Theory of Collusion

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1 Two notions of collusion

Collusion occurs when some or all firms in a market coordinate their market strategies in a way that leads to higher prices, lower output and higher profits. Collusion may concern other dimensions of competition, such as quality or innovation, provided that coordination leads to higher profits and lower consumer or social welfare. Collusion may be explicit, or tacit.

Explicit collusion involves some form of direct or indirect communication ("a meeting of the minds"). Explicit collusion may occur through an agreement or, more likely a "concerted practice". According to the ECJ a concerted practice is a "co-ordination between undertakings which, without having reached the stage of concluding a formal agreement, have knowingly substituted practical co-operation for the risks of competition". Explicit collusion is illegal. Sometimes lawyers or legal scholars use "collusion" to refer to this illegal behavior.

Tacit collusion is a coordination that occurs without any form of explicit communications among firms. It is the result of a mutual understanding of firms' interdependence, or of the intelligent adapatation to the conditions of the market and to the anticipated competitors´ behavior and reactions. Tacit collusion may occur in oliopoly markets. It is sometimes referred to as "conscious parallelism". Under conditions of oligopoly, the pricing and output actions of one firm have a significant impact upon that of its rivals. Firms, after some periods of repeated interaction, may become conscious or aware of this fact and, without an explicit agreement, coordinate their behavior as if they were engaged in a cartel to fix prices and restrict output. The fear that departure from such behaviour may lead to a costly price war, lower profits and market share instability may further create incentives for firms to maintain such an implicit arrangement amongst themselves. Tacit collusion is legal.

It is very difficult to draw a clear line between the existence or absence of communication. Indeed, firms can communicate through messages sent to their clients or market participants other that their rivals or to the press. Tacit collusion occur if it is based only on **market signals** that emerge from the normal course of business and that have not been artificially altered by the competing firms.

Explicit and tacit collusion may have the same economic effects in terms of allocative efficiency, consumer and social welfare.

2 Economic theory of collusion

The economic theory of collusion does not clearly distinguish between explicit and tacit collusion. Since explicit collusion is illegal, firms cannot rely on enforceable contracts to reach and sustain a collusive equilibrium. Hence, also in the case in which firms communicate to coordinate their behavior, this coordination has to be enforced through a market mechanism.

Firms that want to collude (explicitly or tacitly) must solve three problems.

Coordination problem: firms must define the terms of coordination, i.e. a profile of market strategies that increases firms' profit and that all firms agree with. This is equivalent to say that firms must identify (possibly tacitly) an acceptable way to share among them the benefits of collusion and a practical method to achieve the agreed repartition of benefits.

Enforcement problem: when firms collude each firm has a (short run) incentive to deviate (cheat) from the terms of coordination. Hence, firms must identify a market-based mechanism that prevents them from deviating (cheating).

External stability: impeding disruptive actions by fringe or potential competitors and/or buyers.

We will focus mostly on the enforcement problem in order to understand when collusion is feasible and what market condition or firms' conducts can facilitate it.

3 Collusion in a repeated game

Collusion is ineherently a dynamic phenomenon. Indeed, it can also be defined as a departure from the market equilibrium that would prevail in a static game. Therefore we consider a repeated game in which the same firms play the same identical stage game an infinite number of times.

We will use a very simple market model to explore some relevant factors.

There are n firms that compete in price. The vector $p^c = (p_1^c, ..., p_n^c)$ is the unique strategy profile that forms an equilibrium in the stage game. The vector $p^m = (p_1^m, ..., p_n^m)$ is a possible collusive profile, for instance the combination of firms' prices that maximize firms' joint profits. Finally, p_i^d is the price that mazimizes firm i's profit in the stage game if all the other players set the price defined in the collusive profile, i.e. $p_i^d = \arg\max_{p_i} \Pi_i(p_i, p_{-i}^m)$. By definition for at least one firm $p_i^d \neq p_i^m$ otherwise p^m is a Nash equilibrium of the stage game.

Let us define firms' profits:

$\Pi_i^c = \Pi_i(p^c)$ = firm's i competitive profit (i.e. its profit in the equilibrium of the stage game);

$\Pi_i^m = \Pi_i(p^m)$ = firm's i collusive profit (i.e. its profit if all firms, including i, respect the terms of coordination);

$\Pi_i^d = \Pi_i\left(p_i^d, p_{-i}^m\right)$ = firm's i deviation profit (i.e. its profit if all firms respect the terms of coordination but i deviates and adopts a strategy that maximizes its profit in the stage game, given its rivals' strategy). The following inequalities hold for any i:

$$\Pi_i^d \geq \Pi_i^m > \Pi_i^c.$$

Let us consider the following strategy in the repeated game: in the first period play according to the terms of coordination (p_i^m), in the second period and in any following period, play according to the terms of coordination (p_i^m) if in any previous repetition, all firms respected the same terms of coordination, otherwise play as in the equilibrium profile of the stage game (p_i^c). We refer to this strategy as "grim strategy". We want to understand if an equilibrium profile in which all firms adopt the grim strategy is an equibrium of the repeated game. The notion of equilibrium we use is the Subgame Perfect Nash Equilibrium (SPNE) which means that for each player the grim strategy must be the best response to the other players' grim strategy in any possible node of the game.

We can start the analysis from the first repetition of the stage game. We have to check whether the grim strategy is the best response for i given all the other future repetitions of the stage game (i.e. the "continuation game"). If i plays according to the grim strategy its discounted profit over the entire game is:

$$G = \Pi_i^m + \delta_i \Pi_i^m + \delta_i^2 \Pi_i^m + ... = \sum_{t=0}^{\infty} \delta_i^t \Pi_i^m = \frac{\Pi_i^m}{1-\delta_i}$$

where $\delta_i \in (0,1)$ is the discount factor firm i uses to compute the present value of future profits.

Since any other strategy will trigger rivals' reversal to the competitive strategy profile, the most profitable alternative is to play p_i^d in the first repetition and revert to the competitive price in the following periods. All the other possible strategies are dominated by this strategy and therefore we can neglect them.

So, if firm i deviates, it will obtain the deviation profit in the first period and competitive profits in the continuation game:

$$D = \Pi_i^d + \delta_i \Pi_i^c + \delta_i^2 \Pi_i^c + ... = \Pi_i^d + \sum_{t=1}^{\infty} \delta_i^t \Pi_i^c = \Pi_i^d + \frac{\delta_i}{1-\delta_i} \Pi_i^c.$$

Deviation is not profitable if $G \geq D$:

$$\frac{\Pi_i^m}{1-\delta_i} \geq \Pi_i^d + \frac{\delta_i}{1-\delta_i} \Pi^c \tag{1}$$

Condition 1 can be rearranged so that we have:

$$\delta_i \geq \frac{\Pi_i^d - \Pi_i^m}{\Pi_i^d - \Pi_i^c} \equiv \delta_i^*. \tag{2}$$

The variable δ_i^* is called the **critical discount factor**. If condition 2 is satisfied, firm i's "promise" that it will "cooperate" (i.e. collude) is credible because it is in its interest to play acording to the collussive profile, if the other firms do the same. This promise is credible in the first period and in all the other periods as they do not differ from the first one in any respect. The grim strategy contains also a "threat", that is a "punishment" in case one firm deviates. This punishment consists in reverting to the static equilirbium. The punishment is also credible because it implies a best response by each firm to the price charged by all the other players by definition.

Therefore condition 2 is the only one that is required for the collusive profile formed by the grim strategies profile to be a SPNE (of course condition 2 must be satisfied for all firms).

4 Factors that facilitate collusion

To describe the market characteristics and firms' conducts that facilitate collusion, we use the result of the previous section. First we have to notice that collusion is more stable (i.e. easier) if the critical discount factor is lower. Hence condition 2 reveals that collusion is facilitated by any factor that (ceteris paribus) either increases collusive profits, or decreases deviation profits or decreases competitive profits (i.e. increases the severity of the punishment). Indeed, we have:

$$\frac{\partial \delta^*}{\partial \Pi^m} = -\frac{1}{\Pi^d - \Pi^c} < 0$$

$$\frac{\partial \delta^*}{\partial \Pi^d} = \frac{\left(\Pi^d - \Pi^c\right) - \left(\Pi^d - \Pi^m\right)}{\left(\Pi^d - \Pi^c\right)^2} = \frac{\Pi^m - \Pi^c}{\left(\Pi^d - \Pi^c\right)^2} > 0; \text{ and}$$

$$\frac{\partial \delta^*}{\partial \Pi^c} = \frac{\Pi^d - \Pi^m}{\left(\Pi^d - \Pi^c\right)^2} > 0.$$

However, many relevant factors affect more than one type of profit. So, we need to go beyond these results. Let us explore some market characteristics (or business practices) with a simpler game. (See the appendix for a more general result.)

Let's consider a Bertrand game with homogeneous products and n identical firms. In this game we know that

$$p_i^c = mc \text{ for all } i;$$

$$p_i^m = p^m \text{ for all } i; \text{ and}$$

$$p_i^d = p^m - \varepsilon,$$

where: mc denotes marginal costs, p^m is the monopoly price and ε is a small but positive value. If demand is divided equally among the n firms when they charge the same price, we have that:

$$\Pi_i^c = 0;$$

$$\Pi_i^m = \frac{\Pi^m}{n}; \text{ and}$$

$$\Pi_i^d = \Pi^m,$$

where Π^m is the profit that would be gained by the monopolist if the market was served by a single firm.

In this setting, condition 2 becomes:

$$\delta_i \geq \frac{\Pi^m - \frac{\Pi^m}{n}}{\Pi^m} = 1 - \frac{1}{n} \equiv \delta^* (n) . \tag{3}$$

We can use 3 to identify some factors that facilitate collusion.

4.1 Number of firms and concentration (and barriers to entry)

Collusion is easier in markets in which there are few competitors. This stems directly from 3. Indeed, we have that:

$$\frac{\partial \delta^*}{\partial n} = \frac{1}{n^2} > 0.$$

We can also compute the critical discount factor in the limit cases:

$$\delta^* (2) = \frac{1}{2}; \text{ and}$$

$$\lim_{n \to \infty} \delta^* (n) = 1.$$

This result can be used also to show the impact on collusion of **barriers to entry** and **potential competition**. If there are no barriers to entry and any potential competitor can swiftly enter the market, new firms will do so to take advantage of the profit opportunities created by the collusive equilibrium. Hence, the number of competitors will grow, up to the point where collusion is no longer feasible. Hence, the existence of barriers to entry is crucial for collusion.

Condition 3 can be used also to say that the degree of concentration is a relevant factor. In this simple setting, given firms' symmetry, the HHI is simply:

$$HHI = \frac{10,000}{n},$$

so that the previous results also show that the more the market is concentrated the more the collusive equilibrium is likely to be stable.

4.2 Firms (a)symmetry (and barriers to expansion)

Consider the previous case and suppose that there are only 2 firms. Let us assume that some factors engender a possible asymmetry between the two competitors, so that if they charge the same price, firm 1 will obtain a market share equal to λ and firm 2 a market share equal to $1-\lambda$. In this setting, the critical discount factor may be different for the two firms. We have:

$$\delta_1^* = \frac{\Pi^m - \lambda\Pi^m}{\Pi^m} = 1-\lambda$$

$$\delta_2^* = \frac{\Pi^m - (1-\lambda)\,\Pi^m}{\Pi^m} = \lambda.$$

Hence, if $\lambda = 1/2$ then $\delta_1^* = \delta_2^* = 1/2$. However if $\lambda > 1/2$, we will have $\delta_1^* < 1/2$ but $\delta_2^* > 1/2$. Since condition 3 must be satisfied for both firms, $\delta_2^* > 1/2$ is the binding constraint. This shows that the more pronounced the asymmetry between the two firms the more collusion becomes unstable.

It is interesting to notice that the firm with the highest critical discount factor is the one with the lowest market share. The explanation is that this firm would gain the most if it deviates as it will grow substantially. If its ability to grow is limited by some **barriers to expansion**, its incentive to deviate wil be curbed. Firms that have a strong potential to grow, possibly because they adopt a novel and disruptive business model, are referred to as "maverick". The existence of one or more mavericks makes collusion particularly unstable.

4.3 Multimarket contacts

Multimarket contacts exist if firms compete simultaneously in more than one market. These might be different geographic or product markets. The informal argument was that multimarket contacts can facilitate collusion because a deviation in one market can be punished more severely as punishment can occur in both markets. However, this argument is flawed, because it does not take into account that the deviator can deviate in all markets, so that multimarket contact increases both deviation profits and the severity of punishment.

The reason why multimarket contacts can faciliatate collusion is that it may reduce an existing asymmetry. To explain this consider the following example. There are two markets, A and B, in which 2 firms compete. In market A firm 1's marlet share is λ; in market B firm 1's marlet share is $1-\lambda$, with $\lambda > 1/2$. If the two markets are separate (in the sense that the second firm in market A is different from the second firm in market B) we have already established that collusion is feasible in both markets if $\delta \geq \delta^* = \lambda > 1/2$.

Now consider that multimarket contacts exist and that the two firms can collude in both markets. To semplify the exposition let us assume that the level of collusive profits is the same in the two markets, so that $\Pi_A^m = \Pi_B^m = \Pi^m$. Let us compute the critical discount factor for firm 1:

$$\delta_1^* = \frac{\left(\Pi_A^m + \Pi_B^m\right) - \left(\lambda \Pi_A^m + (1-\lambda)\, \Pi_B^m\right)}{\Pi_A^m + \Pi_B^m}$$

Given the condition $\Pi_A^m = \Pi_B^m = \Pi^m$, this becomes

$$\delta_1^* = \frac{2\Pi^m - \Pi^m}{2\Pi^m} = \frac{1}{2}.$$

The same holds form firm 2. Therefore, collusion becomes more stable.

4.4 Market transparency

So far we have assumed that the punishment occurs immediately after the deviation. However, it might be that for some reasons firms are able to detect a deviation only after a certain number of periods, u. This means that a deviator is undetected for u periods over which it keeps gaining the deviation profits. In this case the critical discount factor becomes

$$\delta^* \geq \frac{u\Pi^m - \frac{\Pi^m}{n}}{u\Pi^m} = 1 - \frac{1}{un}$$

which shows that the longer the time it takes to detect a deviation the less likely collusion becomes.

Market transparency may be an inevitable feature of the market or the consequence of some firms' conduct. For instance firms can decide to share information about their commercial activity. If the information is disaggregated and concerns the recent past, the information sharing arrangement may help firms monitor each other and spot a deviation from a collusive path more swiftly. This in turn may facilitate collusion. An information sharing arrangement may also alleviate the coordination problem, as it can determine some focal points that easily become the terms of coordination. This is even more likely if the information concerns firms' intention about future market strategies, such as future prices. For this reason the European Commission (and many National Competition Authorities) consider this arrangement an infringement by object of article 101 TFEU.

5 Appendix

Consider a market characteristic (or a business practice) that can be represented by a continuous variable α. Suppoe that α is potentially able to affect collusive, deviation and competitive profits. The critical discount factor for collusion to be stable is

$$\delta^* = \frac{\Pi^d(\alpha) - \Pi^m(\alpha)}{\Pi^d(\alpha) - \Pi^c(\alpha)}.$$

The impact of this factor on collusion can be investigated by computing the derivative of δ^* with respect to α:

$$\frac{\partial \delta^*}{\partial \alpha} = \frac{\left(\frac{\partial \Pi^d}{\partial \alpha} - \frac{\partial \Pi^m}{\partial \alpha}\right)\left(\Pi^d - \Pi^c\right) - \left(\frac{\partial \Pi^d}{\partial \alpha} - \frac{\partial \Pi^c}{\partial \alpha}\right)\left(\Pi^d - \Pi^m\right)}{\left(\Pi^d - \Pi^c\right)^2}.$$

With some simple manipulation we find that $\frac{\partial \delta^*}{\partial \alpha} \geq 0$ iff

$$\frac{\partial \Pi^d}{\partial \alpha}\left(\Pi^m - \Pi^c\right) - \frac{\partial \Pi^m}{\partial \alpha}\left(\Pi^d - \Pi^c\right) + \frac{\partial \Pi^c}{\partial \alpha}\left(\Pi^d - \Pi^m\right) \geq 0 \tag{4}$$

If condition 4 is satisfied, then the market characteristic represented by α hinders collusion. Condition 4 shows that any factor that makes competition more intense in the stage game, if it does not change collusive and deviation profits (i.e. $\frac{\partial \Pi^d}{\partial \alpha} = \frac{\partial \Pi^m}{\partial \alpha} = 0$), will make collusion more stable. This is sometimes referred to as the "**topsy-turvy principle of collusion**".

Another useful result that can be obtained from condition 4 concerns the case in which in any case firms are able to impose the harshest punishment to deviators which entails $\Pi^c = 0$. If so condition 4 boils down to:

$$\frac{\partial \Pi^d}{\partial \alpha}\Pi^m - \frac{\partial \Pi^m}{\partial \alpha}\Pi^d \geq 0. \tag{5}$$

Since $\Pi^d \Pi^m > 0$, condition 5 is equivalent to:

$$\frac{\frac{\partial \Pi^d}{\partial \alpha}}{\Pi^D} \geq \frac{\frac{\partial \Pi^m}{\partial \alpha}}{\Pi^m}$$

which can be easily intepreted as follows: Given the market characteristic α, if $\Pi^c = 0$ for any α, α hinders collusion if it determines a growth rate of deviation profit higher than the growth rate of the collusive profit or a decline rate of deviation profit lower than the decline rate of the collusive profit. In more simpe words α hinders collusion if it increases the ratio between deviation and collusive profits, which may be called the "**relative incentive to deviate**".