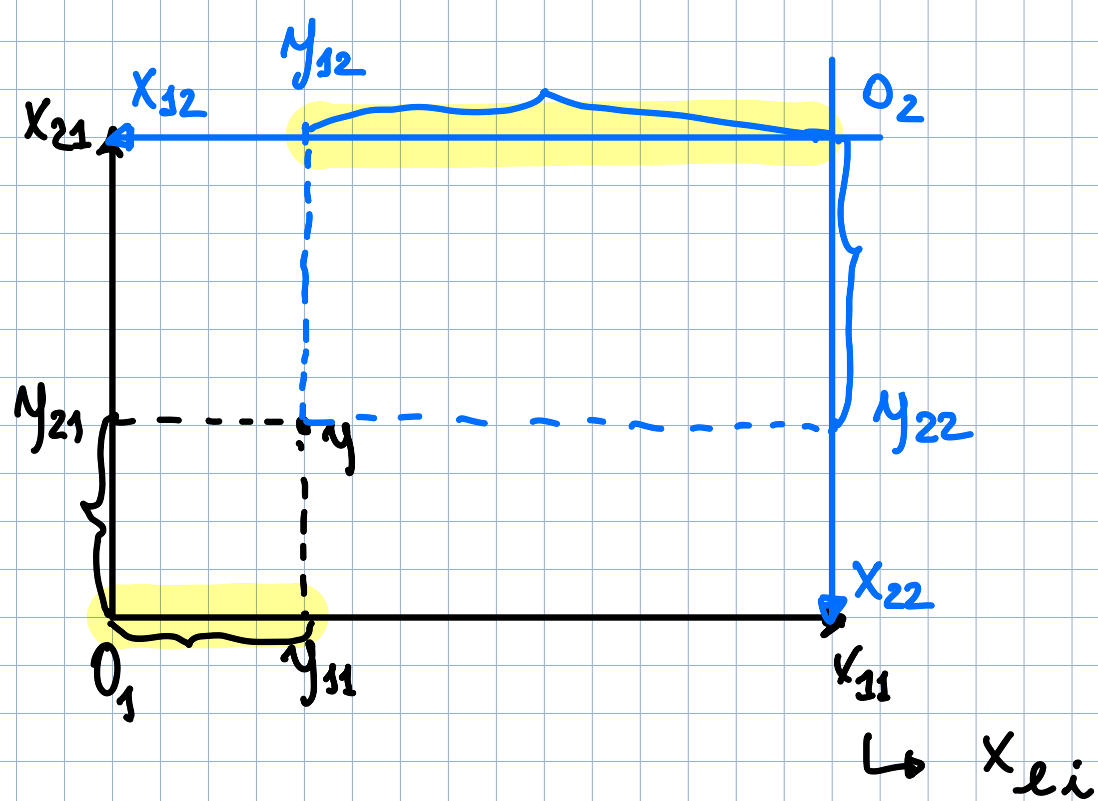
GE SETTING WITH $I = L = 2$

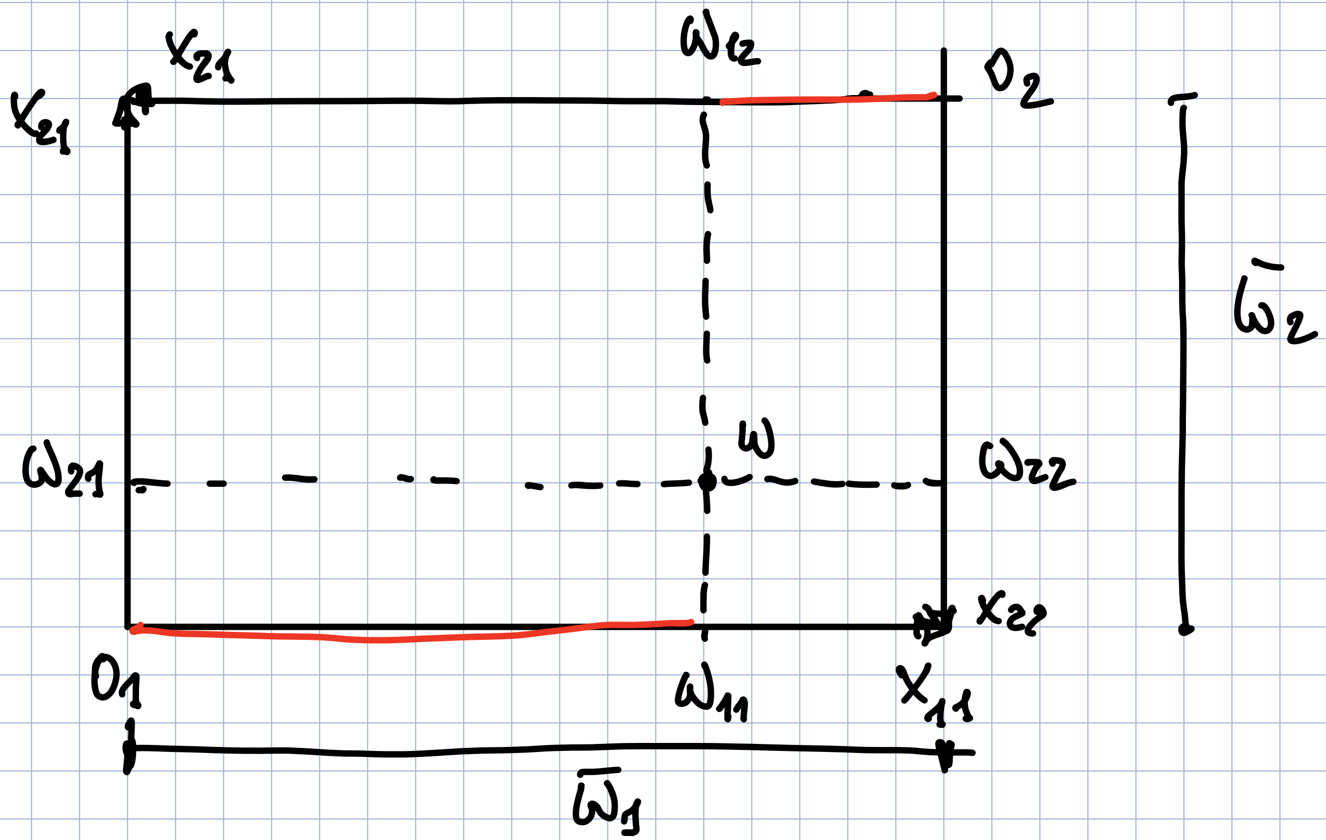




$y = (y_1, y_2) = ((y_{11}, y_{21}), (y_{12}, y_{22}))$ is an allocation

$\bar{\omega}_1 = \omega_{11} + \omega_{12}$ aggregate endowments of commodity 1 and 2

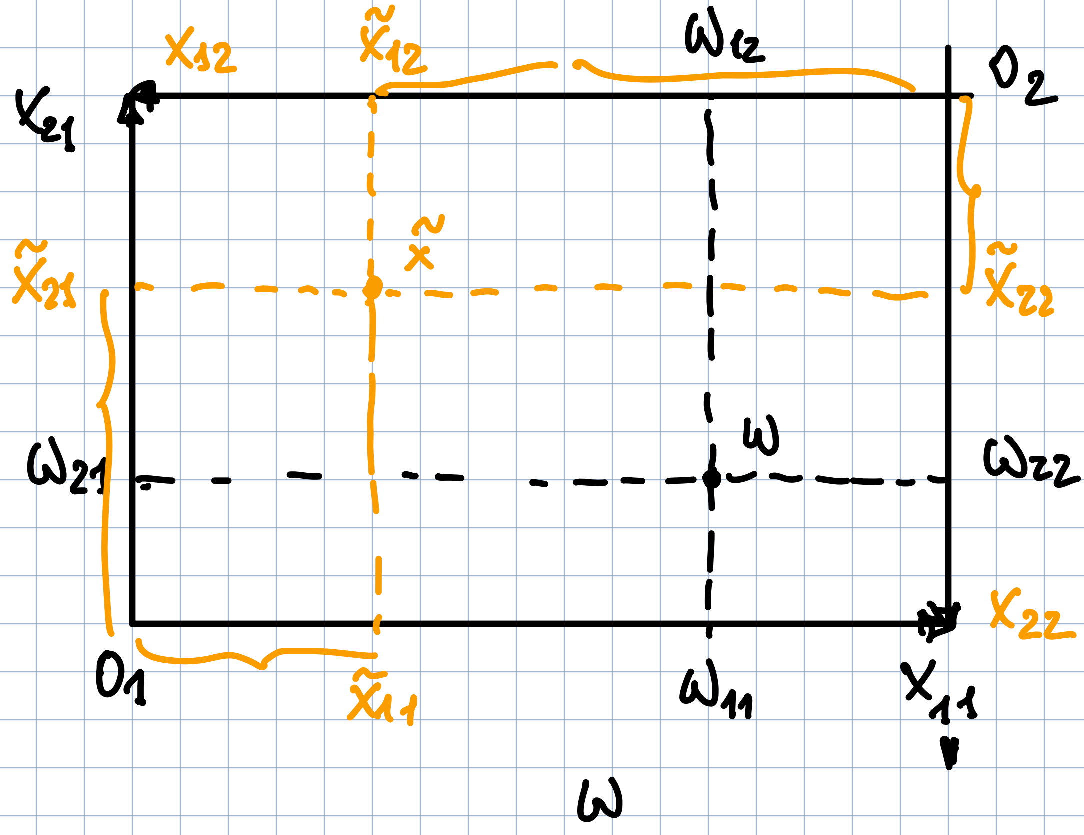
$\bar{\omega}_2 = \omega_{21} + \omega_{22}$ $\omega_{\ell i}$



EDGEWORTH BOX is a tool to analyse exchange economies $\mathcal{E} = \{u_i, \omega_i\}_{i=1}^2$ with $I = L = 2$

Each allocation in the Box satisfies the no-waste property, i.e.:

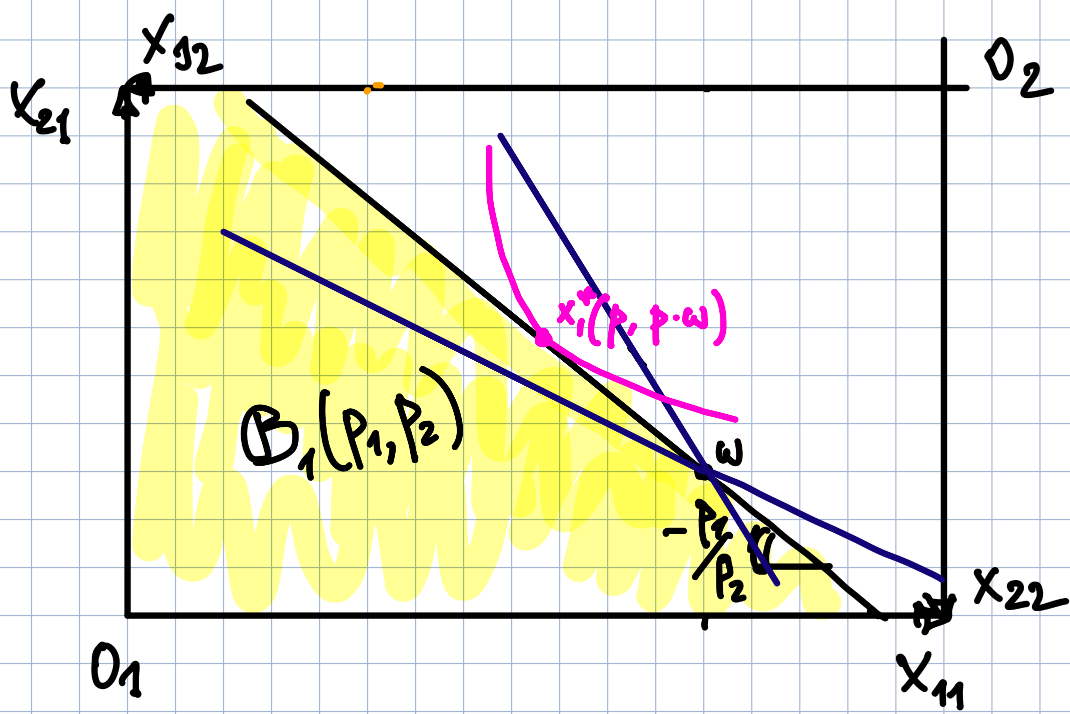
$$x_{\ell 1} + x_{\ell 2} = \bar{\omega}_\ell \qquad \forall\, \ell = 1,2$$



$$\tilde{x} = (\tilde{x}_1, \tilde{x}_2) = ((\tilde{x}_{11}, \tilde{x}_{21}), (\tilde{x}_{12}, \tilde{x}_{22}))$$

$\forall \ell, \forall i \qquad \tilde{x}_{12} = \bar{\omega}_1 - \tilde{x}_{11}$

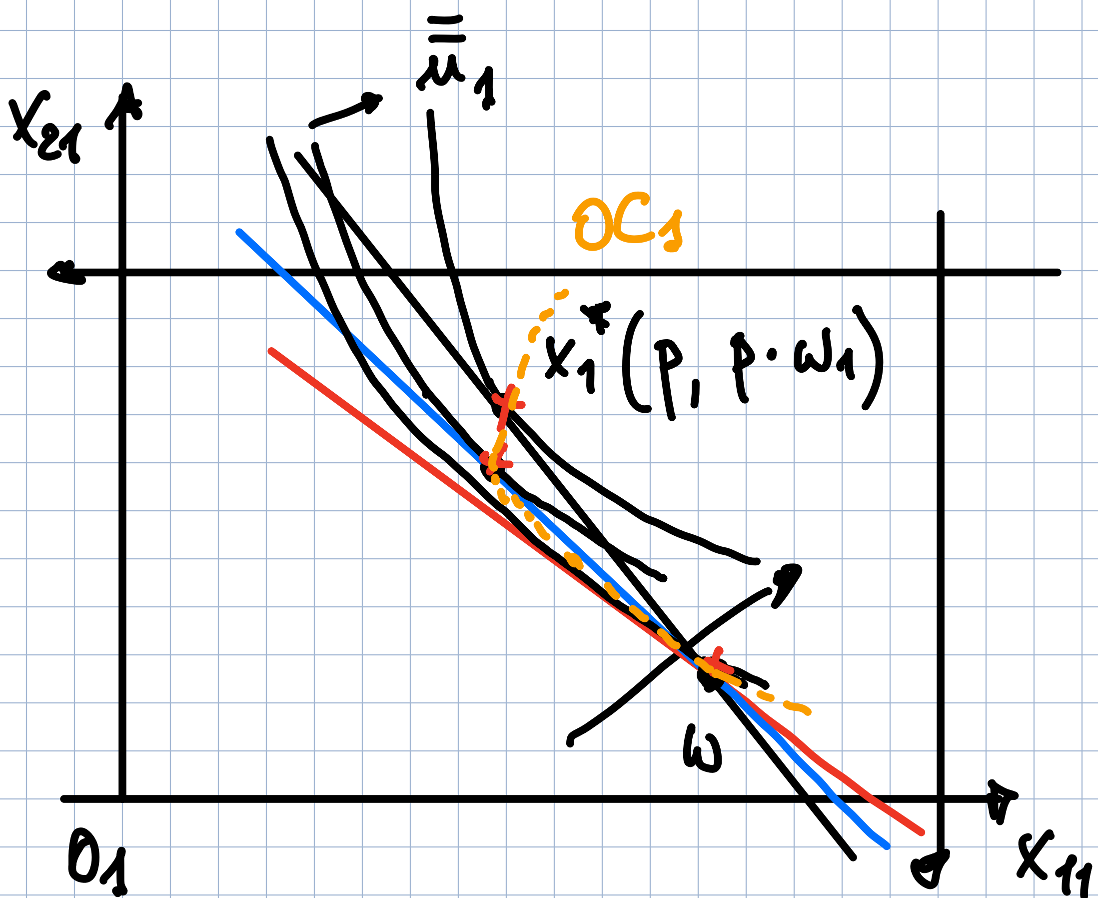
Let $\succsim_i$ be continuous, strictly convex and strongly-monotone



$$B_i(p_1, p_2) = \{ x_i \in \mathbb{R}^2_+ : p \cdot x_i \leq p \cdot \omega_i \}$$

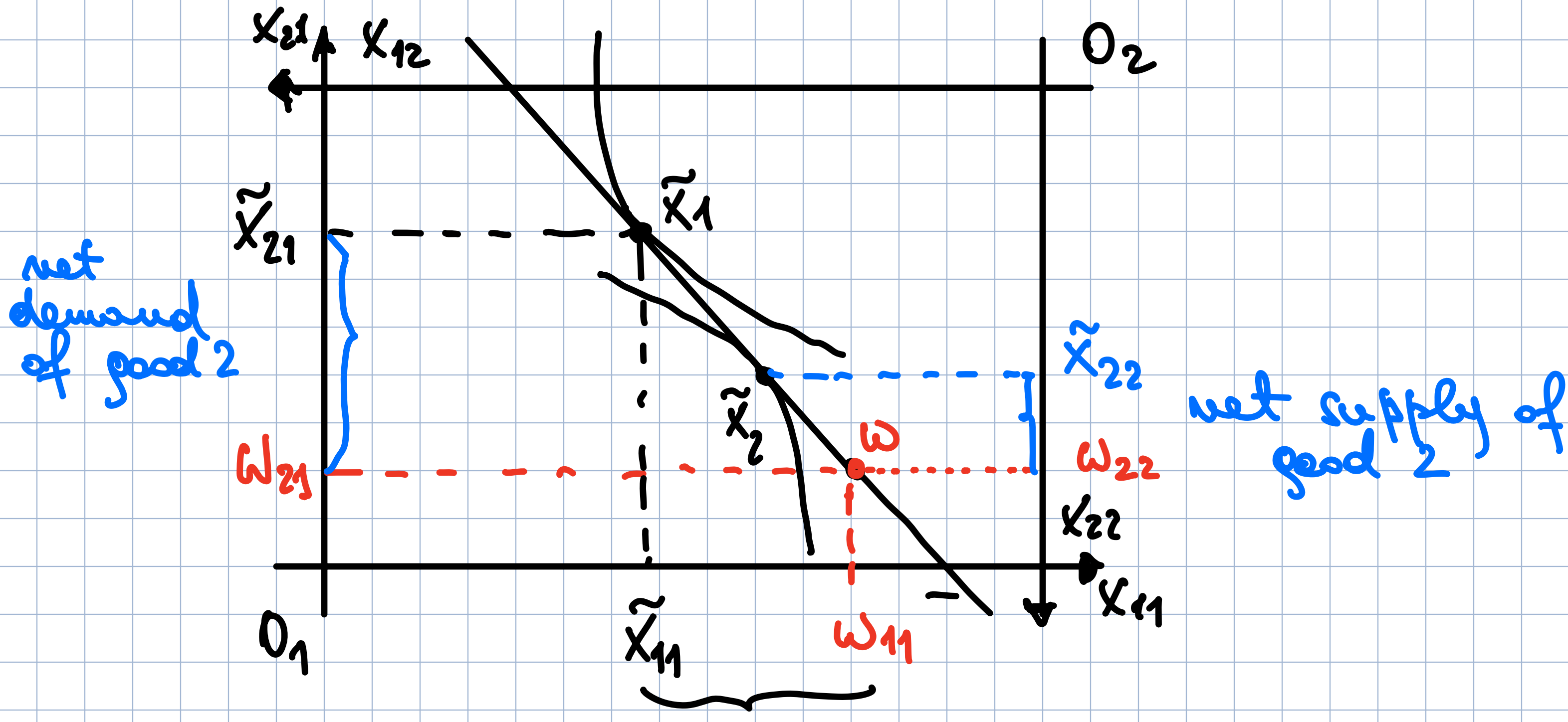
$$p_1 x_{1i} + p_2 x_{2i} \leq p_1 \omega_{1i} + p_2 \omega_{2i} \quad \forall\, i = 1,2$$

Consider how $x_1^*(p, p\cdot\omega_1)$ changes as p change $\Rightarrow$ the collection/set of $x_1^*(p, p\cdot\omega_1)$ $\forall\ p \gg 0$ is the Offer curve of consumer 1 (OC_1).



Since consumer 1 can always consume her endowment the OC_1 is in the upper-contour set of ω_1, and it is tangent to the indifference curve that passes through the endowment ω_1.

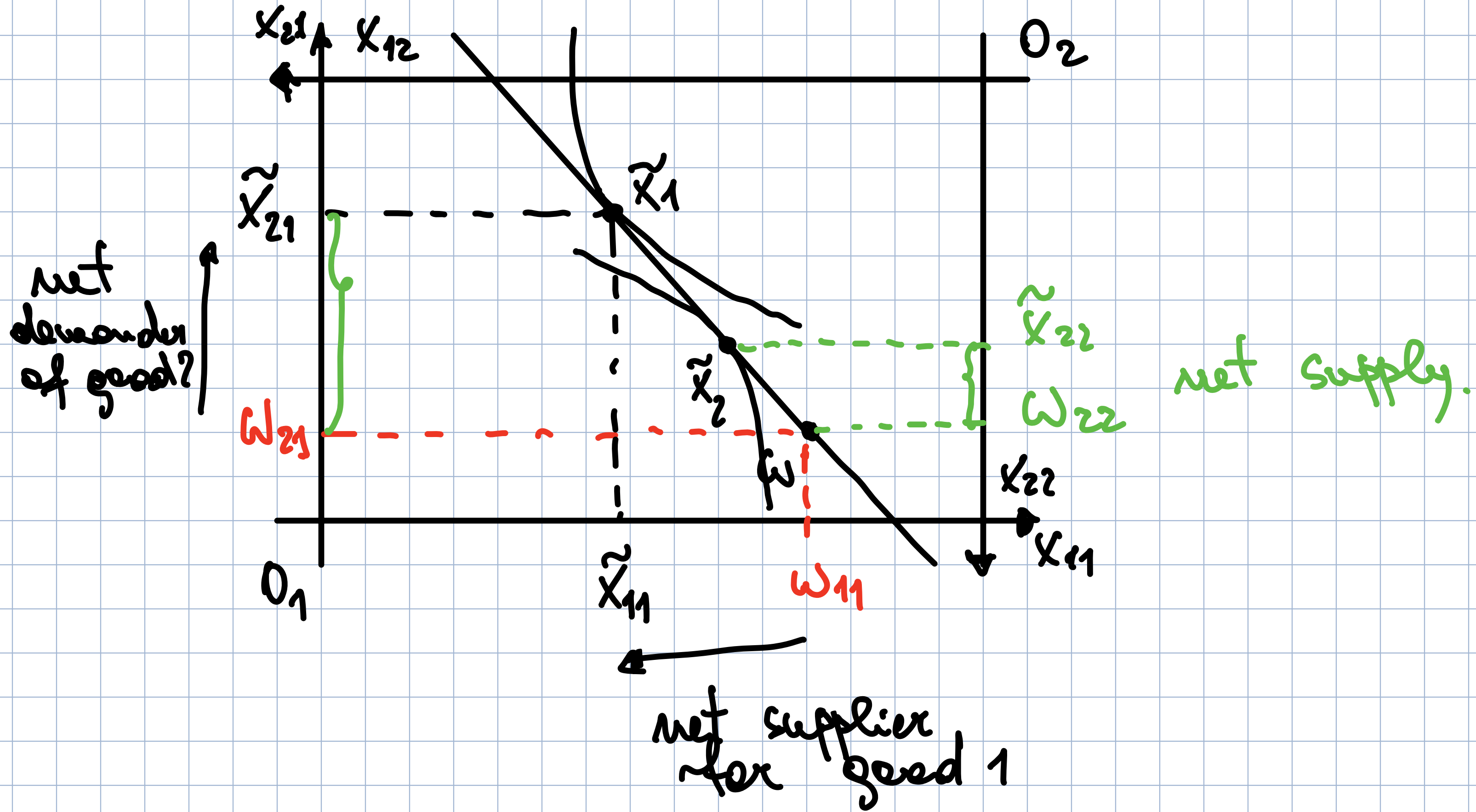
Consider now both consumers and a possible allocation $\tilde{x} = (\tilde{x}_1, \tilde{x}_2)$ as represented below, with prices $(\tilde{p}_1, \tilde{p}_2)$.



since $\tilde{x}_{11} - \omega_{11} < 0$

Consumer 1 is a net supplier of good 1

Is the pair $(\tilde{p}, \tilde{x})$ a Walrasian equilibrium?



A Walrasian equilibrium in the Edgeworth is a profile of prices p^* and an allocation $x^* = (x_1^*, x_2^*)$ s.t. $\forall\, i = 1,2$

$$x_i^* \succsim x_i' \quad \forall \quad x_i' \in B_i(p^*)$$

At x_i^* : $p^* \cdot x_i^* = p^* \cdot \omega_i \quad \forall\, i = 1,2$

$$\begin{cases} p_1^* x_{11}^* + p_2^* x_{21}^* = p_1^* \omega_{11} + p_2^* \omega_{21} \\ p_1^* x_{12}^* + p_2^* x_{22}^* = p_1^* \omega_{12} + p_2^* \omega_{22} \end{cases}$$

Walras' Law holds $\forall i$

Summing the budget constraints over i, we get

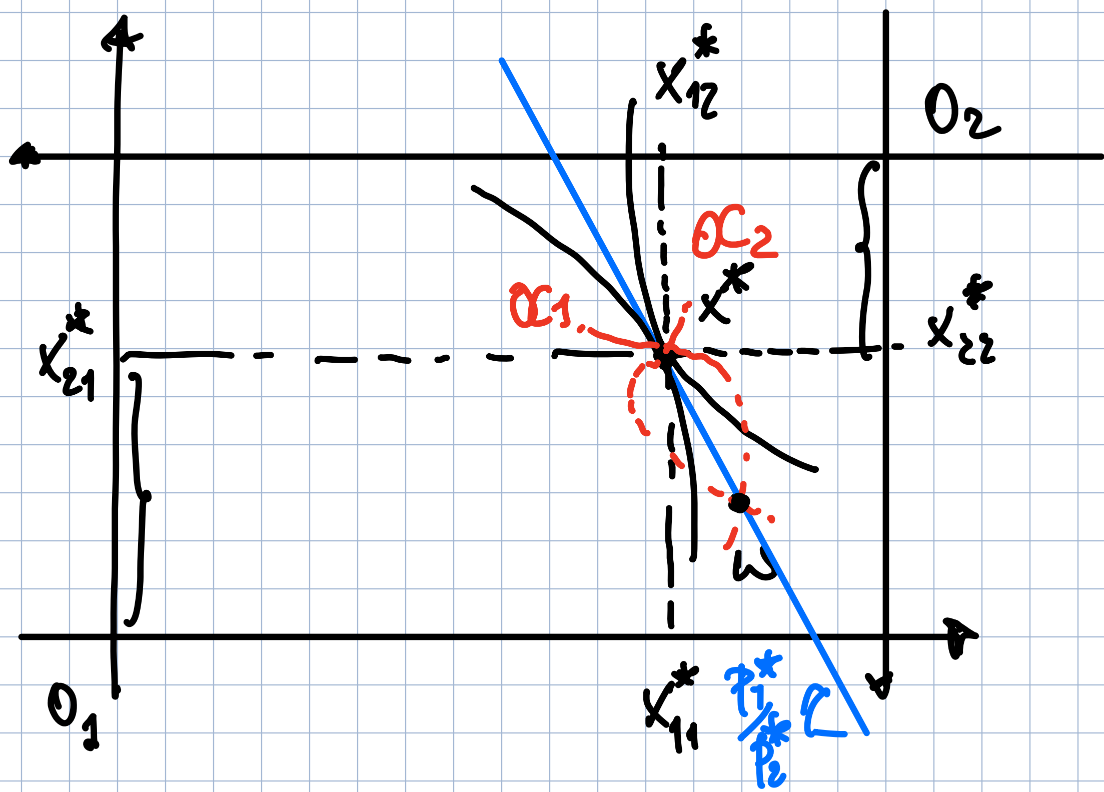
$$p_1^* (x_{11}^* + x_{12}^*) + p_2^* (x_{21}^* + x_{22}^*) = p_1^* (\overbrace{\omega_{11} + \omega_{12}}^{\bar{\omega}_1}) + p_2^* (\underbrace{\omega_{21} + \omega_{22}}_{\bar{\omega}_2})$$

$$p_1^* \underbrace{\left(x_{11}^* + x_{12}^* - \bar{\omega}_1 \right)}_{\text{no excess demand}} + p_2^* \left(x_{21}^* + x_{22}^* - \omega_2^* \right) = 0$$

or mkt clearing condition.

At $(\tilde{x}, \tilde{p})$ the mkt clearing condition does not hold.

(x^*, p^*) are a Walrasian equilibrium



If (x^*, p^*) is a Walrasian eq. then $(x^*, \alpha p^*)$ with $\alpha > 0$ is also a Walrasian eq.

↳ relative prices matter

Ex. $I = L = 2$

$$u_i(x_{1i}, x_{2i}) = x_{1i}^{\alpha} \, x_{2i}^{1-\alpha} \qquad \alpha \in (0,1)$$

$$\omega_1 = (1, 2)$$

$$\omega_2 = (2, 1)$$

For the exchange economy above identify a Walrasian equilibrium.

for $\ell = 1$ $\quad x_{1i}(p, w_i) = \dfrac{\alpha w_i}{p_1}$ $\quad$ with $w_i = p \cdot \omega_i \ \forall i$

$$w_1 = p_1 \cdot 1 + p_2 \cdot 2 = p_1 + 2p_2$$

$$w_2 = p_1 \cdot 2 + p_2 \cdot 1 = 2p_1 + p_2$$

$$x_{11}^*(p, p \cdot \omega_1) = \frac{\alpha (p_1 + 2p_2)}{p_1}$$

$$x_{21}^*(p, p \cdot \omega_1) = \frac{(1-\alpha)(p_1 + 2p_2)}{p_2}$$

$$x_{12}^*(p, p \cdot \omega_2) = \frac{\alpha (2p_1 + p_2)}{p_1}$$

$$x_{22}^*(p, p \cdot \omega_2) = \frac{(1-\alpha)(2p_1 + p_2)}{p_2}$$

$$x_{11}^*(p, p\cdot\omega_1) + x_{12}^*(p, p\cdot\omega_2) = \bar{\omega}_1$$

$$\frac{\alpha(p_1 + 2p_2)}{p_1} + \frac{\alpha(2p_1 + p_2)}{p_1} = 3$$

$$\alpha + 2\alpha\frac{p_2}{p_1} + \alpha 2 + \alpha\frac{p_2}{p_1} = 3$$

$$\cancel{3}\alpha + \cancel{3}\alpha\frac{p_2}{p_1} = \cancel{3}^{\,1}$$

$$\frac{p_2^*}{p_1^*} = \frac{1-\alpha}{\alpha}$$

check that the market for good 2 also clears at $\frac{p_2^*}{p_1^*} = \frac{1-\alpha}{\alpha}$.

What is a Pareto efficient allocation in the Edgeworth box?

An allocation x is Pareto efficient in the Edgeworth box if there exists no other allocation x' (in Edgeworth box) such that

$$x_i' \succsim x_i \quad \forall\ i = 1,2$$

and

$$x_i' \succ x_i \quad \text{for some } i$$

- RECALL -

A Walrasian equilibrium in the Edgeworth is a profile of prices p^* and an allocation $x^* = (x_1^*, x_2^*)$ st. $\forall\ i = 1,2$

$$x_i^* \succsim x_i' \quad \forall \quad x_i' \in B_i(p^*)$$

$$x_1^*(p^*, p^* \cdot \omega_1) \in OC_1$$

$$x_2^*(p^*, p^* \cdot \omega_2) \in OC_2$$

An allocation x is Pareto efficient in the Edgeworth box if there exists no other allocation x' (in Edgeworth box) such that

$$x_i' \succsim x_i \quad \forall \ i = 1,2$$

and

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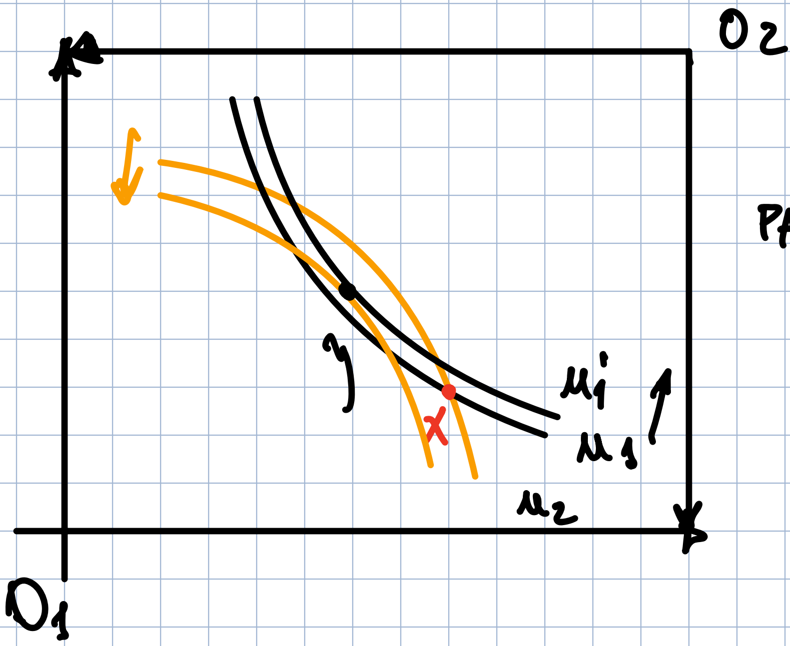
INEFFICIENT ALLOCATIONS



x is NOT PARETO EFFICIENT

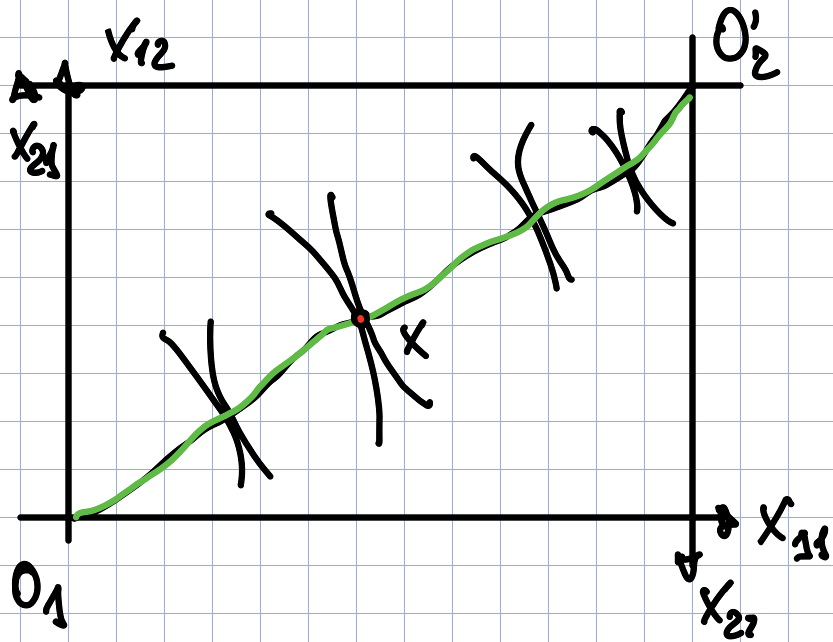
y PARETO DOMINATES x

$$u_1(y_{11}, y_{21}) > u_1(x_{11}, x_{21})$$

&

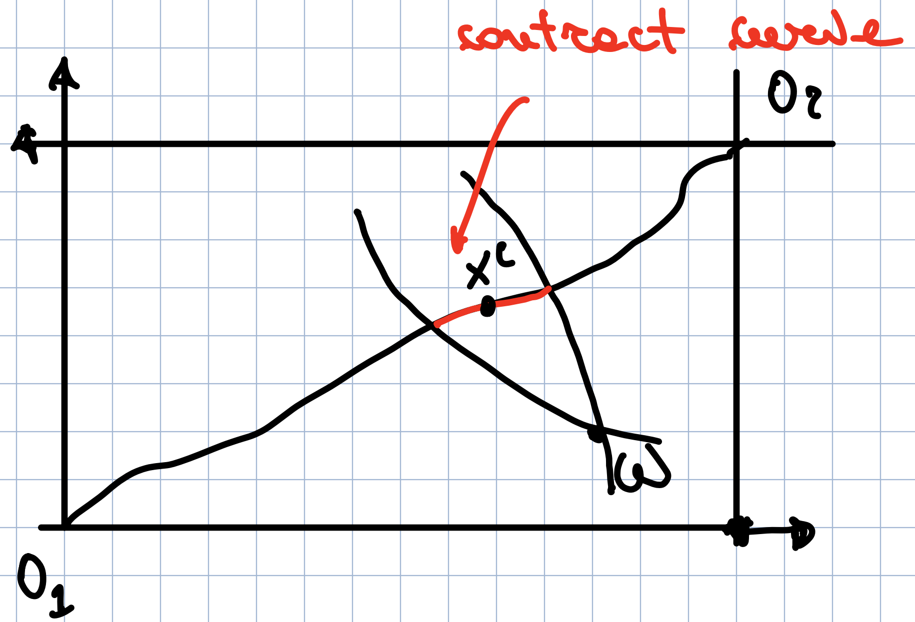
$$u_2(y_{12}, y_{22}) > u_2(x_{12}, x_{22})$$

y Pareto dominates x $\Rightarrow$ x is NOT Pareto efficient.



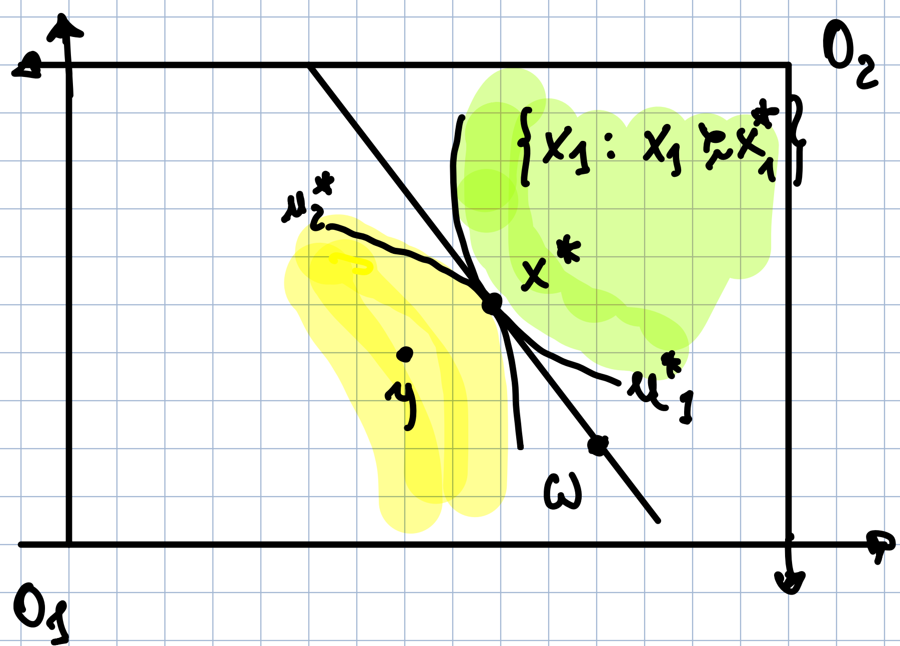
x is Pareto efficient.

the Pareto set is the set of Pareto efficient allocations of an economy.



$x_i^c \succsim_i \omega_i$

The contract curve is the subset of the Pareto set in which each consumer obtains a utility which is at least equal to her autarkic utility, $u_i(\omega_i)$.



Any Walrasian equilibrium allocation x^* belongs to the Pareto set.

Def. An allocation x^* in the Edgeworth box is supportable as an equilibrium with transfers if $\exists$

- a price vector p^*,
- a system of transfers T_1, T_2 s.t.

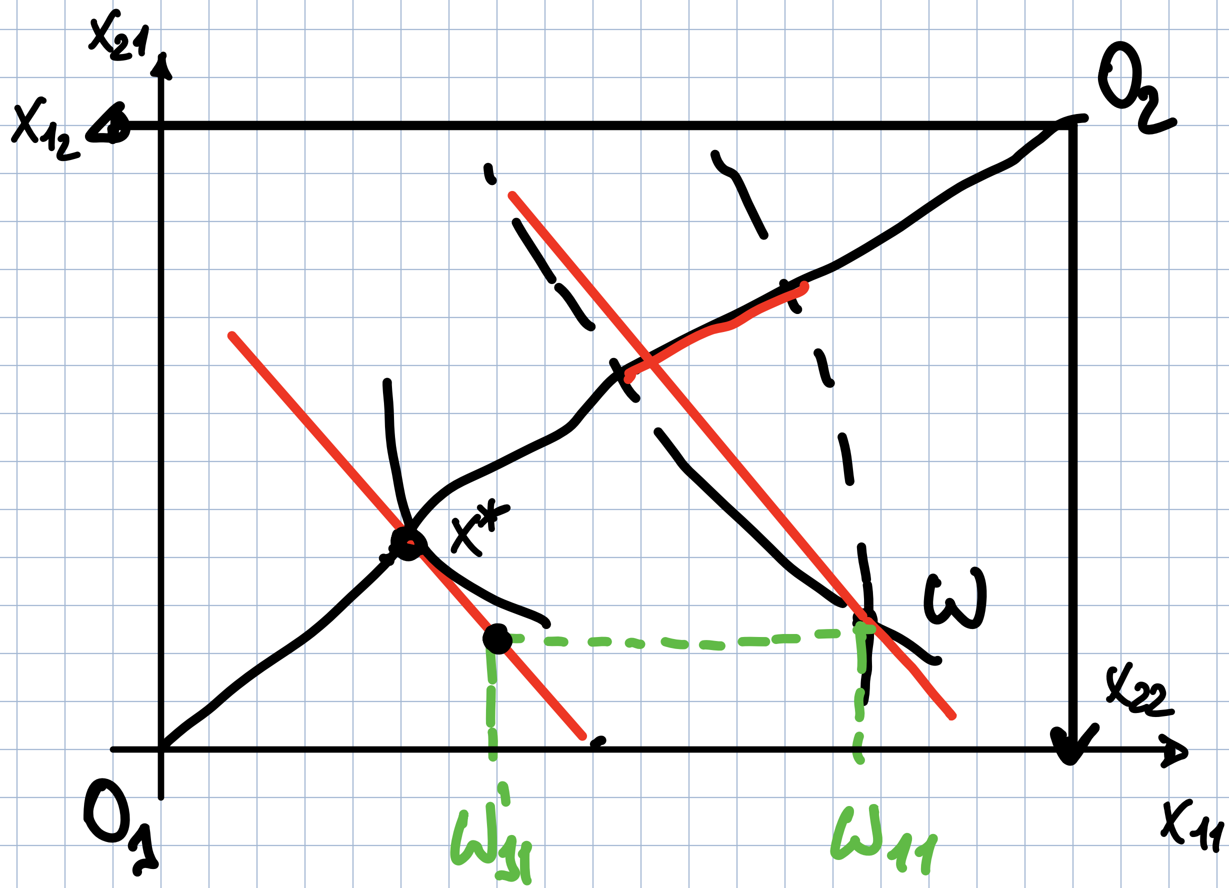
$$T_1 + T_2 = 0$$

such that

$$x_i^* \succsim x_i' \qquad \forall\ x_i' \in \mathbb{R}^2_+$$

$$\text{s.t. } p^* \cdot x_i' \leq p^* \cdot \omega_i + T_i \quad \forall i$$

If $\succsim_i$ are continuous, strictly convex and strongly monotone, then any Pareto efficient allocation is supportable as an equil. with transfers.



Exercise.

Find the contract curve / Pareto set in an Edgeworth box, with preferences given by

$u_i(x_{1i}, x_{2i}) =$

and endowments equal to

$\omega_1 = (\omega_{11}, \omega_{21}) = ($

$\omega_2 = (\omega_{12}, \omega_{22}) = ($