

EXERCISE 31.1 (Contributing to a public good) Each of n people chooses whether or not to contribute a fixed amount toward the provision of a public good. The good is provided if and only if at least k people contribute, where $2 \leq k \leq n$; if it is not provided, contributions are not refunded. Each person ranks outcomes from best to worst as follows: (i) any outcome in which the good is provided and she does not contribute, (ii) any outcome in which the good is provided and she contributes, (iii) any outcome in which the good is not provided and she does not contribute, (iv) any outcome in which the good is not provided and she contributes. Formulate this situation as a strategic game and find its Nash equilibria. (Is there a Nash equilibrium in which more than k people contribute? One in which k people contribute? One in which fewer than k people contribute? (Be careful!))

Contributing to a public good

The following game models the situation.

Players The n people.

Actions Each person's set of actions is $\{\text{Contribute}, \text{Don't contribute}\}$.

Preferences Each person's preferences are those given in the problem.

An action profile in which more than k people contribute is not a Nash equilibrium: any contributor can induce an outcome she prefers by deviating to not contributing.

An action profile in which k people contribute is a Nash equilibrium: if any contributor stops contributing then the good is not provided; if any noncontributor switches to contributing then she is worse off.

An action profile in which fewer than k people contribute is a Nash equilibrium only if no one contributes: if someone contributes, she can increase her payoff by switching to noncontribution.

In summary, the set of Nash equilibria is the set of action profiles in which k people contribute together with the action profile in which no one contributes.