Ex 1

Consider a public good provision game, with *n* individuals. Each individual must choose whether or not to contribute to the public good, the public good is provided if and only if at least one individual contributes by paying a tax *t*. Once public good is provided all individuals benefit from it. In particular, the value of the good, when provided, is *v* to each individual.

The utility of each individual is given by the value of the good (if provided) minus the tax *t* (if the individual contributes). When the public good is not provided each individual has utility equal to zero. Assume *v>t*.

   a) Show whether there exists a Nash equilibrium in which the public good is provided and all *n* individuals contribute.

       The case of n individual contributing cannot be an equilibrium because there exists an incentive to deviate for at least one individual, that is stop contributing and getting the good as well.

   b) Show whether there exists a Nash equilibrium in which none contributes and the public good is not provided.

       This case cannot be an equilibrium because at least one individual has the incentive to deviate from the prescribed action and start contributing. In this case his/her net utility is positive, given v>t

   c) Answer question b) under the condition *v<t*.

       When instead v<t there is not incentive to deviate and start contributing because the net utility will be negative thus the case in which none contributes is an equilibrium.

Ex. 2

Assume *3* individuals (*i=1,...,3*) can vote for a level of public good (*G*) under a majority voting rule. Individuals have preferences over the public and private good (*Z*) described by the following utility function:

$$U_i = \beta_i \ln(G) + Z_i$$

Assume that $\beta_1=0.4$, $\beta_2=0.1$, $\beta_3=0.5$. The income for each individual is w_i. The cost of the public good, *G*, is *c=1* and it is equally shared among individual, then each individual pays *1/3* of the total provision. The price of the private good Z_i is normalized to *1*.

   a) Discuss the necessary conditions for the Media Voter Theorem to hold.
   b) Find the Pareto Efficient level of public good.
   c) Find the level of public good provided by a majority voting.
   d) Verify whether the level of public good provided by a majority voting implies an over or under-provision of public good