Q 3

a) Assume three consumers of a public good. The total willingness to pay is given by the vertical sum of the individual demand functions:

$$P_1 + P_2 + P_3 = 280 - 3G$$

the optimal provision of public good is when W equalizes the marginal cost:

$$280 - 3G = 160$$

then $G = \frac{120}{3}$

b) For any price below 280 the demand for the public good is positive but no indvidual is willing to pay more than 140. Thus it is possible to achieve a price of 160 only is at least two consumers contribute. Clearly this induces the consumers to free ride, but this implies that no public good will be provided.

c) The benefit from the povision of public good is the are above the marginal cost and the aggregate demand function, that is

$$\frac{280 - 160}{2} \frac{120}{3} = 2400$$

Q 4

a) the **net** benefits are $v_1 = -35$, $v_2 = -15$, $v_3 = 45$. In the VCG mechanism the net payoff of the player 1 is:

$$U_1 = \begin{cases} v_1 + r_2 + r_3 & \text{if } r_1 + r_2 + r_3 \geq 0 \\ 0 & \text{if } r_1 + r_2 + r_3 < 0 \end{cases}$$

where $r_2 + r_3$ are the side transfers to player 1, the same argument holds for player 2 and 3

b) Now assume that 2 and 3 truthfully reveal their valuations. Then the payoff for player 1 becomes:

$$U_1 = \begin{cases} v_1 + 30 & \text{if } r_1 + 30 \geq 0 \\ 0 & \text{if } r_1 + 30 < 0 \end{cases}$$

It is possible to see that only $r1 = v1$ is dominant for player 1, where r_1 does not directly enter the his payoff, it only affect indirectly his payoff by means of the condition for the provision of the public good not to be provided $r_1 + 30 < 0$.

b) The provision of public is not optimal because $v_1 + v_2 + v_3 < 0$

c) If 3 reports 45, then 1 and 2 may collude by jointly setting $r_1 = -28 > v_1$ and $r_2 = -8 > v_2$. These reports will affect the provision of the public good, in fact $-28 - 8 + 45 > 0$. Moreover their payoffs would be:

$$U_1 = -35 - 8 + 45 = 2$$

$$U_2 = -15 - 28 + 45 = 2$$

Hence if 3 truthfully reveals his valuation, 2 and 1 have the incentive to collude. In particular, their own misreport does not directly increase their own

payoff, is the other's misreport that increases this payoff but each player need a collusive strategy to induce the other player to misreport and allow this increase in the payoff.