Rock - Paper - Scissors 12/4/202

Strategic or normal-form game: any game in which players do not know their opponents' choices when taking their action.

$i =$ Edoardo, Emanuela $(N=2)$

$A_i = \{R, P, S\} \quad \forall i$

$u_{Edo} : A_{Edo} \times A_{Eme} \to \mathbb{R}$

$$\{R, P, S\} \times \{R, P, S\}$$

$$\Downarrow$$

$$\{\underset{a_{Edo}\ a_{Eme}}{RR}, RP, RS, PR, PP, PS, SR, SP, SS\}$$

take Edoardo

$u_{Edo}(R,R) = 0 \ 1$ (draw) $\to$ 1

$u_{Edo}(R,P) = -1 \ 0$ (lose) $\to$ 3

$u_{Edo}(R,S) = +1 \ 3$ (win) $\to$ 0

$u_{Edo}(P,R) = 3$ ... $\to$ 0

$u_{Edo}(P,P) = 1$ $\rightsquigarrow$

$u_{Edo}(P,S) = 0$

$u_{Edo}(S,R) = 0$

$u_{Edo}(S,P) = 3$

$u_{Edo}(S,S) = 1$

This is an example of a game of pure competition. (zero-sum game)

$(+1,-1)$ $(0,0)$

A strategic game

$$G = \{N, A_1, A_2, \dots, A_N, u_1, \dots, u_N\}$$

with $u_i : A_1 \times A_2 \times \dots \times A_N \to \mathbb{R}$

$$G = \{N, (A_i)_{i=1}^N, (u_i)_{i=1}^N\}$$

Bi-matrix of Rock-Paper-Scissors (especially with N=2)

$P_1 \backslash P_2$	R	P	S
R	$u_1(R,R)$, $u_2(R,R)$		
P			$u_1(P,S)$ $u_2(P,S)$
S			

$u_1(a_1, a_2, \dots)$

action of P1 · action of P2

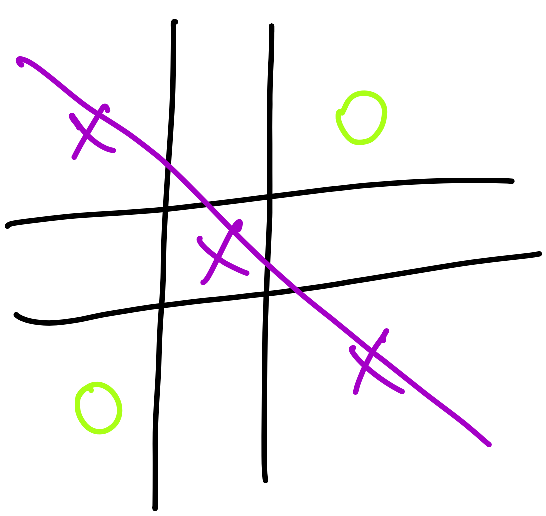
$P_1 \backslash P_2$	R	P	S
R	1,1	0,3	3,0
P	3,0	1,1	0,3
S	0,3	3,0	1,1

Rewrite the bimatrix for the case of $\{+1, 0, -1\}$.

A strategic game G in which N is finite and A_i is a finite set for every player i is a FINITE game

Example Tic - Tac - Toe

dynamic game with $N=2$ in which players move sequentially, and after each move, every player perfectly observes all past moves of the play.

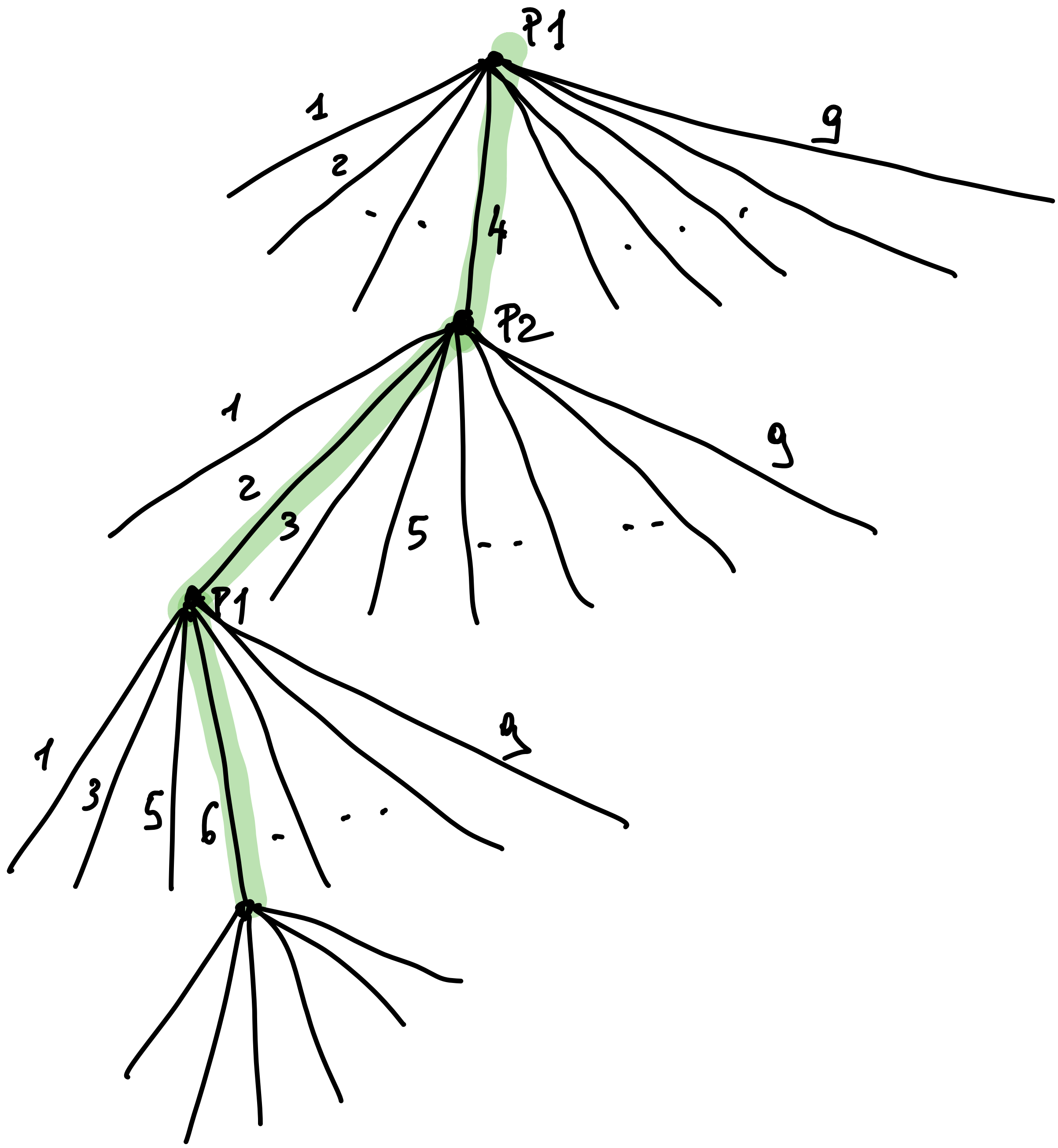


Player 1 uses X

Player 2 uses O

Who is the player who plays first.

Assume this is Player 1.



this is a game tree with nodes, branches. it gives a way to represent extensive-form game.

Focus on strategic game.

$$G = \{ N, (A_i)_{i=1}^N, (u_i)_{i=1}^N \}$$

Can we predict the result of this game? How players will play?

PRISONER'S DILEMMA (trade)

$N = 2$

Each players owns a fruit: orange and apple. They like fruits.

P1 owns apple
P2 owns orange.

P1's preferences are such that
consuming A+O ≻ consuming O ≻
≻ consuming A ≻ consuming no fruit.

Symmetric $\succsim$ for P2.

Each player can choose between
$(\text{Trade}, \text{Keep}) = A_1 = A_2$

For P1

$$u_1(K,T) > u_1(T,T) > u_1(K,K) > u(T,K)$$

$$3 > 2 > 1 > 0$$

$$u_2(T,K) > u_2(T,T) > u_2(K,K) > u_2(K,T)$$

P1 \ P2	K	T
K	1, 1	3, 0
T	0, 3	2, 2

For P1, for every action that P2 may choose, the payoff from "Keep" is greater than the payoff from "trade"

$$u_1(K, a_2) > u_1(T, a_2) \quad \forall a_2 \in \{K,T\}$$

P1 chooses Keep.

the same is true for P2.
Indeed,

$u_2(a_1, K) > u_2(a_1, T) \quad \forall a_1 \in \{K, T\}$

$\rightarrow u_2(K, K) > u_2(K, T)$
$u_2(T, K) > u_2(T, T)$

Hence, P2 would also strictly prefer Keep to trade, irrespectively of P1's choice.
P2 chooses Keep.

13/4/2022

Recall the prisoner's dilemma example.

For each player (1 or 2), the action K strictly dominates the action T. That is, T is a strictly dominated action $\forall$ player in the prisoner's dilemma.

Notice: no rational player plays strictly dominated actions.

Definitions.

Take a strategic game G, a player i, and two actions $a_i, b_i \in A_i$. Then we say that a_i strictly dominates b_i if

$$u_i(a_i, a_{-i}) > u_i(b_i, a_{-i}) \quad \forall a_{-i} \in A_{-i}$$

Ae , b_i is a strictly dominated action for player i in the game G, if there exists an action in A_i, that strictly dominates it.

Example

1 \ 2	L	C	R
U	1,0	1,2	0,1
D	0,3	0,1	2,0

$A_1 = \{U, D\}$

$A_2 = \{L, C, R\}$

C strictly dominates R $\Rightarrow$ R is a strictly dominated action, P_2 would not choose it.

Since R will never be chosen by P2, we find that (in the reduced game in which $A_2' = A_2 \setminus \{R\}$) D is strictly dominated by U for P1. => P1 would not choose D.

In the newly reduced game (with action sets A_2' and $A_1' = A_1 \setminus \{D\}$)
L is also strictly dominated by C.

↳ Prediction is that P2 chooses C and player 1 chooses U
↳ yielding payoffs. (1,2).

this is the iterated elimination of strictly dominated actions.

↳ we need to assume common knowledge of rationality for all players.

WEAKLY DOMINATED ACTIONS

1 \ 2	L	C	R
U	1, 0	1, 2	0, 2
D	0, 3	0, 1	2, 0

P2 is indifferent between C, R if P1 plays U

R is not strictly dominated now!

Definition.

Consider a strategic game G, player i, and a pair of actions $a_i, b_i \in A_i$ we say that a_i weakly dominates b_i

if
$$u_i(a_i, a_{-i}) \geq u_i(b_i, a_{-i}) \quad \forall a_{-i} \in A_i$$

and
$$u_i(a_i, a_{-i}) > u_i(b_i, a_{-i})$$ for at least one $a_{-i} \in A_{-i}$

$\Rightarrow$ Same as before to claim that b_i is a weakly dominated action for player i

Can we use iterated elimination of "weakly" dominated actions as a safe solution method? No.
there can be problems.

↳ Example.

1 \ 2	x_2	y_2
x_1	3,2	2,2
y_1	3,1	0,0
z_1	0,0	2,1

For P1, observe that x_1 weakly dominates y_1.

By contradiction. Let's eliminate y_1, by claiming rationality ⇒ not playing weakly dominated actions.

the reduced game is

1 \ 2	x_2	y_2
x_1	3,2	2,2
z_1	0,0	2,1

for P2, x_2 is weakly dominated by y_2

Prediction

→ P2 chooses y_2
→ P1 is indifferent between z_1 and x_1

↳ (x_1, y_2) ; (z_1, y_2)

Let's now observe that x_1 weakly dominates z_1 in original G.

the reduced game would be

P1 \	x_2	y_2
x_1	3,2	2,2
y_1	3,1	0,0

for P2, y_2 is weakly dominated by x_2

Prediction

(x_1, x_2) ; (y_1, x_2)

→ P2 chooses x_2
→ P1 is indifferent between x_1, y_1

the order of elimination of weakly dominated actions may affect the final outcomes!!

Predictions made relying on this methodology would not be reliable.

Hence we cannot conclude that players' rationality $\Rightarrow$ not playing weakly dominated actions.!!

Which methodology to use to solve strategic games if strict domination does not appear, as in case below?

P1 \ P2	L	C	R
T	0,4	4,0	5,3
M	4,0	0,4	5,3
B	3,5	3,5	6,6