11/4/22

MICROECONOMICS II - 2022

F. Campioni & R. Pezzuto

References

- Gibbons "Game Theory : A Primer"
- Osborne "Introduction to Game theory"

↳ Osborne & Rubinstein "A course in Game theory".

- Myerson -

Game theory

Non-cooperative games.

↳ static games

↳ dynamic games

Player ≡ decision maker

Single player optimal choice problem (without uncertainty)

Pl. i

identify a set of possible choices as

A_i with $a_i \in A_i$

Preference relation on the set of choices

$\succsim_i$ (complete and transitive)

$\Updownarrow$

utility function $u_i : A_i \to \mathbb{R}$

Preferences are represented by a utility function when

$\forall (a,b) \in A_i$

$a \succsim_i b \iff u_i(a) \geq u_i(b)$

Player i is RATIONAL

choice of Mr. i is simply to select within the set A_i the alternative $a_i \in A_i$ which gives him the highest utility.

~

In a game we have at least <u>two</u> players $i = 1,2$

for $i = 1$

A_1 is the set of choices of player 1
$a_1 \in A_1$

$\boxed{u_1 : A_1 \times A_2 \longrightarrow \mathbb{R}}$ payoff function

$u_1(a_1, a_2)$ represents the utility that player 1 obtains if she chooses a_1 and player 2 chooses a_2

the same can be said for player 2.

for $i = 2$

A_2 , $a_2 \in A_2$

$u_2 : A_1 \times A_2 \rightarrow \mathbb{R}$

$(u_1(a_1, a_2), u_2(a_1, a_2))$ is an outcome of the game

Both player 1 and 2 want to make a choice $a_1 \in A_1$ and $a_2 \in A_2$ that maximizes her payoff. Notice that the payoff of each player depends on the choice of her opponent and on her own choice.

Let $N \geq 2$ be the number of players
$i = 1, 2, 3, \dots, N$

For each $i = 1, 2, \dots, N \Rightarrow A_i$
$A_1, A_2, \dots, A_N$ choice sets

For each $i = 1, \dots, N$ a payoff function
$u_i : A_1 \times A_2 \times \dots \times A_N \to \mathbb{R}$

Each players wants to select within A_i an alternative that maximizes her utility/payoff $u_i(a_1, a_2, \dots a_N)$.

Notation : Consider player 1.

$$u_1(a_1, a_2, a_3, \dots, a_N)$$

$\Downarrow$ we usually write

$$u_1(a_1, a_{-1})$$

$\Downarrow$

with $a_{-1} = (a_2, a_3, \dots, a_N)$

$$a_{-1} \in A_{-1} \equiv A_2 \times A_3 \times \dots \times A_N$$

For a generic player i, we let

$$u_i(a_i, a_{-i}) = u_i(a_i, \underbrace{a_1, a_2, \dots a_{i-1}, a_{i+1}, \dots a_N}_{a_{-i}})$$

a profile of choices of all players except Mr. i — a_{-i}

We also let

$$a \equiv (a_1, a_2, \dots, a_N) \in A \equiv A_1 \times A_2 \times \dots \times A_N$$

be a profile (vector) of players' choices.