The gravity equation in international trade

Feenstra, chapters 5, 6
Allen and Arkolakis (Class notes), chapters 3, 4

Lecture outline

- The empirical gravity equation
- The theoretical foundations
  - The gravity equation under perfect competition (the Armington model)
  - The gravity equation under monopolistic competition (the Krugman model)
  - The gravity equation under monopolistic competition with firm heterogeneity (the Melitz model)
  - The gravity equation in the Ricardian models (the Eaton and Kortum model)

The empirical relation

Newton's universal law of gravitation

- The force of gravity (F) between two objects 1 and 2 is given by

$$\mathbf{F} = \mathbf{G}\,\frac{\mathbf{M_1 \cdot M_2}}{\mathbf{d^2}},$$

where G is the gravitational constant, M_1 and M_2 are the masses of the two objects, and d is the distance between them

- "*The larger each of the objects is, or the closer they are to each other, the greater the force of gravity between them*"

The simplest gravity equation

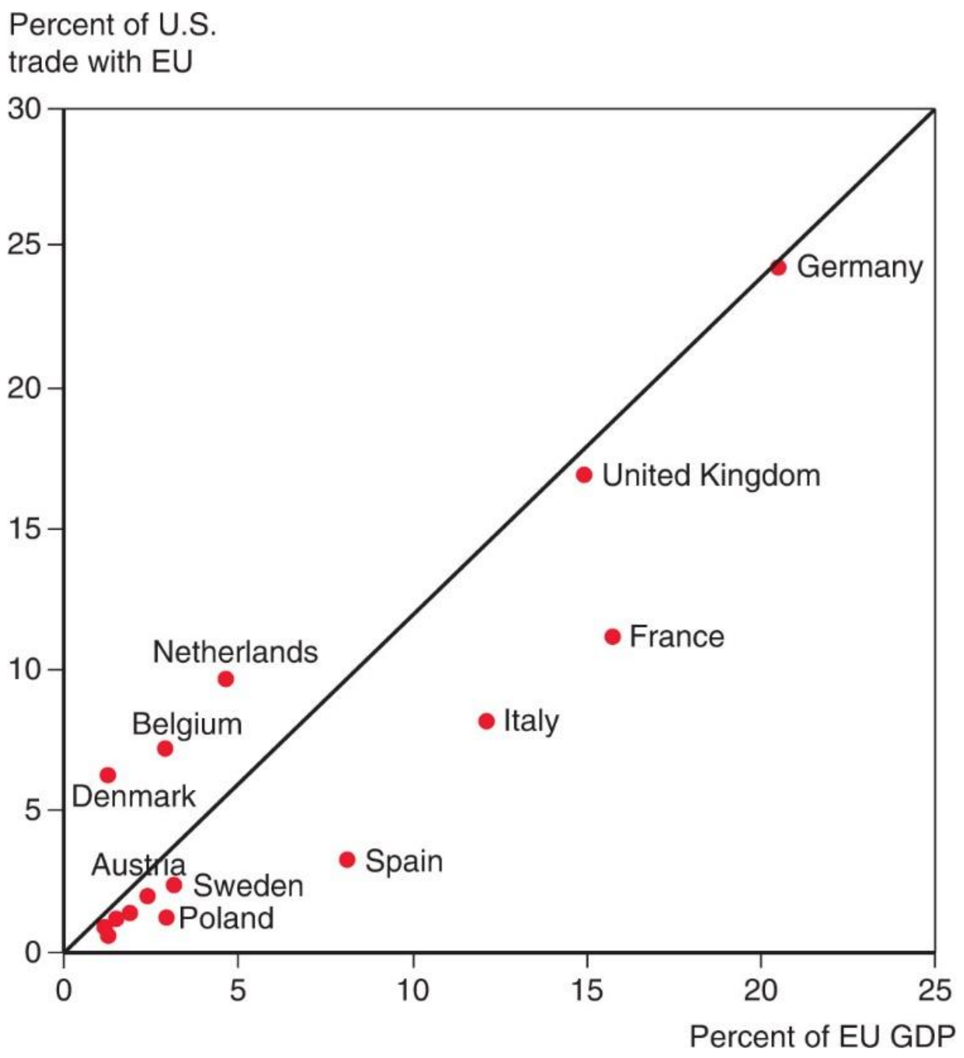
- Timbergen (1962) suggested that the volume of trade between two countries is similar to the force of gravity between two objects, that is,

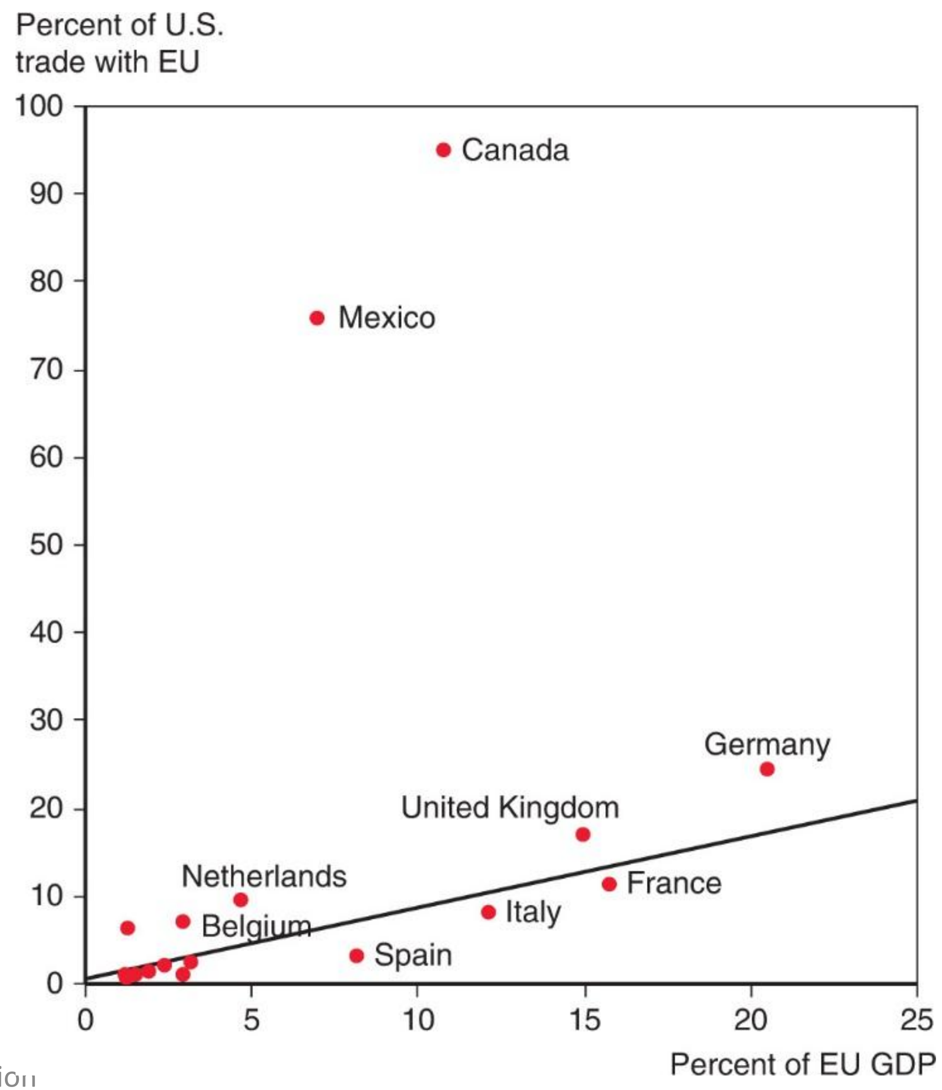
$$\mathbf{X}_{\mathrm{ij}} = \mathbf{A}\,\frac{\mathbf{Y}_{\mathrm{i}} \cdot \mathbf{Y}_{\mathrm{j}}}{\mathbf{d}^{\boldsymbol{\rho}}} \quad (\textit{Gravity})$$

where X_{ij} is the amount of trade (imports, exports or their sum) between country i and country j, Y_{i} and Y_{j} are their GDPs, d is the distance between them, A and ρ are two positive constant

- *The (economically) larger each of the countries is, and/or the closer they are to each other, the greater the amount of trade between them*

Size and distance matter





The simplest gravity equation

- Log-linearizing (*gravity*) we obtain the expression for the simplest regression that can be estimated

$$lnX_{ij} = \alpha + \beta_i \ln Y_i + \beta_j lnY_j + \rho\ ln\ d_{ij} + \varepsilon_{ij}\ ,$$

where we have added coefficients β_i and β_{j} to the GDPs

- More complex versions have been estimated to account for the existence of international trade barriers (such as transport costs or tariffs) (see, e.g., Anderson, 1979; Redding and Venables, 2004)
- When applied to regions (rather than countries), one should take into account the existence of powerful border effects (McCallum, 1995), that is, the idea that, other things equal (e.g., distance, size etc.), intranational trade is much larger than international trade
    - McCallum (1995) has introduced a dummy for intra/international trade for Canada vis a vis the US: the dummy is significant and big

The generalized gravity equation

- A generalized version of the gravity equation writes as

$$\mathbf{X_{ij} = K_{ij}\gamma_i\delta_j}$$

where $\boldsymbol{\gamma}_{\mathrm{i}}$ measures the size of the origin (exporting country), $\boldsymbol{\delta}_{\mathrm{j}}$ measures the size of the destination (importing country) and $\mathrm{K_{ij}}$ is a measure of resistance of trade between i and j

- Note that resistance here captures not only distance but also transport costs, trade policy measures and other frictions to international trade

The generalized gravity equation

Once we introduce several measures of trade resistance as controls, such as:

- Distance between markets
- Cultural affinity: close cultural ties, such as a common language
- Geography: ocean harbors and a lack of mountain barriers
- Political borders: crossing borders involves formalities, often different currencies, tariffs etc.
- Trade agreements

the gravity model explains bilateral trade patterns remarkably well!

But what are the theoretical foundations of this equation?

Theoretical foundations:
the gravity equation under perfect competition (the Armington model)

Foundations of the gravity equation

- The gravity equation presented above has become one of the most successful empirical relationships in economics
  - It explains a large fraction of the variation in observed bilateral trade flows
- It, however, represents a statistical (a-theoretical) relationship
- We now discuss some (successful) attempts to provide a solid theoretical foundation to that relationship
- We introduce the Armington model as formalized by Anderson (1979)
- Essentially, the gravity equation is obtained from the solution to the consumption maximization problem under the hp. of CES preferences

Environment and hypotheses

- S countries and $\omega \in \Omega$ varieties
- Hp1: perfect specialization: each country produces a different good
  - This hp. is compatible with both classical (Ricardian or H-O) models and new trade (Krugman or Melitz-like) models with intra-industry trade, where each variety is produced by no more than a single country (complete specialization)
- Hp2: CES preferences over varieties
  - This hp. also is compatible with both classical models and new trade models
  - Remark: the original Anderson (1979) actually assumed Cobb-Douglas preferences
- Hp3: perfect competition
  - Later we present a micro-foundation of the gravity equation in a monopolistic competition context (Krugman- or Melitz-like)

The demand side

Reminder: the consumption maximization problem under CES preferences

- In country j (the destination country), agents derive utility from consuming goods shipped from any country $\mathrm{i} \in S$ (the origin country)
- The optimal problem is that of finding the consumption bundle $\{\mathrm{c_{ij}}(\omega)\}_{\omega\in\Omega}$ maximizing

$$\mathrm{U_j} = \left[\sum\nolimits_{\omega\in\Omega} \mathrm{c_{ij}}(\omega)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \quad \text{s.t.} \ \sum\nolimits_{\omega\in\Omega} \mathrm{c_{ij}}(\omega)\mathrm{p_{ij}}(\omega) \leq \mathrm{X_j}$$

where $\mathrm{X_j}$ denotes total spending in country j. The usual steps yield the CES demand for each variety as

$$\boldsymbol{c_{ij}(\omega) = p_{ij}(\omega)^{-\sigma} X_j P_j^{\sigma-1}}$$

where

$$\mathrm{P_j} = \left[\sum\nolimits_{\omega\in\Omega} \mathrm{p_{ij}}(\omega)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

is the Dixit-Stiglitz price index

The demand side

Reminder: the consumption maximization problem under CES preferences

- Hence, the value of trade of good ω from country i to country j is price times quantity, that is

$$\boldsymbol{X_{ij}(\omega) = p_{ij}(\omega)c_{ij}(\omega) = p_{ij}(\omega)^{1-\sigma}X_jP_j^{\sigma-1}} \qquad (1)$$

- To derive the gravity equation from the previous expression, we are only left to substitute for the optimal price and aggregate across all varieties

The supply side: pricing under perfect competition

- From now on, we drop the ω index because each country $\mathrm{i} \in S$ is assumed to produce a distinct variety of good (i.e., countries map one-to-one to varieties)
- Under perfect competition, firms price at marginal cost
- Denoting by $\mathrm{w_i}$ and $\mathrm{A_i}$ the wage and the marginal productivity of workers (one worker can produce $\mathrm{A_i}$ units of output), perfect competition implies $\mathrm{w_i} = \mathrm{p_i A_i}$ (workers are paid the value of their marginal product) or, equivalently $\mathrm{p_i} = \mathrm{w_i}/\mathrm{A_i}$ (firms price at marginal cost)

The supply side: trade costs

- Armington (1969) assumes the existence of trade "iceberg costs" τ (Samuelson, 1952):
  - In order for 1 unit of good to arrive at destination, τ>1 units must be produced and shipped (while τ−1 "melts" along the away)
- Hence, the price in country j of consuming one unit from country i is

$$p_{ij} = \tau_{ij} \frac{w_i}{A_i}, \qquad (2)$$

thus implying that $p_{ij} = \tau_{ij} p_i$ (p_{ij} is usually interpreted as the price at destination, p_i is the price at origin, or "at the factory door")

Deriving the gravity equation

- Substituting for (2) into (1) we obtain

$$X_{ij} = p_{ij}c_{ij} = \left(\frac{w_i}{A_i}\right)^{1-\sigma} \tau_{ij}^{1-\sigma} X_j P_j^{\sigma-1} \qquad (3)$$

- As a last step, total income in country i (again, the origin country) is equal to its total sales, i.e.,

$$Y_i = \sum_j X_{ij} = \sum_j \left(\frac{w_i}{A_i}\right)^{1-\sigma} \tau_{ij}^{1-\sigma} X_j P_j^{\sigma-1} \text{ from which}$$

$$\left(\frac{w_i}{A_i}\right)^{1-\sigma} = \frac{Y_i}{\sum_j \tau_{ij}^{1-\sigma} X_j P_j^{\sigma-1}}$$

- Substituting for the last expression back into (3), we finally obtain →

The gravity equation

$$\mathbf{X_{ij}} = \boldsymbol{\tau_{ij}^{1-\sigma}} \frac{\boldsymbol{Y_i}}{\sum_j \boldsymbol{\tau_{ij}^{1-\sigma} X_j P_j^{\sigma-1}}} \frac{\boldsymbol{X_j}}{\boldsymbol{P_j^{1-\sigma}}} \qquad (4)$$

- Expression (4) shows that the bilateral trade spending is related to the product of the GDPs of the two countries, the distance/trade cost, and a general equilibrium component (capturing the idea that, in a GE model with more than two countries, bilateral trade flows between i and j do not only depend on the features of i and j but also on all other countries)
- Expression (4) is about as close as we can get to the original gravity equation

Theoretical foundations:
the gravity equation under monopolistic competition (Krugman, 1980)

The environment

- We now keep hp1 (complete specialization) and hp2 (CES preferences) but remove hp3 (perfect competition) to embrace a monopolistically competitive supply side
  - In this framework, firms play a role, and each (exporting) country can produce (and export) more than one variety

- As we will see, going through analogous steps, we obtain the same expression for the gravity equation

The demand side

- As before, CES preferences imply that the demand for each variety writes

$$c_{ij}(\omega) = p_{ij}(\omega)^{-\sigma} X_j P_j^{\sigma-1}$$

where P_j is the Dixit-Stiglitz price index

- The amount spent on each variety ω is obtained from multiplying price and quantity demanded, that is

$$x_{ij}(\omega) = p_{ij}(\omega) c_{ij}(\omega) = p_{ij}(\omega)^{1-\sigma} X_j P_j^{\sigma-1}$$

- Aggregating across all firms (each producing a different variety) in country i (the exporter), we get the bilateral trade flows from i to j as

$$X_{ij} = \int_{\omega} x_{ij}(\omega)\, d\omega = X_j P_j^{\sigma-1} \int_{\omega} p_{ij}(\omega)^{1-\sigma}\, d\omega \qquad (5)$$

The supply side

- Profit maximization under monopolistic competition implies that, for any $\omega \in \Omega$, optimal pricing (MR=MC) for a firm selling from country i to country j is

$$\mathrm{p_{ij}}(\omega) = \frac{\tau_{\mathrm{ij}} \mathrm{w_i}}{\mathrm{A_i}} \frac{\sigma}{\sigma - 1} \text{ for any } \omega \in \Omega \qquad (6)$$

where $\mathrm{A_i}$ → marginal productivity of labor (inverse of marginal labor input β in the original Krugman (1979) model)

- Note: we can drop the index ω in (6) because firms are equally productive in the Krugman model ($\mathrm{A_i}$ is not ω - specific)

Deriving the gravity equation

- Substituting for (6) into (5) we obtain

$$X_{ij} = X_j P_j^{\sigma-1} \int_{\omega} \left[\frac{\tau_{ij} w_i}{A_i} \frac{\sigma}{\sigma-1}\right]^{1-\sigma} d\omega = \left(\frac{w_i}{A_i}\right)^{1-\sigma} \tau_{ij}^{1-\sigma} \left[\frac{\sigma}{\sigma-1}\right]^{1-\sigma} N_i X_j P_j^{\sigma-1} \qquad (7)$$

where N_i is the measure of firms producing in country i (number of varieties in country i)

- Again, as a last step, total income in country i is equal to its total sales across all destinations, i.e.

$$Y_i = \sum_j X_{ij} = \left(\frac{w_i}{A_i}\right)^{1-\sigma} \left[\frac{\sigma}{\sigma-1}\right]^{1-\sigma} N_i \sum_j \tau_{ij}^{1-\sigma} X_j P_j^{\sigma-1} \text{ from which}$$

$$\left(\frac{w_i}{A_i}\right)^{1-\sigma} = \frac{Y_i}{\left[\frac{\sigma}{\sigma-1}\right]^{1-\sigma} N_i \sum_j \tau_{ij}^{1-\sigma} X_j P_j^{\sigma-1}}$$

- Substituting for the last expression back into (7), we finally obtain →

Deriving the gravity equation

$$\mathbf{X_{ij}} = \boldsymbol{\tau_{ij}^{1-\sigma}} \frac{\boldsymbol{Y_i}}{\sum_j \boldsymbol{\tau_{ij}^{1-\sigma} X_j P_j^{\sigma-1}}} \frac{\boldsymbol{X_j}}{\boldsymbol{P_j^{1-\sigma}}}$$

- Again, bilateral trade spending is related to the product of the GDPs of the two countries, the distance/trade cost, and a general equilibrium component

Theoretical foundations:
the gravity equation under monopolistic competition with firm heterogeneity (Melitz, 2003)

The environment

- We now remove the hp. that firms are identical
- As we will see, going through analogous steps we obtain, once again, a similar expression for the gravity equation

The demand side

- As before, CES preferences imply that the demand for each variety writes

$$c_{ij}(\omega) = p_{ij}(\omega)^{-\sigma} X_j P_j^{\sigma-1} \quad (8)$$

where P_j is the Dixit-Stiglitz price index

- The amount spent on each variety ω is obtained from multiplying price and quantity demanded, that is

$$x_{ij}(\omega) = p_{ij}(\omega) c_{ij}(\omega) = p_{ij}(\omega)^{1-\sigma} X_j P_j^{\sigma-1} \; (9)$$

- Aggregating across all firms (varieties) in country i (the exporter), we get the bilateral trade flows from i to j as

$$X_{ij} = \int_{\omega} x_{ij}(\omega)\, d\omega = X_j P_j^{\sigma-1} \int_{\omega} p_{ij}(\omega)^{1-\sigma}\, d\omega \quad (10)$$

The supply side

- Profit maximization under monopolistic competition implies that, for any $\omega \in \Omega$, optimal pricing (MR=MC) for a firm selling from country i to country j is

$$p_{ij}(\varphi) = \frac{\tau_{ij} w_i}{\varphi} \frac{\sigma}{\sigma - 1} \qquad (11)$$

where φ is the firm's productivity distributed according to a density function $g(\varphi)$

- Combining (9) with (11), total revenue of a firm writes as

$$x_{ij}(\omega) = p_{ij}(\omega) c_{ij}(\omega) = \left(\frac{\tau_{ij} w_i}{\varphi} \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} X_j P_j^{\sigma - 1}$$

Aggregation

- Once again, we could substitute for (11) into (10). Only this time the integral in (10) is harder than in the previous case (Krugman), because firms with different productivities now charge different prices
- In fact,

$$\int_{\omega} \mathrm{p}_{\mathrm{ij}}(\omega)^{1-\sigma}\,\mathrm{d}\omega = \int_0^{\infty}\left(\frac{\tau_{\mathrm{ij}}\mathrm{w}_{\mathrm{i}}}{\varphi}\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\mu_{\mathrm{ij}}(\varphi)\mathrm{d}\varphi =$$

$$= \mathrm{M}_{\mathrm{ij}}\left(\tau_{\mathrm{ij}}\mathrm{w}_{\mathrm{i}}\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\int_0^{\infty}\varphi^{\sigma-1}\,\mu_{\mathrm{ij}}(\varphi)\mathrm{d}\varphi = \mathrm{M}_{\mathrm{ij}}\left(\tau_{\mathrm{ij}}\mathrm{w}_{\mathrm{i}}\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\tilde{\varphi}_{\mathrm{ij}}\right)^{\sigma-1} \qquad (12)$$

where

$\mu_{\mathrm{ij}}(\varphi)$ is the (equilibrium) prob. density function of the productivities of firms in j selling to j

M_{ij} → is the (equilibrium) measure of firms in j selling to j

$\tilde{\varphi}_{\mathrm{ij}} \equiv \int_0^{\infty}\varphi^{\sigma-1}\,\mu_{\mathrm{ij}}(\varphi)\mathrm{d}\varphi$ captures the aggregate productivity of firms in i selling to j

The gravity equation

- Substituting for (12) into (10), we finally obtain an expression for **total trade flows** as

$$X_{ij}(\omega) = X_j P_j^{\sigma-1} \int_{\omega} p_{ij}(\omega)^{1-\sigma} \, d\omega =$$

$$= \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \tau_{ij}{}^{1-\sigma} w_i{}^{1-\sigma} X_j P_j^{\sigma-1} M_{ij} \left(\tilde{\varphi}_{ij}\right)^{\sigma-1},$$

which resembles the Krugman (1980) gravity equation (expression (7)), except that we now need to keep track of both the number of firms selling to j (M_{ij}) and their average productivity ($\tilde{\varphi}_{ij}$)

Theoretical foundations:

the gravity equation in Ricardian models (Eaton and Kortum, 2002)

Introduction

- The Ricardian model (both the original one and the DFS (1977) extension to a *continuum* of goods) has proved to be hard to be extended to several countries and arbitrary trade costs (which made it impossible to deliver a proper gravity equation)
- Eaton and Kortum (2002) consider a Ricardian structure (where trade is driven by technological differences across countries) in a world with many countries and arbitrary trade costs, thus delivering a first Ricardian gravity model

Introduction

- Main questions:
  - What explains bilateral trade flows? How does distance affect such trade patterns? How can we link productivity differences to trade flows?
- Core assumptions:
  - Countries differ in technology/productivity
    - In particular, each country draws productivity for each good from a probability distribution (Fréchet)
  - Perfect competition: prices equal unit costs, buyers source each good from the lowest-cost supplier globally
  - Trade costs (iceberg costs): they increase with distance and barriers

Introduction

- Key mechanisms:
  - Trade is shaped by comparative advantage (Ricardian logic): countries specialize in (and thus export) goods where they are relatively more productive (lower costs or higher productivity)
  - Who supplies a good? The country with the lowest delivered cost, which is a combination of production cost (technology) and trade cost (distance/barriers)
- Implications and main results:
  - Trade patterns emerge from a "race" between costs of production, productivity and distance
  - A gravity equation emerges naturally: bilateral trade flows follow a gravity-like structure:
    - Larger economies → more trade; Greater distance → less trade
- Legacy:
  - It provides a clear micro-foundation for the gravity equation
  - It is the backbone of modern quantitative trade

The environment

- Finite (and discrete) number N of countries, $i \in S$
- *Continuum* of goods, $\omega \in \Omega$
- Countries vary (exogenously) in their productivity of each good
  - $z_i(\omega)$ denotes the productivity of country i in producing ω
- CRS technology:
  - the same bundle of inputs is necessary to produce any good ω (one might think of labor)
  - The cost of this bundle for country i is denoted by c_i (if labor is the only input, then c_i → w_i

  →The cost of producing one unit of good ω is $c_i/z_i(\omega)$
- Iceberg costs for trade: $\tau_{ij} \geq 1$ to ship a good from country i to j

Supply

- Perfectly competitive industries imply that pricing (p=MC) occurs according to

$$p_{ij}(\omega) = \frac{c_i}{z_i(\omega)} \tau_{ij}$$

- However, as in DFS (1977), consumers in j purchase good ω from the country that can provide it at the lowest price. Hence:

$$p_j(\omega) = \min_{i \in S} p_{ij}(\omega) = \min_{i \in S} \frac{c_i}{z_i(\omega)} \tau_{ij}$$

- The previous expression tells us that country j is more likely to buy good ω from country i if this country has (i) a lower unit cost c_i, (ii) a higher productivity $z_i(\omega)$, (iii) lower trade costs τ_{ij}

Demand

The demand side is standard

- CES preferences over the *continuum* of existing varieties $\omega \in \Omega$:

$$\mathrm{U_j} = \left[\int_{\Omega} \mathrm{q_j}(\omega)^{\frac{\sigma-1}{\sigma}} \mathrm{d}\omega \right]^{\frac{\sigma}{\sigma-1}}$$

where

$$\mathrm{P_j} = \left[\int_{\Omega} \mathrm{p_j}(\omega)^{1-\sigma} \, \mathrm{d}\omega \right]^{\frac{1}{1-\sigma}}$$

is the ideal price index for the CES demand system

Productivity

- The major innovation of Eaton and Kortum (2002) is that productivity $z_i(\omega)$ is a random variable drawn independently and identically for any $\omega \in \Omega$
- Denoting by $F_i(z) = \Pr\{z_i(\omega) \leq z\}$ the cumulative distribution function, Eaton and Kortum (2002) assume that $F_i(z)$ is a Frechet distribution:

$$F_i(z) = \exp\{-T_i z^{-\theta}\}$$

where $T_i > 0$ is a measure of aggregate productivity of country i (absolute advantage), while $\theta > 1$ measures the heterogeneity (variance) of productivity across goods ω (comparative advantage)

Towards the gravity equation

- The probability that country i exports good ω to country j at a price lower than p is

$$G_{ij}(p) = \Pr\{p_{ij}(\omega) \le p\} = \Pr\left\{\frac{c_i}{z_i(\omega)}\ \tau_{ij} \le p\right\} =$$

$$= \Pr\left\{z_i(\omega)\ \ \le \frac{c_i}{p}\tau_{ij}\right\} = 1 - F_i\left(\frac{c_i}{p}\tau_{ij}\right) = 1 - \exp\left\{-T_i\left(\frac{c_i}{p}\tau_{ij}\right)^{-\theta}\right\}$$

Towards the gravity equation

- On the other hand, the probability that the price of a good ω in country j is at least as low as p (i.e., the probability that country j pays a price less than p for good ω) equals one minus the probability that all exporting countries have prices greater than p, that is:

$$\mathrm{G_j(p) = Pr\{min_{i \in S}\, p_{ij}}(\omega) \leq \mathrm{p\} = 1 - Pr\left\{min_{i\in S}\, p_{ij}}(\omega) \geq \mathrm{p}\right\} =$$

$$= 1 - \prod\nolimits_{\mathrm{i} \in S} (1 - \mathrm{G_{ij}(p)}) = 1 - \prod\nolimits_{\mathrm{i} \in S} \exp\left\{-\mathrm{T_i}\left(\frac{\mathrm{c_i}}{\mathrm{p}}\tau_{\mathrm{ij}}\right)^{-\theta}\right\} =$$

$$1 - \exp\left\{-\mathrm{p}^{\theta} \sum\nolimits_{\mathrm{i} \in S} \mathrm{T_i}\left(\mathrm{c_i}\tau_{\mathrm{ij}}\right)^{-\theta}\right\} = 1 - \exp\{-\mathrm{p}^{\theta}\Phi_{\mathrm{j}}\}$$

where $\Phi_{\mathrm{j}} \equiv \sum_{\mathrm{i} \in S} \mathrm{T_i}\left(\mathrm{c_i}\tau_{\mathrm{ij}}\right)^{-\theta}$

Towards the gravity equation

- We are now ready to solve for the probability that country i is in fact the lowest cost-cost supplier to country j
- It is possible to prove that

$$\pi_{ij} \equiv \Pr[\, p_{ij} \leq \min\{p_{kj}(\omega);\; k \neq i] \;= \frac{T_i\left(c_i\tau_{ij}\right)^{-\theta}}{\Phi_j}$$

(the proof is left as an exercise, see p. 55 of AA. Hint: you need to exploit the expressions of the two previous slides)

Towards the gravity equation

- Because all goods receive i.i.d draws (and there is a continuum of varieties), by the law of large numbers, probability π_{ij} will be equal to the share of goods that i sells to j
- Hence, the value of exports $\mathrm{X_{ij}}$ from country i to country j over total purchases $\mathrm{X_j}$ in country j will be

$$\frac{\mathrm{X_{ij}}}{\mathrm{X_j}} = \frac{\mathrm{T_i}\left(\mathrm{c_i}\tau_{\mathrm{ij}}\right)^{-\theta}}{\Phi_{\mathrm{j}}}$$

where $\Phi_{\mathrm{j}} \equiv \sum_{\mathrm{i}\in\mathrm{S}} \mathrm{T_i}\left(\mathrm{c_i}\tau_{\mathrm{ij}}\right)^{-\theta}$

→ The fraction of goods exported from i to j depends on i's share in j's Φ_{j}

Towards the gravity equation

$$\frac{X_{ij}}{X_j} = \frac{T_i \left(c_i \tau_{ij}\right)^{-\theta}}{\Phi_j}$$

The more productive countries (high T_i), countries with lower unit costs (low c_i), and countries with lower bilateral costs (low τ_{ij}), all relative to other countries (Φ_j), will comprise a larger fraction of the goods sold to country j

References (specific to the topics tackled in this lecture)

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