

# Provision of Public Good

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*Main textbooks:*

*Advanced Microeconomic Theory, Jehle, G. A., and P. J. Reny*

*Intermediate Public Economics, Hindriks, J and Myles G. D. (Ch. 5 first edition 2006, Ch 6 second edition 2013)*

# Summary

- Private market (provision) of public good leads to Pareto-inefficient level because of the *free-riding*
- Only Government intervention (or voluntary cooperation) may lead to an efficient provision:
  - **1)** *voting*
  - **2)** *personalized prices*
  - **3)** *mechanism design (preferences revelation mechanism)*

**Nonrivalry** and **nonexcludability** induces **free riding**: each consumer has an incentive to rely on others to make purchase (consume) of the public good

**Standard assumptions** required for the efficient provision of private goods **do not hold**

### Free riding: a formal approach

- Two-consumer ( $h=1,2$ )
- Two-good (private,  $x$ , and public good,  $g$ )
- Consumers choose the optimal level of private and public good, given their prices normalized to 1.

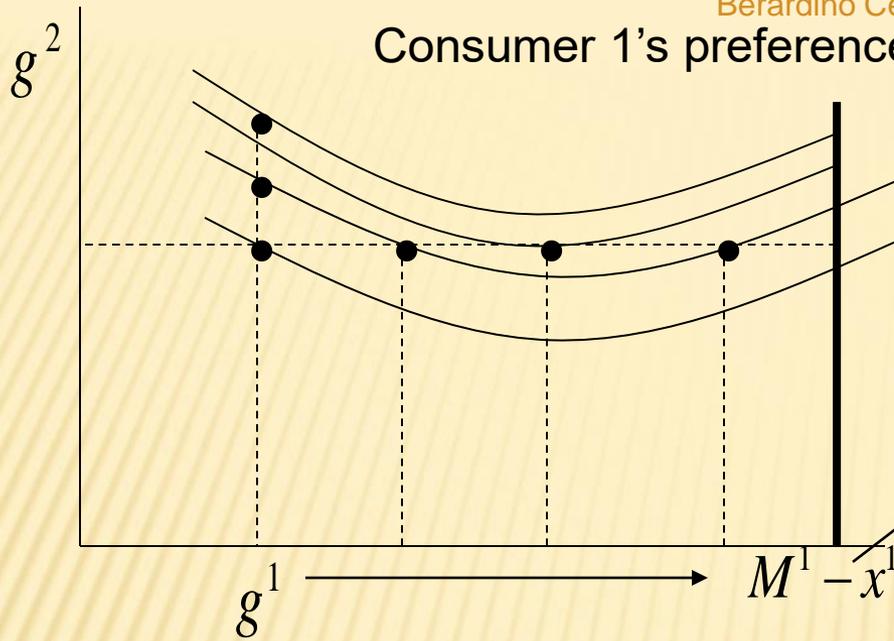
$$\text{➤ } U^h(x^h, G^h)$$

$$G^h = g^1 + g^2$$

$$M^h = x^h + g^h$$

$$U^h(M^h - g^h, g^1 + g^2) \longrightarrow \text{Strategic interaction (game theory approach)}$$

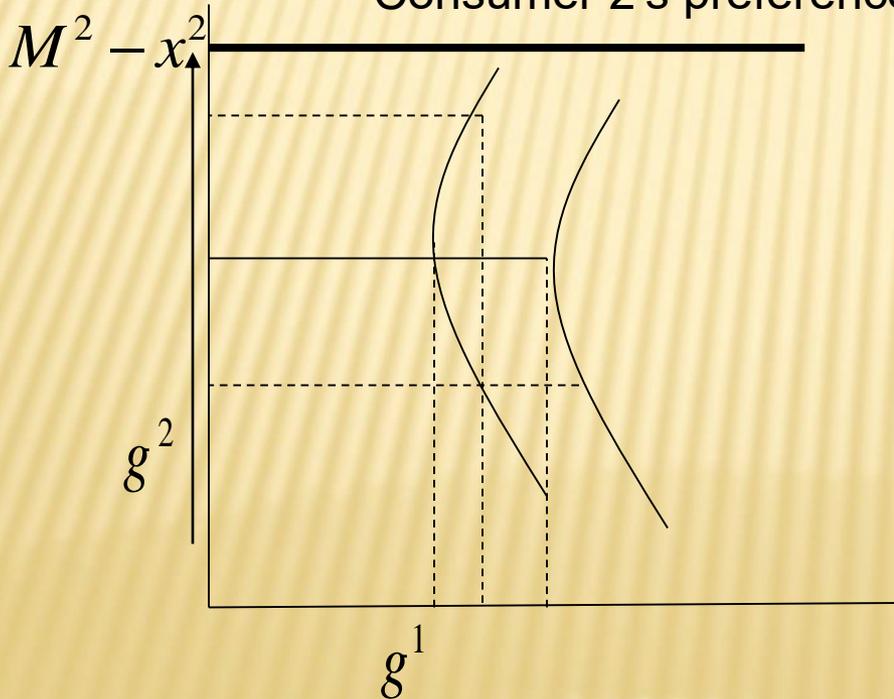
### Consumer 1's preferences



$$U_{g^2}^1 > 0 \quad U_{g^1 g^1}^1 < 0$$

BC  
Income places an upper limit for  $g^1$

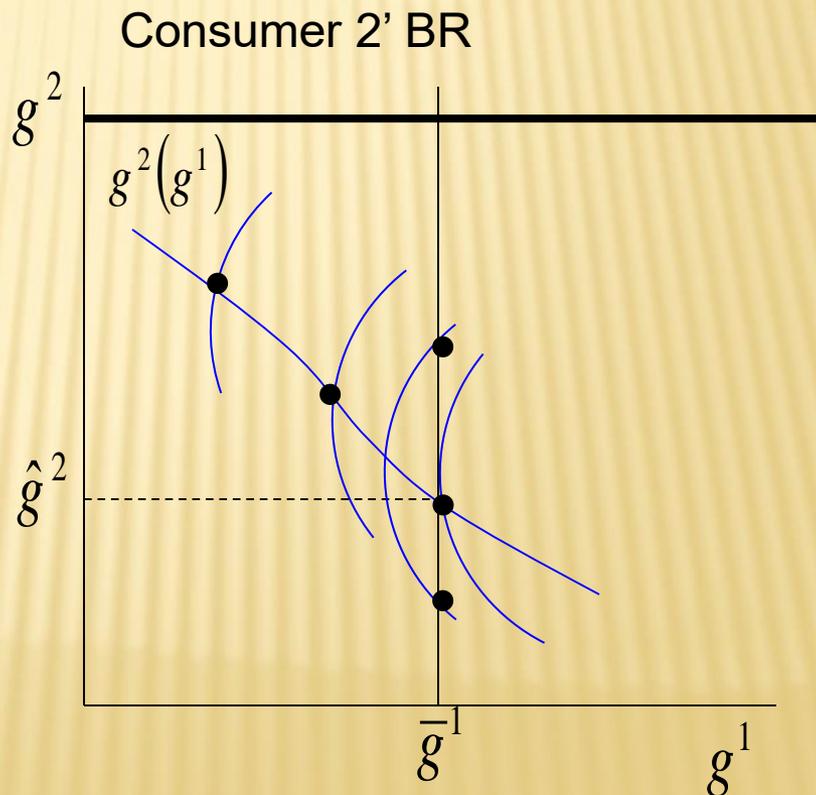
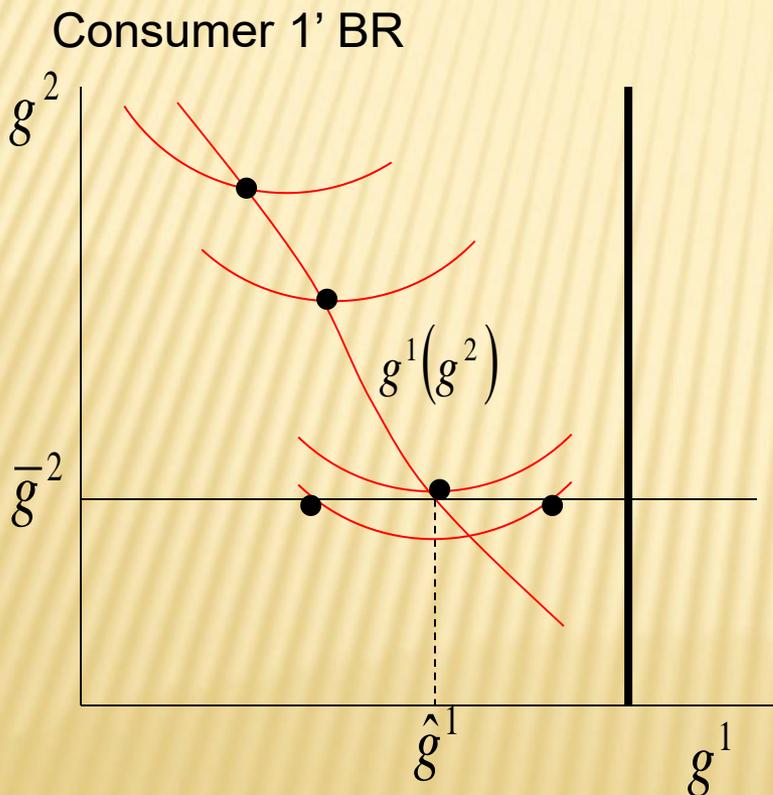
### Consumer 2's preferences

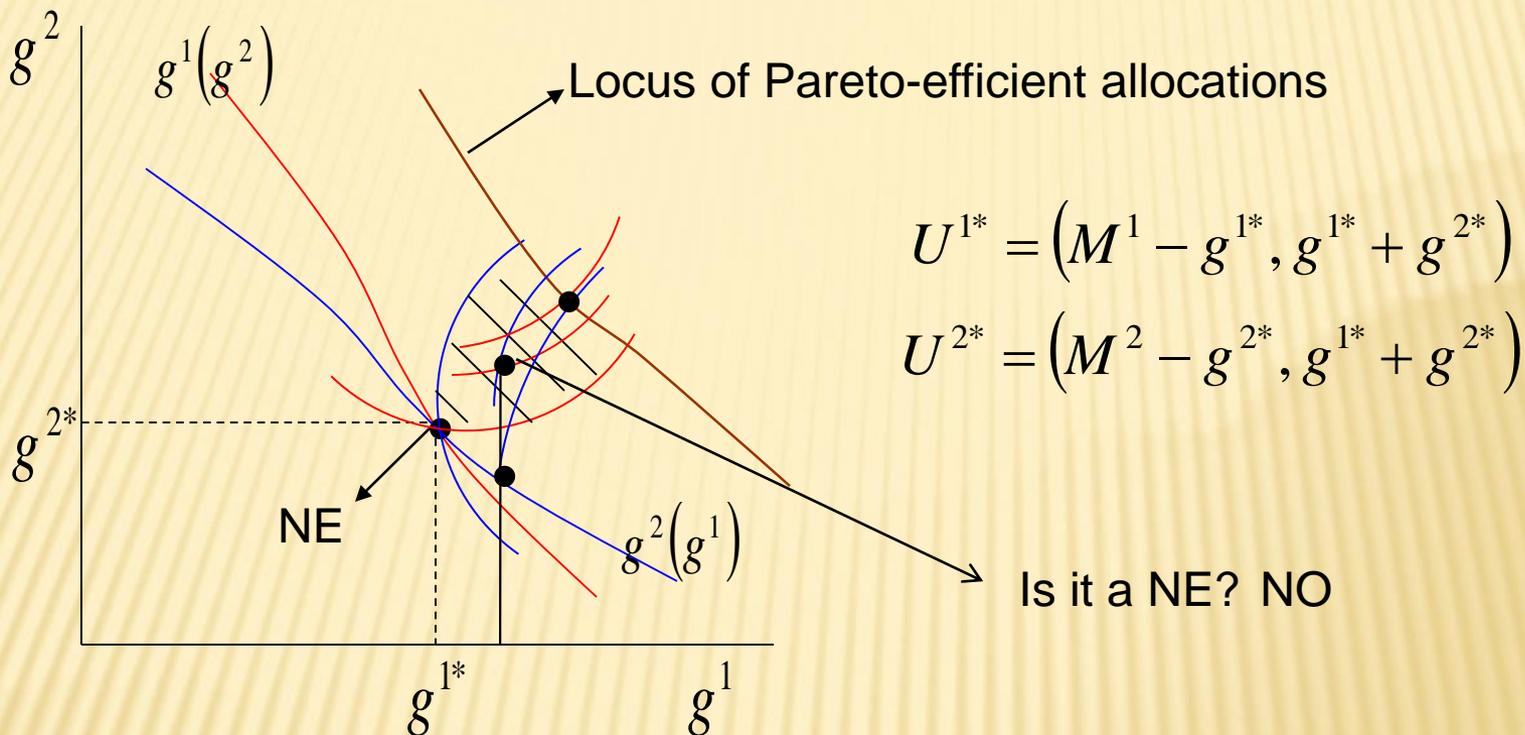


$$U_{g^1}^2 > 0 \quad U_{g^2 g^2}^2 < 0$$

**Nash equilibrium (simultaneous one-shot choice)** : any pair  $(g^{1*}, g^{2*})$  such that  $g^{1*}, g^{2*}$  solve:

$$\begin{cases} BR_1 = g^1(g^2) \\ BR_2 = g^2(g^1) \end{cases} \quad \text{s.t.} \quad \begin{aligned} g^1(g^2) &\equiv \arg \max_{g^1} U^1 \\ g^2(g^1) &\equiv \arg \max_{g^2} U^2 \end{aligned}$$





The NE is not Pareto-efficient.

There always exists a strict Pareto-improvement at every feasible combination

$$g^{1*} < \tilde{g}^1, g^{2*} < \tilde{g}^2 \text{ such that } U^{1*}(g^{1*}, g^{2*}) < U^1(\tilde{g}^1, \tilde{g}^2) \text{ and } U^{2*}(g^{1*}, g^{2*}) < U^2(\tilde{g}^1, \tilde{g}^2)$$

$\tilde{g}^1, \tilde{g}^2$  are **Pareto efficient** only when indifference curves are tangent, but **not NE**

- ❑ Given the strategic incentive to free ride, the equilibrium quantity is always lower than the Pareto preferred allocation
- ❑ Each consumer relies on the other's consumption to achieve a sufficient level of public good avoiding to provide themselves.
- ❑ Strategic interaction implies that despite indifference curves are tangent, equality between marginal rates of substitution (as in competitive two private goods case) does not hold.

As long as consumers make their choice noncooperatively and simultaneously in a one-shot game, only government intervention can provide an efficient level of public good...

..under the following condition:

Samuelson rule (1954 RES) for the efficient provision of public goods

$$MRS_{G,x}^1 + MRS_{G,x}^2 = 1$$

Obtained as follows:

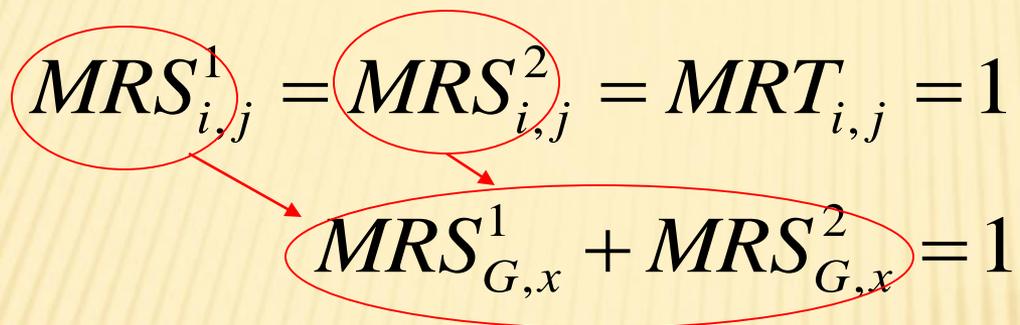
Tangency between indifference curves at the  
Pareto Frontier implies

$$\underbrace{MRS_{G,x}^1 + MRS_{G,x}^2}_{\text{Marginal benefit of 1 more unit of public good in the economy}} = \underbrace{1}_{\text{Marginal cost of one more unit of public good (1 unit of private good)}}$$

Marginal benefit of 1 more unit of  
public good in the economy

Marginal cost of one more unit of  
public good (1 unit of private good)

## Efficient provision of private goods vs Samuelson condition

$$MRS_{i,j}^1 = MRS_{i,j}^2 = MRT_{i,j} = 1$$
$$MRS_{G,x}^1 + MRS_{G,x}^2 = 1$$


- Without public good the further private good can be given to anybody (no matters who) and efficiency keeps holding because marginal benefit of all consumers equalize (*distributive concerns arise*)
- With public good, equality among marginal benefits is not necessary for efficiency because an extra unit of public good benefits all consumers (*no redistributive concerns arise*)

## **Drawbacks of Samuelson`s rule:**

- Tax distortion prevents efficiency as well (standard inefficiency from taxation, in terms of welfare loss)
- Asymmetric information about the consumers` preferences (utilities, MRS)

## **Alternative methods for providing public good ( NO free riding):**

1. Voting
2. Personalized prices (Lindahl Prices)
3. Mechanism design:
  - Preference revelation mechanism
  - Clarke-Groves Mechanism

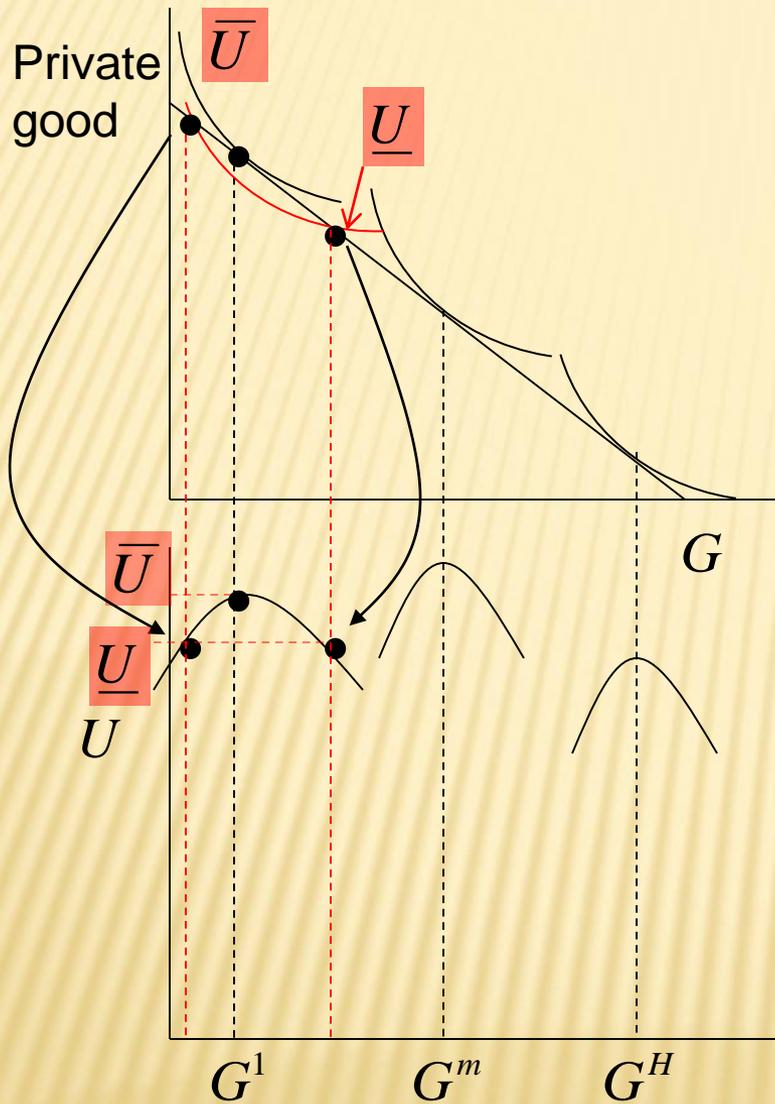
## 1. Voting

Comparison between the voting outcome and the efficient provision of public good

Basic remark about voting: strictly **concave utility** functions imply **single-peaked preferences**

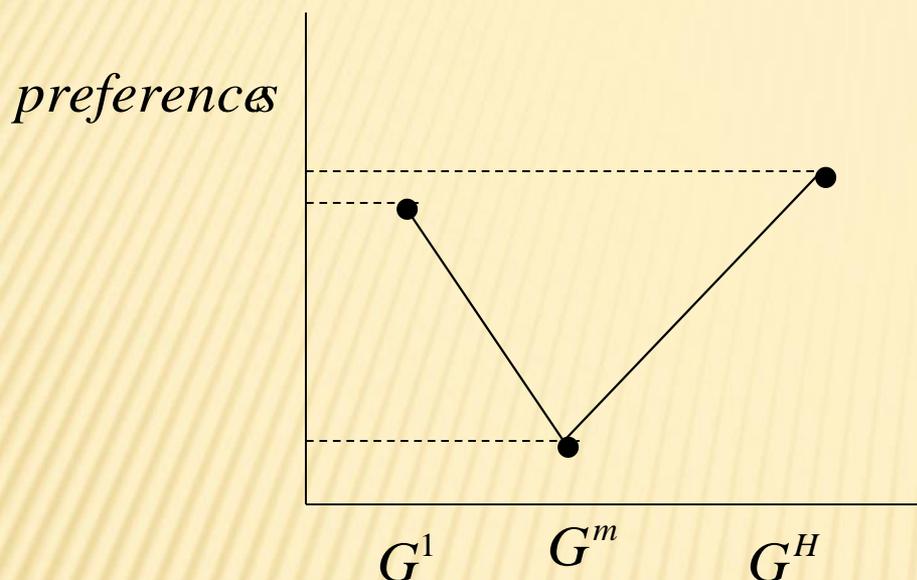
- $H$  Consumers (voters) are differentiated according to one dimension (income) and are asked to vote for public good  $G$  in a majority voting (or alternatively they can vote for a tax used to finance public good)
- The cost of public good is equally shared among consumers, with each unit of public good costs  $\frac{1}{H}$
- Given the budget constraint  $M^h = x^h + \frac{G}{H}$ :

$$U^h\left(M^h - \frac{G}{H}, G\right)$$



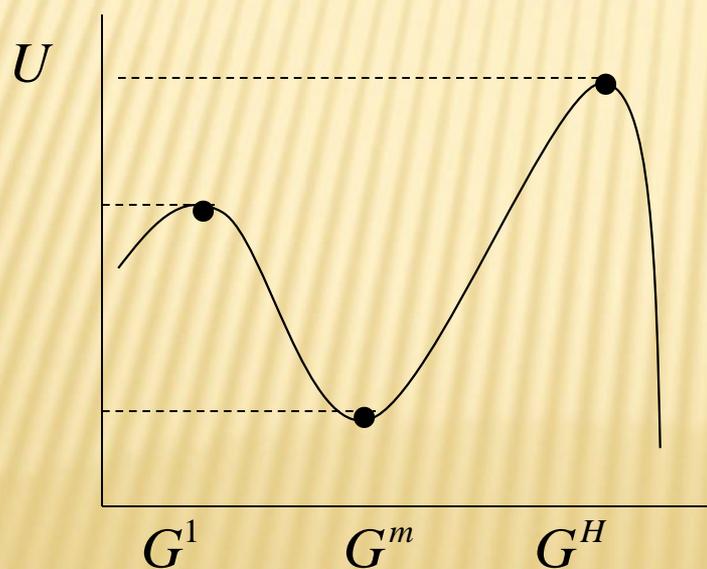
1. utilities are concave then preferences are single peaked (only a single value of  $G$  maximizes utility for voter  $h$ ).
2. The variable the voters vote for is *one dimensional* and ordered in a transitive way
3. Assume  $H > 2$  (odd)

No single peaked preferences, no strictly concave function



Black's (1958) median voter theorem applies: the voter with the median preference for public good,  $G^m$ , is decisive (pivotal) in the majority voting..

.. the equilibrium level of public good **resulting from voting** is the median among all the most preferred levels



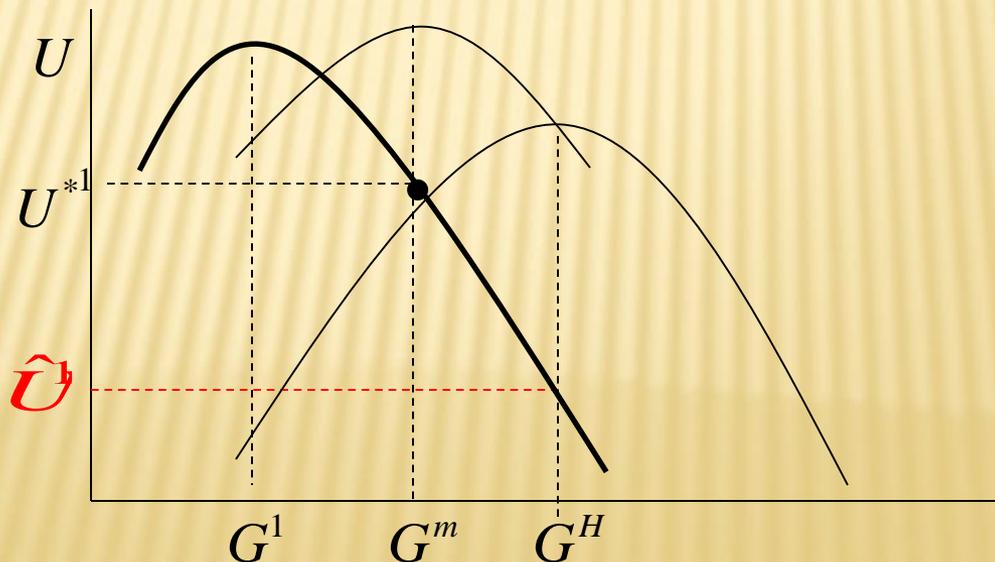
Formally the equilibrium strategy for any player  $h$  is *voting sincerely and disclosing his true preferences*.

**Proofs by contradiction:** simple case with three voters and three potential equilibrium candidate levels of  $G$ .

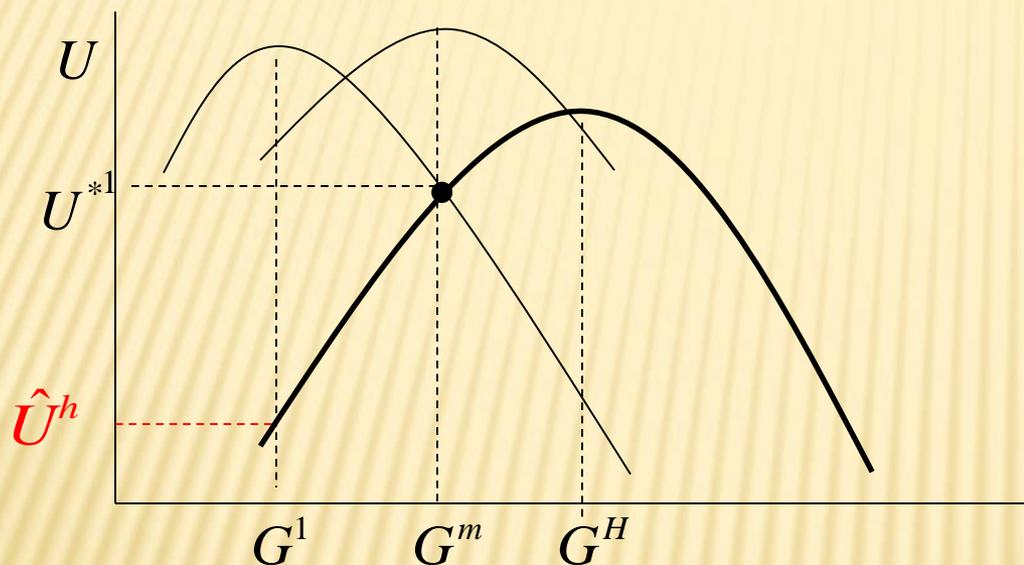
We construct an alternative strategy entailing that:

*Each voter  $h$  votes strategically (not sincerely) in order to change the result of the voting equilibrium.*

- Incentive to deviate from equilibrium strategy for the voter on the left of the median voter: to defeat the median voter he has to vote the best level for the voter at the right of the median one. But, this is not an optimal choice. He would be better off with the level of median voter



- Incentive to deviate from equilibrium strategy for the voter on the right of the median voter: to defeat the median it needs to vote the best level for the voter at the left. Again, not optimal.



- Median voter's incentive to deviate from the equilibrium strategy: he never deviates because by definition  $G^m$  is its best choice

Then solving the maximization problem of the median voter

$$\max_{\{G\}} U^m \left( M^m - \frac{G}{H}, G \right)$$

$$MRS^m = \frac{1}{H}$$

Then the level of the median voter is efficient (we have the Samuelson rule) only if

$$MRS^m = \sum_{h=1}^H \frac{MRS^h}{H} \longrightarrow \text{Mean MRS of voters}$$

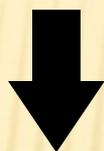
Voting does not provide the efficient level of public good unless we have this particular case: only median voter's preferences matter

$$MRS^m < \sum_{h=1}^H \frac{MRS^h}{H}$$

Under provision of public good

$$MRS^m > \sum_{h=1}^H \frac{MRS^h}{H}$$

Over provision of public good



Drawback: changing the voters' preferences do not alter the majority voting outcome but it affects the efficient level of the public good