Basic of games
Nash Equilibrium

Preliminaries.

Rational individual, acting to maximize

•Objective function (payoff)

by

•Choosing Variable (action/strategy)

Strategic interaction ➔ the payoff of one agent depends on its own actions (strategies) as well as on the actions (strategies) of other agents

Strategic behaviour ➔ the optimal choice (strategy) of each agent depends of what he conjectures other agents will choose

Strategic interaction and ***strategic behaviour*** are widely diffused in any field of the social life: economics, business, politics, diplomacy……….

……….**and in our day life**:

Suppose you are invited to a party

You know that many other (nice) people have been invited, but you don't know how many of them will eventually go

If you go, you enjoy the party only if many of them come (otherwise the party will turn out quite boring).

Your payoff (i.e. how much you will enjoy the party) not only depends on your decision (i.e. whether to accept the invitation), but also on the decisions of the other people invited

In taking your decision, it is very likely that you will try to guess how many people will accept the invitation

Furthermore

other people invited will act like you (i.e. they will try to guess if you will go to the party)

when conjecturing what other people will do, you may need to conjecture what other people conjecture regarding your choice, and so forth.

Also, if the strategic interaction evolves over time (e.g. a sequence of parties with the same people invited), you should also take into account that your decision today can have an impact on the other people conjectures and decisions in the future.

Payoff interdependence (strategic interaction) ➔ a host of possibilities for **strategic behaviour**

Game theory ➔ strategic interaction and behaviour as a game among players

Basic elements of a Game

A game consists of:

- a set of **players**: 1,2,3,…., N

- a set of **strategies** (actions) **for each player**

- a set of **rules**:   who can do what and when;   players' information

- a set of **payoff functions** (one for each player):

 utility (payoff) each player gets as a result of each possible
 combination of strategies (strategy profile)

  ➔ a strategy profile is a specific combination of N
    strategies (one strategy for each player)

  ➔ a player's payoff function shows the payoff the
    player would get from each possible strategy profile

- Players are **rational** ➔ aim at maximising their payoff
          pursue this aim in a consistent way
          rational conjectures (beliefs) on other players

Generic definition of a game solution

A solution of a game is a combination of strategies (one for each player, i.e. a strategy profile) such that each player chooses to play his specified strategy according to a sensible notion of rational behaviour.

We call a solution **equilibrium of the game**.

As we will see, we need to specify **different notions** of equilibrium which better apply to different classes of games

For all of them we will stress the corresponding notion of rational behaviour

Game representation – normal form: payoff matrix

Player 2

Two players: 1, 2

Two strategies for each player

Player 1 strategies:
Top, Bottom

Player 2 strategies:
Left, Right

	L	R
T	5 5	6 3
B	3 6	4 4

Player 1

Four possible strategy profiles: (*T*, *L*) (*T*, *R*) (*B*, *L*) (*B*, *R*)

Each cell gives the payoffs associated to the corresponding profile

(*T*, *L*) ➔ Player 1 gets 5, Player 2 gets 5

(*T*, *R*) ➔ Player 1 gets 3, Player 2 gets 6

….and so on

Simultaneous moves game

The **normal form** representation is the most appropriate when players move simultaneously

Each player must decide his strategy before knowing the other players' decisions

Each player knows all the information summarised in the payoff matrix ➡ other players' strategy sets, all possible strategy profiles and the associated payoffs

First solution concept: equilibrium in dominant strategies

Applicable only to a limited class of games where each player has a dominant strategy

Dominant strategy ➡ **a strategy** giving the player a **higher payoff** than **any other strategy** whatever the rivals will choose

Player 2

	L	R
T	5 / 5	6 / 3
B	3 / 6	4 / 4

Player 1

Point of view of Player 1
B is better than *T either*
if Player 2 plays *L* or
if Player 2 plays *R*

B *dominant strategy* for Player 1

Point of view of Player 2
R is better than *L either*
if Player 1 plays *T* or
if Player 1 plays *B*

R *dominant strategy for* Player 2

In this game each player has a dominant strategy

Rational behaviour ➜ play the dominant strategy
robust notion of individual rational behaviour
each player does not need to assume that the rival is
rational (and even to know the rival's payoffs)

Equilibrium in dominant strategies ➜ (*B*, *R*)

Player 2

	L	R
T	5 / 5	6 / 3
B	3 / 6	4 / 4

Player 1

Famous game known as *Prisoner's dilemma*

Playing their respective dominant strategies, (*B*, *R*) players end up with (*4*, *4*)

But **both players** would be better off with (*T*, *L*) giving payoffs (*5*, *5*)

Why don't they go for (*T*, *L*) then?

Recall players **cannot decide jointly** what to do (i.e. **cannot cooperate** in selecting a preferred strategy profile). They behave individually.

For each player, the individual incentive is to play his dominant strategy
If player 1 played *T,* player 2 would respond by playing *R, not L*

Conflict between individual incentives and joint incentives

Nash Equilibrium (general solution concept)

What can we do if there are no dominant or dominated strategies?

In such cases, it is more apparent that each player's optimal strategy depends on his conjectures on which strategies the rivals will choose

Accordingly, the solution concept focuses on players' conjectures

Nash equilibrium (NE) ➔ most powerful solution concept ➔ provides a solution for almost all games (all games we will see in this course)

1. **Intuitive definition of NE ➔ a situation where:**

• players choose an optimal strategy given their conjectures on what the other players will do

2. Equivalent definition (more formal and operative)

NE is a strategy profile (i.e. one strategy for each player) such that the **specified (i.e. equilibrium)** strategy of **each player** is the **best reply** to the **specified (i.e. equilibrium)** strategies of the other players.

Notice ➔ in **equilibrium** each player gets the highest possible payoff conditional on the other players' **equilibrium** strategies

3. Third equivalent definition (no incentive to deviate)

NE is a strategy profile (i.e. one strategy for each player) such that **no player can unilaterally** change his strategy in a way that **improves** his payoff.

Best Reply Functions (*BRF*)

The second definition, based on the idea of **best reply**, allows to find the NE of a game in a simple way

Consider a game with two players: *A* and *B*

Two strategies for each player: Player *A* ➜ (a_1, a_2) Player B ➜ (b_1, b_2)

Suppose that the strategy profile (a_1, b_2) is a Nash Equilibrium

This means that: a_1 is the **best strategy (reply) of *A*** when *B* plays b_2

b_2 is the **best strategy (reply) of *B*** when *A* plays a_1

Then, to find a NE:

• *find the best reply of player A to each strategy of player B   (BRF of A)*

• *find the best reply of player B to each strategy of player A   (BRF of B)*

• *look for a strategy profile that appears simultaneously in the BRF of both players ("intersection of the Best reply functions of the two players")*

Nash Equilibrium – example

P2

	L	C	R
T	1 / 2	2 / 2	1 / 0
M	1 / 1	2 / 1	1 / 1
B	2 / 0	0 / 0	1 / 2

P1

BRF of P1

If P2 plays *L* ➔ *T*

If P2 plays *C* ➔ *T*

If P2 plays *R* ➔ *B*

BRF of P2

If P1 plays *T* ➔ *C*

If P1 plays *M* ➔ *C*

If P1 plays *B* ➔ *L*

Only the strategy profile (*T*, *C*) appears in the reply function of both players

(*T*, *C*) is the **only cell** where we find both a red **and** a blue circle

(*T*, *C*) is the unique Nash Equilibrium of this game.

Multiple Nash Equilibria

NE solves almost all games. But the solution is not always unique. One example of multiple Nash equilibria is the following *coordination game*

Pl 2

	Yes	No
Yes	1 / ②	0 / 0
No	0 / 0	② / 1

Pl 1

Players must (separately) answer *Yes* or *No* to the same question

Both are better off if the answer is the same (*coordination*)

But

P1 prefers *coordination* on *No*, P2 prefers *coordination* on *Yes*

(*disagreement on the point of coordination*)

Solving the game:
Both (*Yes*, *Yes*) and (*No*, *No*) are Nash equilibria