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Practice 1

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Exercise 1 Players 1 and 2 are bargaining over how to split one dollar. Both players simultaneously name the amounts they would like to have, $s_1 > 0$ and $s_2 > 0$. If $s_1 + s_2 \leq 1$, then the players receive the amounts they named; if $s_1 + s_2 > 1$ then both players receive zero.

1. Describe the strategic form game.
2. Find all pure-strategy Nash equilibria of this game.
3. Are there equilibria in weakly dominated strategies? Explain.

Exercise 2 Each hunter in group of n hunters has two alternatives for the hunt: to the pursuit of a stag or catch a hare (rabbit). If all hunters pursue the stag, they catch it and share it equally; if any hunter devotes her energy to catching a hare, the stag escapes, and she secures herself the hare. Each hunter prefers $1/n$ of the stag to on ehare.

1. For the case where $n = 2$, provide the strategic-form representation of the game and find the Nash Equilibria.
2. Suppose now that the hunters are $n > 2$ and that they must all go together to catch the stag. Find the Nash equilibria of this variant, in which the hunters' preferences remain the same of the original game.
3. Find the Nash equilibria of the variant of the previous game, with $n > 2$ players, when m hunters are enough to catch the stag, with $2 \leq m < n$.
4. Assume that each hunter prefers the fraction $1/k$ of the stag to a hare, but prefers the hare to any smaller fraction of the stag, where k is an integer with $m \leq k \leq n$. Find the Nash equilibria of the strategic game that models this situation.

Exercise 3 (Cournot oligopoly) Consider the following oligopoly model. There are n firms. Let q_i denote the quantity produced by firm i. The market clearing price p depends on the total output:

$$p(q_1, ..., q_n) = \begin{cases} a - b\sum_{i=1}^{\infty} q_i & \text{if } a - b\sum(q_i) > 0 \\ 0 & \text{otherwise} \end{cases}$$

where $a > 0$, $b > 0$. The total cost of firm i for producing quantity q_i is $C_i(q_i) = cq_i$. That is, there are no fixed costs and the marginal cost is constant and equal to c. Assume that $c < a$. Suppose that all firms choose their quantities simultaneously.

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1. Define actions and payoffs for the players.

2. Find the symmetric Nash equilibrium.

3. What happens as n approaches infinity? What is the economic interpretation of the limit result?

Exericise 4 Consider the oligopoly model described in Exercise 3 and suppose that $n = 2$. Let each duopolist have constant average and marginal costs, but suppose that $0 < c_1 < c_2$. Show that firm 1 will have greater profits and produce a greater share of the market output than firm 2 in the Nash equilibrium.

Exericise 5 Consider the following duopoly model with differentiated products, where two firms compete on the price of their product. Let the demand for firm $i = 1, 2$ be:

$$q_i(p_i, p_j) = \begin{cases} \alpha - p_i + \beta p_j & \text{if } p_i < \alpha + \beta p_j \\ 0 & \text{otherwise} \end{cases}$$

with $j = 1, 2,\ j \neq i$, and $\beta > 0$. Let $C_i(q_i) = cq_i$, with $c < \alpha$, be the cost function of each firm. Assume that each firm chooses the price of its product without knowing its opponent's price decision.

1. Provide the strategic-form representation of this game.

2. Which are the best-reply functions of the duopolist firms?

3. Which conditions define a Nash equilibrium of this duopoly? Find the Nash equilibrium of the game.