Optimal taxation

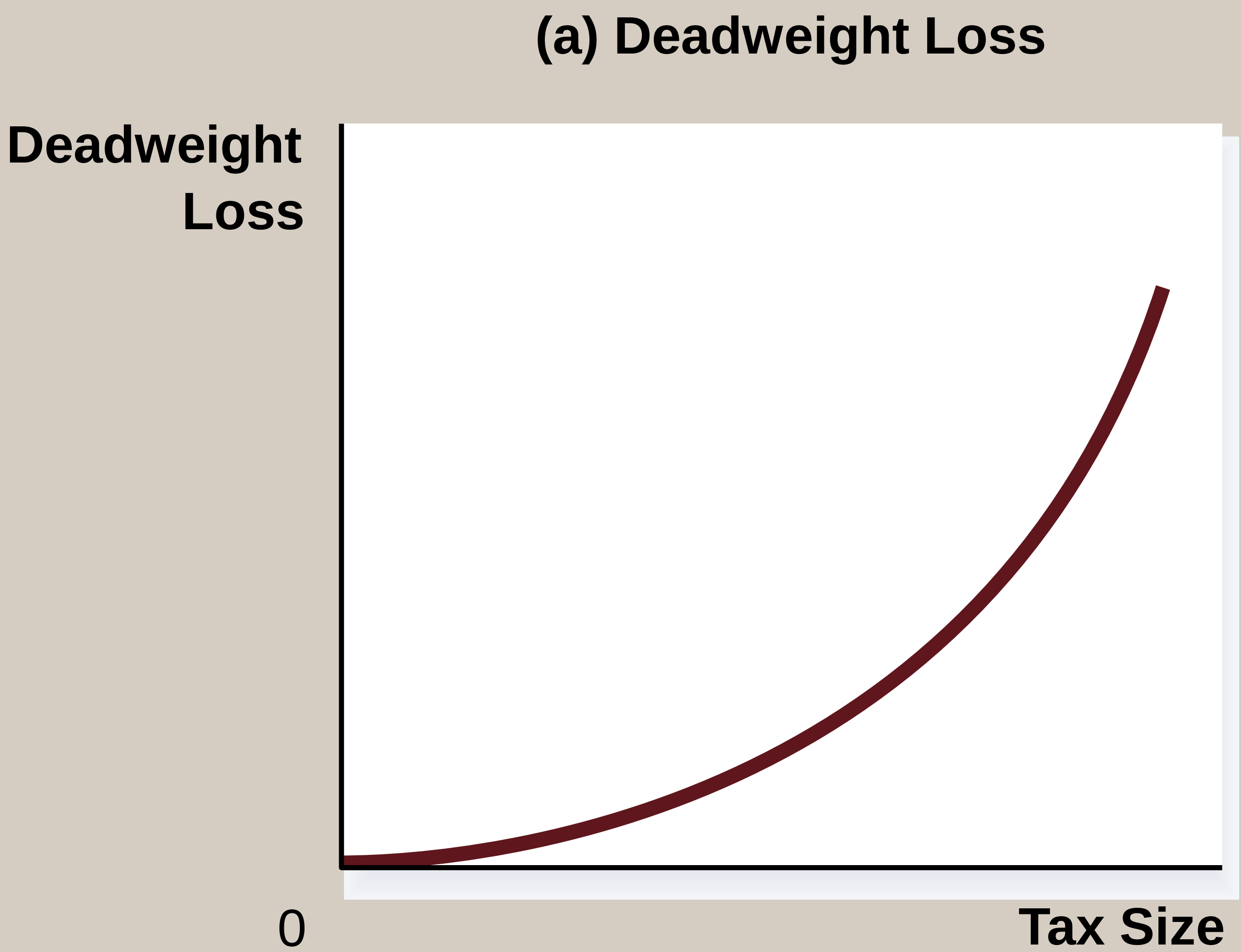
Commodities

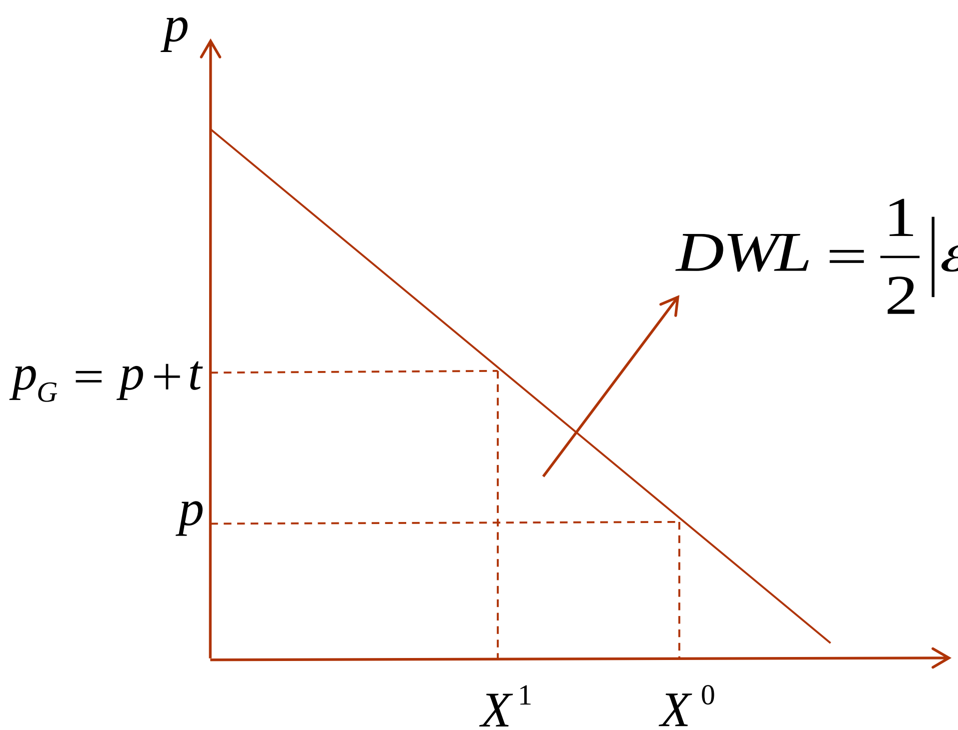
Berardino Cesi, Rome Tor Vergata 2020

Preliminary notes and definitions

- Lump sum taxation (no distortion): it does not affect the consumption behavior of the individual (it does not work through the price mechanism)(First best).
- No distortion implies no change in the behavior <u>that affects the level of the tax</u>.
- No substitution effect implies $\Rightarrow$ No deadweight loss.
- Commodity taxation (distortion: deadweight loss)
- With a deadweight loss, the reduction in the consumer welfare exceeds the tax revenue.

DO YOU REMEMBER ??



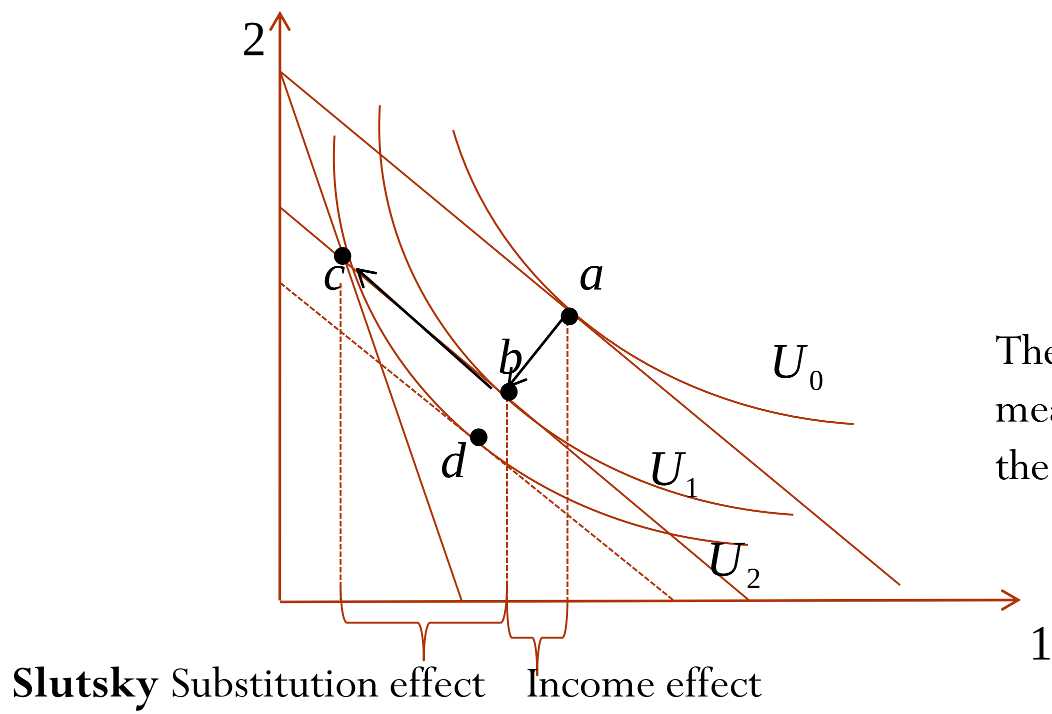


$$DWL = \frac{1}{2}\left|\varepsilon^d\right|\frac{X^0}{p}t^2$$

DWL:

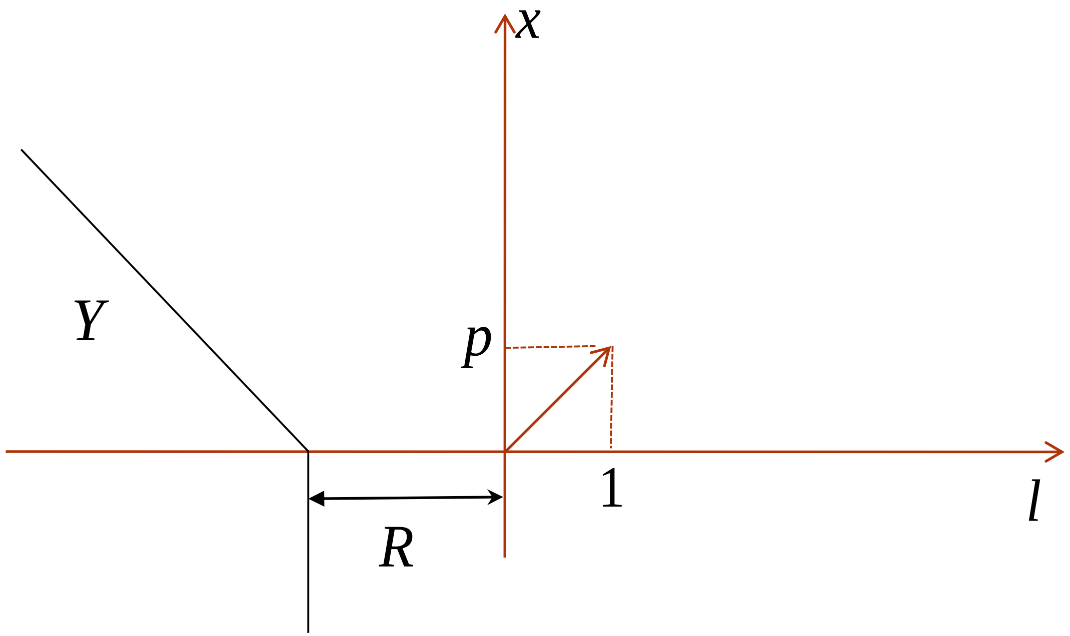
- is a convex function of the tax rate.
- increases with the price elasticity of demand

- Measuring the DWL if the good 1 is taxed:
  - Difference from the utilities under a lump sum tax and a commodity tax, when the same tax revenue is raised ($DWL = U_1 - U_2$)
  - A lump sum leads to *b*, a commodity taxation to *c* (both on the same budget constraint, same tax revenue)
  - The substitution effect is the cause of the $DWL = LS_{a-b} - LS_{a-d} > 0$

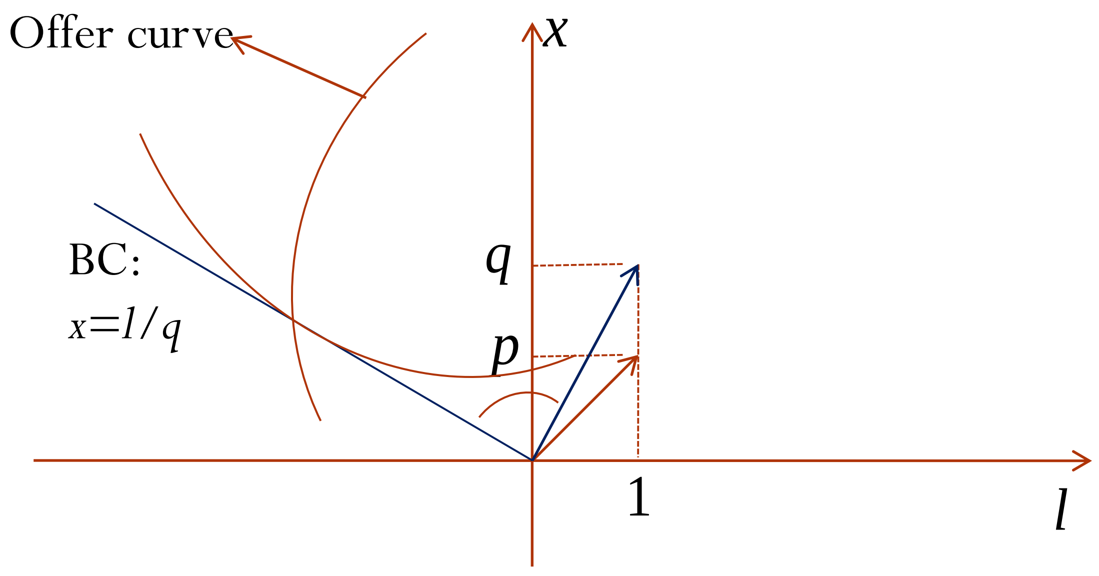


The LS necessary to measure the reduction in the utility from U_0 to U_1

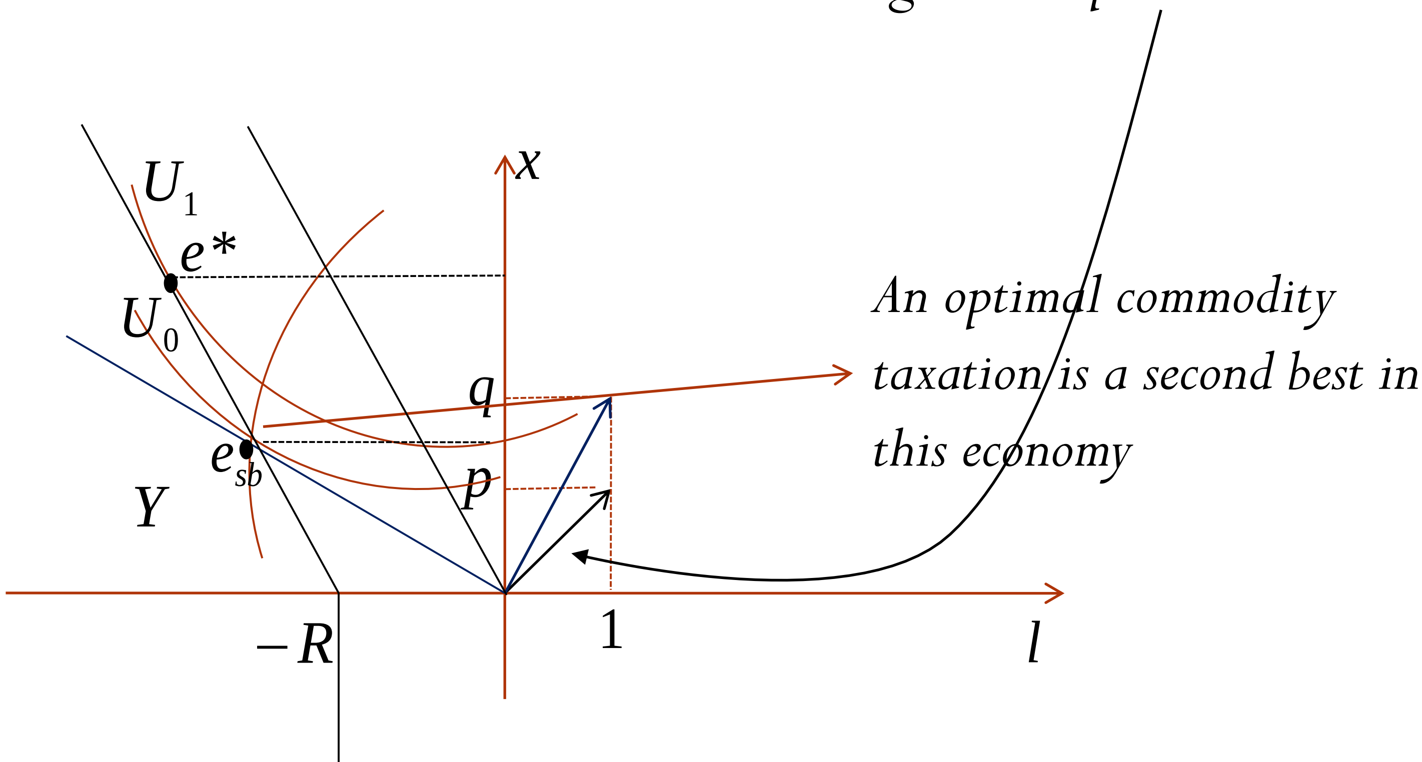
- An Optimal Tax System is the set of taxes inducing the highest level of welfare at a given level of tax revenue.
- An optimal commodity tax is a second best (lower welfare than a LS)
- Assume a simple model with a two-good (l,x) economy, 1 consumer-1 firm (**market is competitive**, $p=w$)
- Production: $Y(l): l \rightarrow x$



- Consumer
  - *w=1 (normalized),q is (gross) price of x*
  - *Budget constraint (BC): qx=wl then qx= 1*
  - *Tax rate: t=q-p*



- At optimal tax rate: $t_{sb} = q - p$, consumer chooses consumption e_{sb} and; $R_{sb} = t_{sb} x_{sb}$, $x(e^*) > x(e_{sb})$
- With a *LS*, budget and production frontier coincide (Pareto improvement), $x(e^*) = x(e_{sb})$, the same tax revenue R raised by the LS but the consumer is better off
- First best: e* achievable with the new LS budget BG: *qx=1-R. LS=R*



Competitive market: the Inverse elasticity rule

- Assume *i=1,..,n* goods with different elasticity
  - Which is the optimal tax?
  - How the tax burden is spread across different goods?

1 consumer-Economy

- *i=1,..,n* goods are produced with constant returns to scale by competitive firms (marginal cost independent from the scale of production).
- Production function: $Y(l)$, wage assumed *numeraire (w=1)*.
- Net tax (producer) price of good is $p_i = c_i$, c_i units of labor (*l*) required to produce good *i*.
- The gross (consumer) price is: $q_i = p_i + t_i$
- Required tax revenue: $R = \sum_{i=1}^{n} t_i x_i$, x_i consumption level of good *i*

  $x_0 =$ Labor (untaxed)

Independent demands $\varepsilon_{i,j}=0$ (our previous intuitive case)

- Assume a three-good economy with the consumer's utility
  $U=\left(x_0,x_i,x_{j\neq i\neq 0}\right)$
- Budget constraint: $x_0=\sum_{i=1}^{2} q_i x_i$
- The tax revenue constraint is:
- $$R=\sum_{i=1}^{2} t_i x_i \quad ,t_i=q_i-p_i \qquad \sum_{i=1}^{2} q_i x_i-\sum_{i=1}^{2} p_i x_i=R$$
- The government infers taxes from the maximization of the consumer's utility

$$\max_{\{x_i\}} L=U\left(x_0,x_i,x_j\right)+\lambda\left[\sum_{i=1}^{2} q_i x_i-\sum_{i=1}^{2} p_i x_i-R\right]$$

Technical advanced note (NOT COMPULSARY BUT USEFULL FOR A MORE ADVANCED STUDY)

- Consumer's maximization anticipated by the government when setting the optimal quantities

$$L : U\left(x_0, x_i, x_{j \neq i \neq 0}\right) + \alpha\left(x_0 - q_1 x_1 - q_2 x_2\right)$$

FOC (First order conditions for the optimal x)

$$U_{x_0} + \alpha = 0 \longrightarrow U_0 = -\alpha$$

$$U_{x_i} - \alpha q_i = 0 \longrightarrow U_i = \alpha q_i$$

$$U_{x_j} - \alpha q_j = 0 \longrightarrow U_j = \alpha q_j$$

with $-\alpha$ (α) the marginal disutility (utility) of labor (income)

- Since demands are independent $q_i = q_i(x_i)$, and using $x_0 = \sum_{i=1}^{2} q_i x_i$ after computing the FOC for any quantity x_i the optimal tax rate of each commodity x_i is:

Tax rate $$\frac{t_i}{p_i + t_i} = -\left[\frac{\lambda - \alpha}{\lambda}\right]\frac{1}{\varepsilon_i^d}$$

λ Marginal cost (utility cost) of one more unit of tax revenue
α Marginal utility of another unit of income for the consumer
$\lambda > \alpha$: since taxes are distortionary

Main property

Efficiency vs. Equity:

- More of the tax burden on the goods with the lower deadweight loss (less elastic) (Efficiency)
- Necessary goods (with low elasticity) are highly taxed
- Lower income consumers bear relatively more of the tax burden than high-income consumers (**no Equity**)
- The same qualitative result about equity holds when demand are not independent $\varepsilon_{i,j} \neq 0$ (**Ramsey Rule**)

Tax rate $$\frac{t_i}{p_i + t_i} = -\left[\frac{\lambda - \alpha}{\lambda}\right]\frac{1}{\varepsilon_i^d}$$

Optimal taxation (Imperfect Competition)

- 2-good economy, 1 labor-good, 1 household.
- Good 1 (competitive market, constant return to scale)
- Consumer price, $q_i = t_i + p_i$

$$U = U\left(x_0\left(q_1, q_2\right), x_1\left(q_1, q_2\right), x_2\left(q_1, q_2\right)\right)$$

$$R = t_1 x_1 + t_2 x_2$$

- Good 2 (Monopolist chooses output $x_2\left(q_1, q_2\right)$)

$$\max \pi_2 = \left[q_2 - c - t_2\right] x_2\left(q_1, q_2\right)$$

$q_2 = q_2\left(q_1, t_2\right)$: Profit-maximizing price

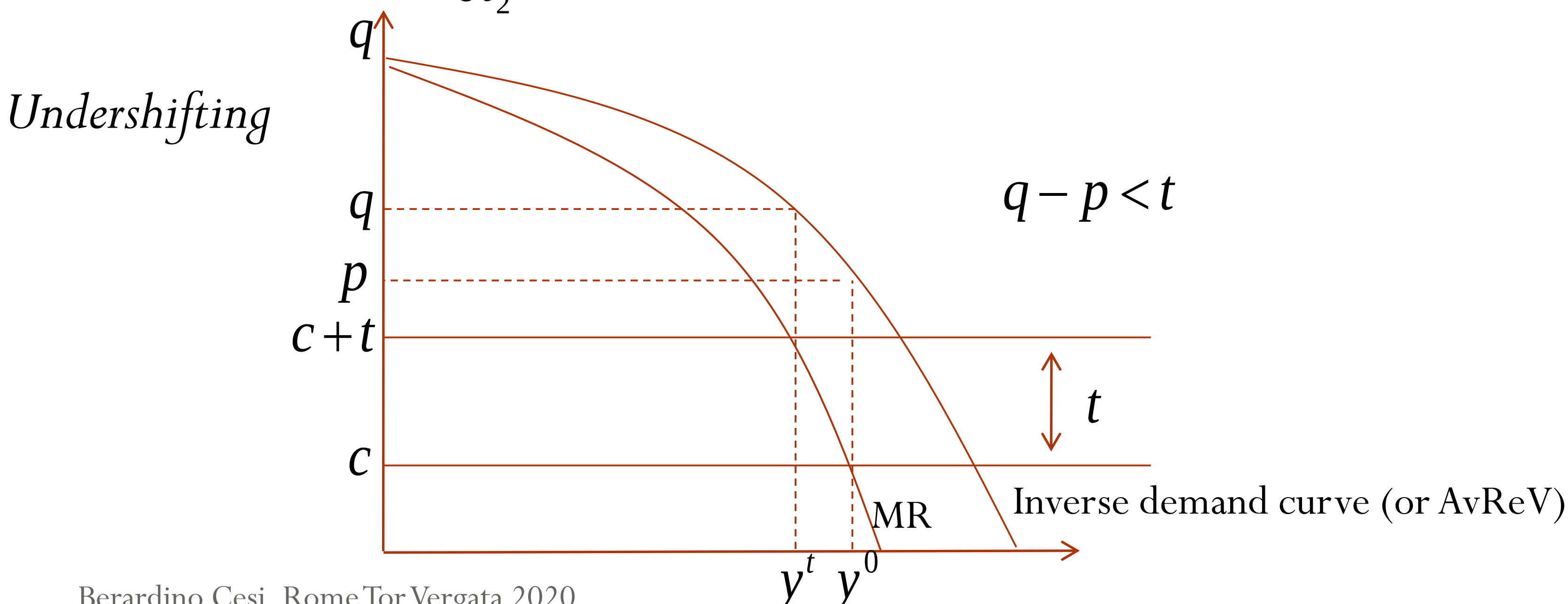
$\dfrac{\partial q_2}{\partial t_2}$: rate of shifting of the tax

Taxation under imperfect competition: a brief recall

$\frac{\partial q_2}{\partial t_2} < 1$ *Undershifting*: not all the tax is passed on to the consumers but a part is absorbed by the monopolist

$\frac{\partial q_2}{\partial t_2} > 1$ *Overshifting*: more than the tax is passed on to the consumers

- Is it possible to show that the sufficient condition for $\frac{\partial q_2}{\partial t_2} < 1$ is $p''(x) < 0$ and $p''(x) > 0$ for $\frac{\partial q_2}{\partial t_2} > 1$



Optimal taxation

Income

- Workers pay an *ad-valorem tax* computed as a share of their labor income

Italy individual income tax rates 2018

Tax (%)	Tax Base (EUR)
23%	0 - 15,000
27%	15,001-28,000
38%	28,001-55,000
41%	55,001-75,000
43%	75,001 and over

New UK Income Tax Rates and Brackets for 2018/19

Tax Rate (Band)	Taxable **Income**	**Tax** Rate
Personal allowance	Up to £11,850	0%
Basic rate	£11,851 to £46,350	20%
Higher rate	£46,351 to £150,000	40%
Additional rate	Over £150,000	45%

France net taxable income earned in 2018 (taxable in 2019)

Income Share	Tax Rate
Up to €9,964	0%
Between €9,964 - €27,519	14%
Between €27,519 - €73,779	30%
Between €73,779 - €156,224	41%
Above €156,224	45%

Examples of a Flat Tax

Russia is the largest nation in the world to use a flat tax. Russia imposes a **13% flat tax** on earnings (residents and non-residents). The nation has considered moving to a progressive tax to boost tax revenue.

Other countries: Estonia, Latvia, and Lithuania

Italy? Who knows..

Ex. Gross income 40.000

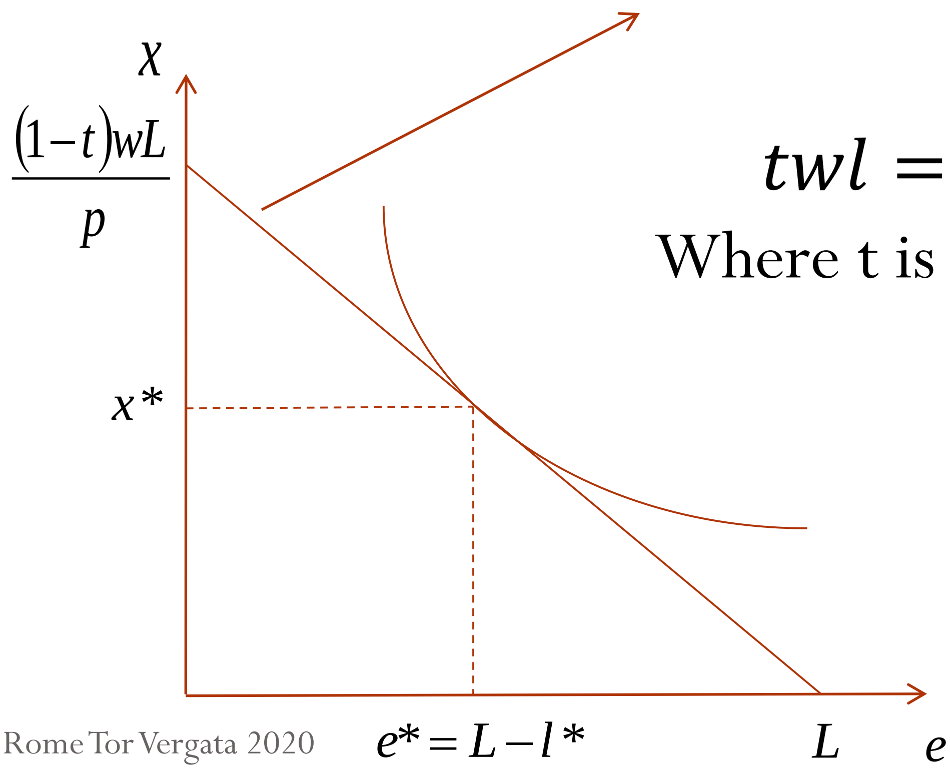
- Tax to pay:
  - 23% of 15,000=3,450 (A)
  - 27% of 28,000-15,001= 3509 (B)
  - 38% of 40000-28001=4559 (C)
- Amount to pay (tax revenue)=A+B+C

Tax and labor supply

- consumption-leisure utility

$$U(x, L-l), U_l < 0 \text{ : disutility from labor supply}$$

- x consumption, l labor supply, L maximum time endowment, leisure: $e = L - l$, budget line $px = (1-t)wl$



$$twl = income\ tax$$

Where t is a % of the income

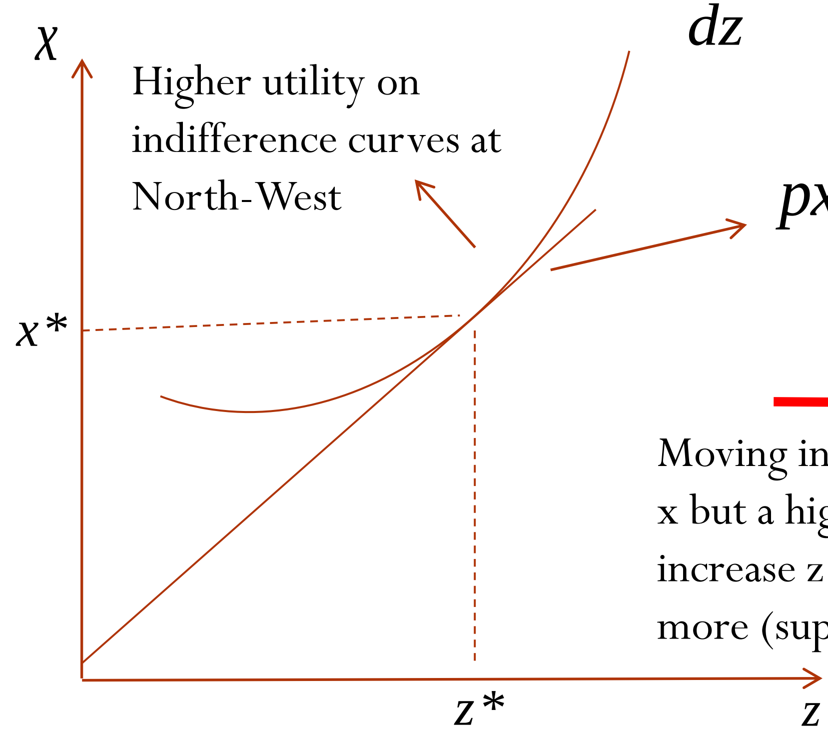
Tax and labor supply: preliminary intuitions

- Consumption (x)/pre-tax income (z) utility

$$z = wl \Rightarrow U\left(x, \frac{z}{w}\right)$$

$$MRS_{zx} = U_z dz + U_x dx = 0$$

$$\frac{dx}{dz} = -\frac{U_z}{U_x} > 0$$

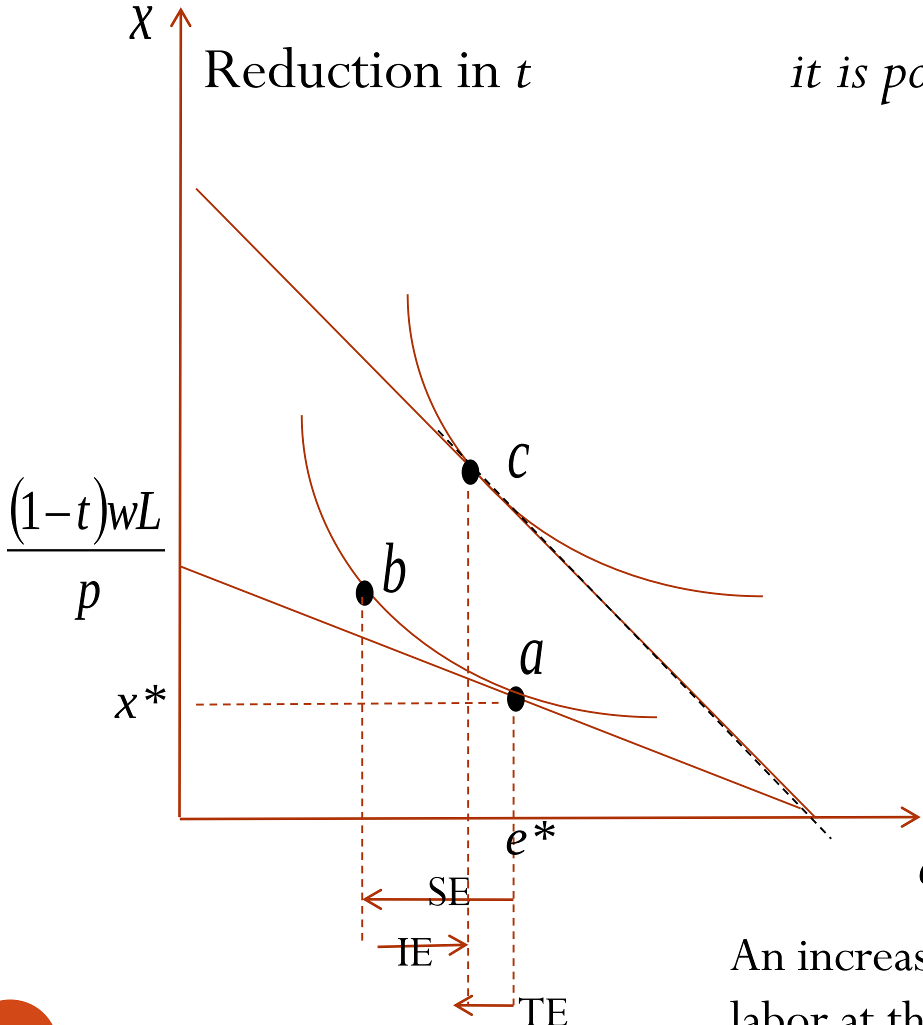


$$px = (1-t)z$$

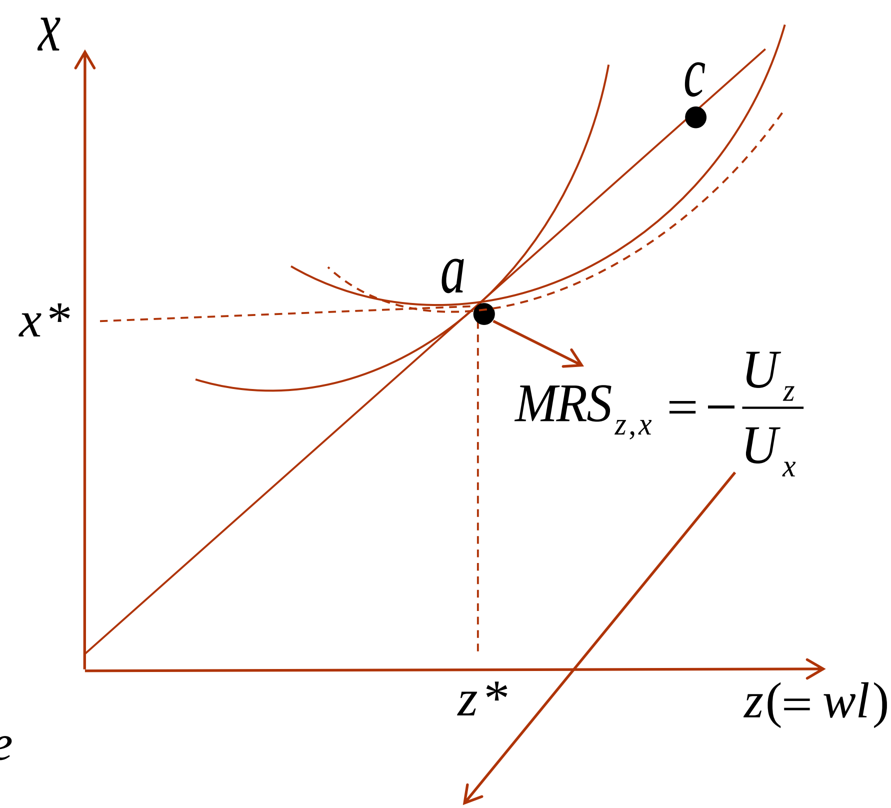
Moving in this direction implies the same x but a higher z, that is costly because to increase z the worker needs to work more (supply more costly l)

Tax and labor supply

Ambiguous effect on labor supply: $x_b > x_a$, $e_b < e_a$, but $l_c \le l_a$??



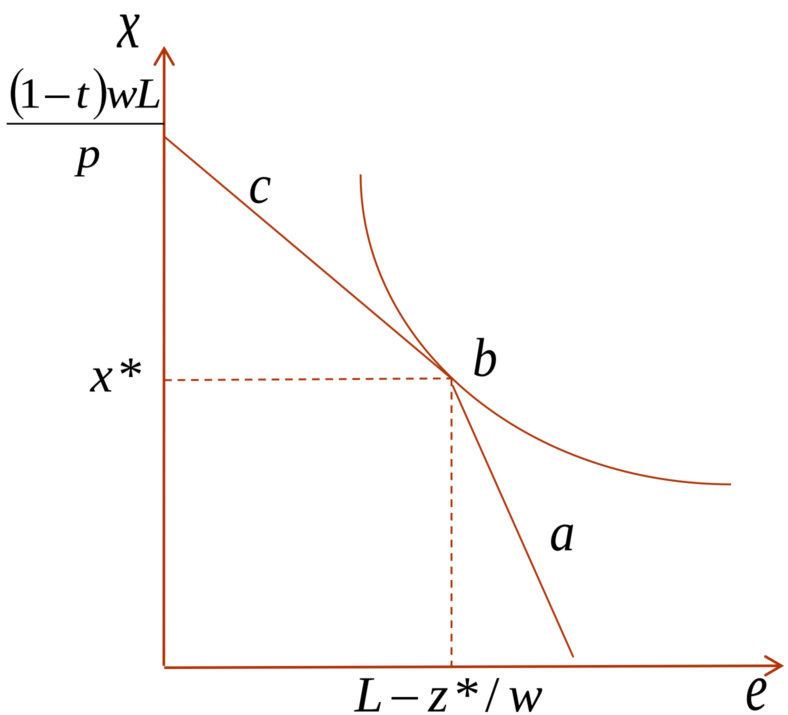
it is possible $z_c > z^*$ *and* $l_c \le l_a$ *if IE dominates*

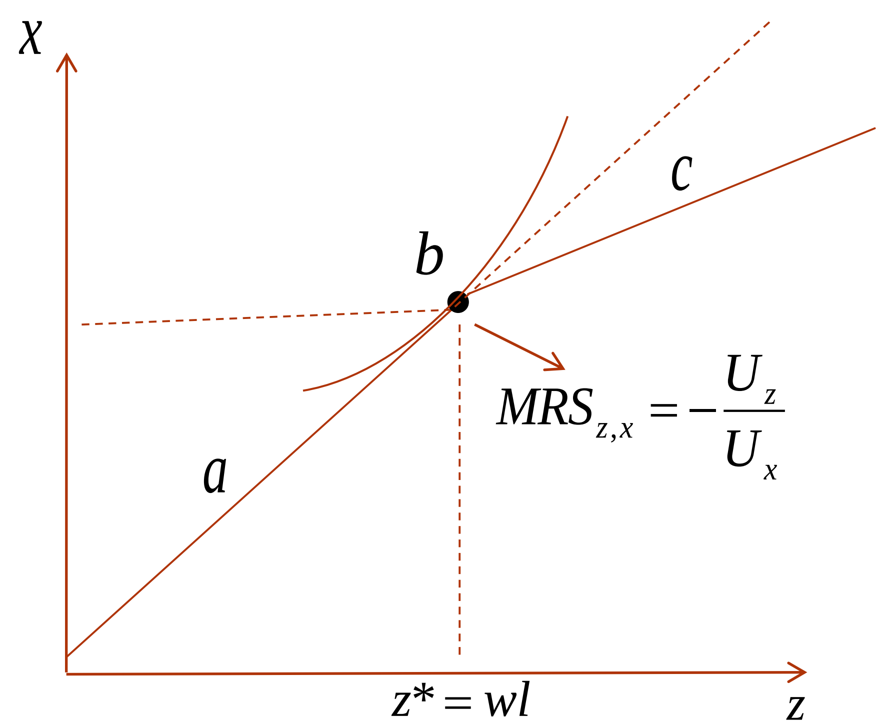


An increase in w, makes the worker less willing to supply labor at the same x (less additional labor is required to achieve any given increase in consumption)

Tax and labor supply

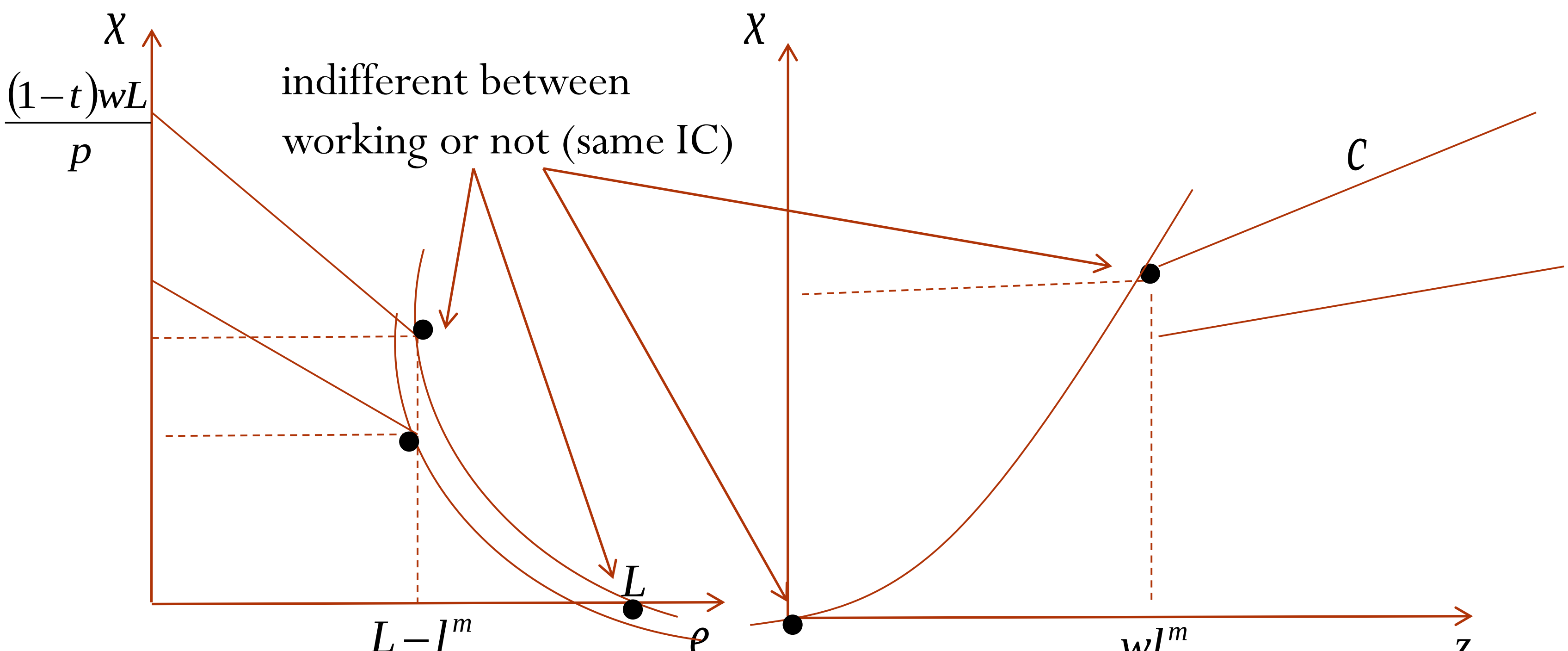
- Practical application (income thresholds) (see IRPEF)
  - $y < \tilde{y} \Rightarrow t = 0$, corner solution should be considered,
  - At the kink point, marginal change in the tax rate has no effect on the labor supply





Tax and labor supply

- Given the minimum working time, l^m, workers could supply zero labor after a tax increase
- The choice for the consumer is then between either undertaking no work or working at least the minimum

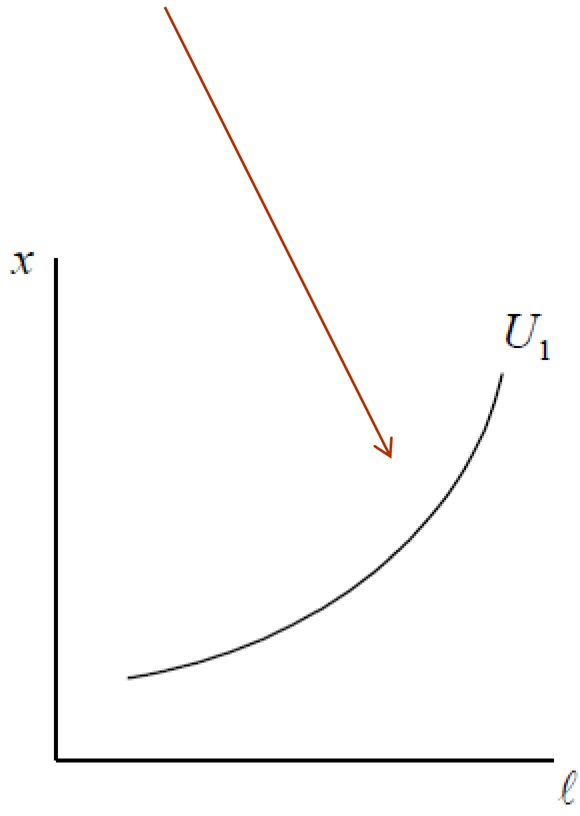


Mirrlees [1971] + Ebert (1992)

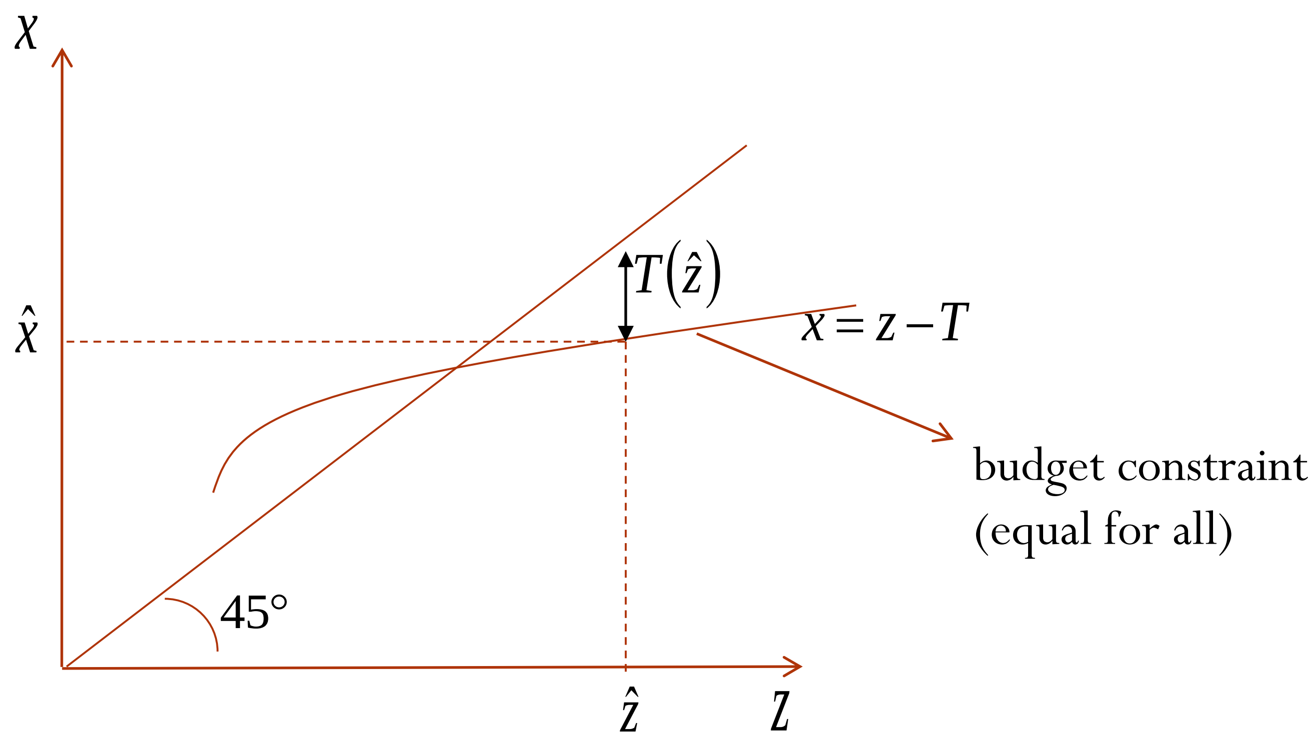
- Main contribution:
  - Unequal distribution of income (equity concerns)
  - Labor supply is introduced in the utility function (efficiency concerns)
  - No prior restriction on the optimal tax function
- Individuals differ according to skill, s (hourly output). Before tax income is $z(s)=sl(s)$, then consumption function is $x(z)=z-T(z)$
- Ability is *private information* of the individuals
- Tax: $T(z)$
- All individuals have the same utility $U(x,l)$, a high-ability individual needs less labor supply to earn any given income

$$U(x,l)\Rightarrow U(x,z/s)=U(x,z,s)$$

Higher utility at North West (low labor supply more consumption)



Tax function

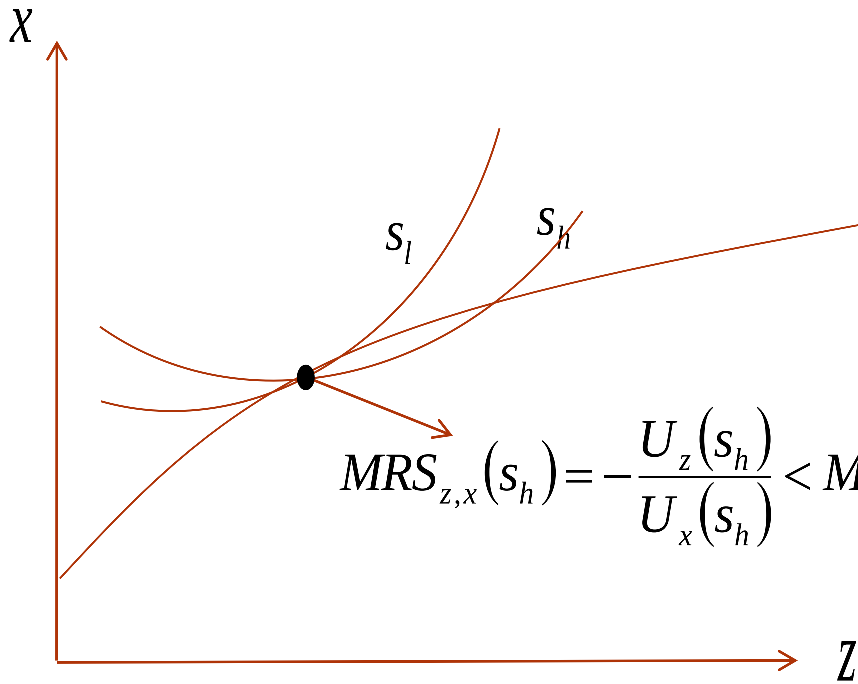


high-ability individuals need less labor supply to earn any given income

- Preferences are assumed to respect the *Agent monotonicity* (Seade 1982) (Single crossing property, *Spence-Mirrlees condition*)

$$DMRS : \frac{\partial MRS_{z,x}(z,x;s)}{\partial s} < 0; \forall (z,x;s)$$

$$MRS_{z,x} = -\frac{U_z}{U_x}$$



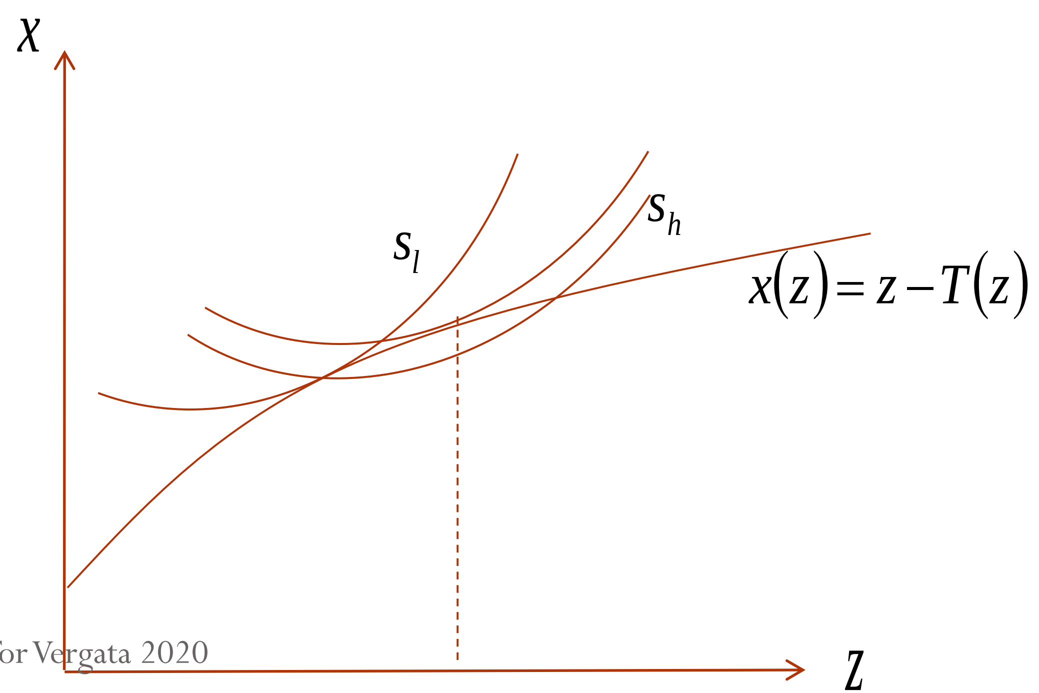
$$MRS_{z,x}(s_h) = -\frac{U_z(s_h)}{U_x(s_h)} < MRS_{z,x}(s_l) = -\frac{U_z(s_l)}{U_x(s_l)}$$

- A marginal increase in *z* reduces the utility because arisen from a higher *l*.
- To remain on the same indifference curve, the individual needs to increase utility by increasing consumption (the disutility from producing more gross income should be compensated by more consumption)
- For high ability individuals this disutility is smaller, they need a smaller increase in consumption (x) to remain on the same indifferent curve (the change in x due to a change in z is smaller)

- This condition ensures that high and low skill individual do not *pool* at the same consumption allocation (same optimal demand of gross income on the budget)
- DMRS (diminishing MRS in s) implies:

$$\frac{\partial z(s)}{\partial s} \geq 0; \frac{\partial x(s)}{\partial s} \geq 0$$

- An utility maximizer individual with high skill earns more gross and net income



- **Individuals**
  - They choose net and gross income (then labor supply):

$$\max_{x,l} U(x,l)$$
$$s.t.$$
$$x(s) = z(s) - T(z(s))$$

  - Define the pair $(x^*(s), z^*(s))$ as the optimal solution
  - Given the tax system, individuals choose the pair $(x^*(s), z^*(s))$ that, given their own skill, maximizes their utility.
- **Government** anticipates the maximization problem of each individual and sets *T(z(s))*

Main Interpretations (hard enough!!!)

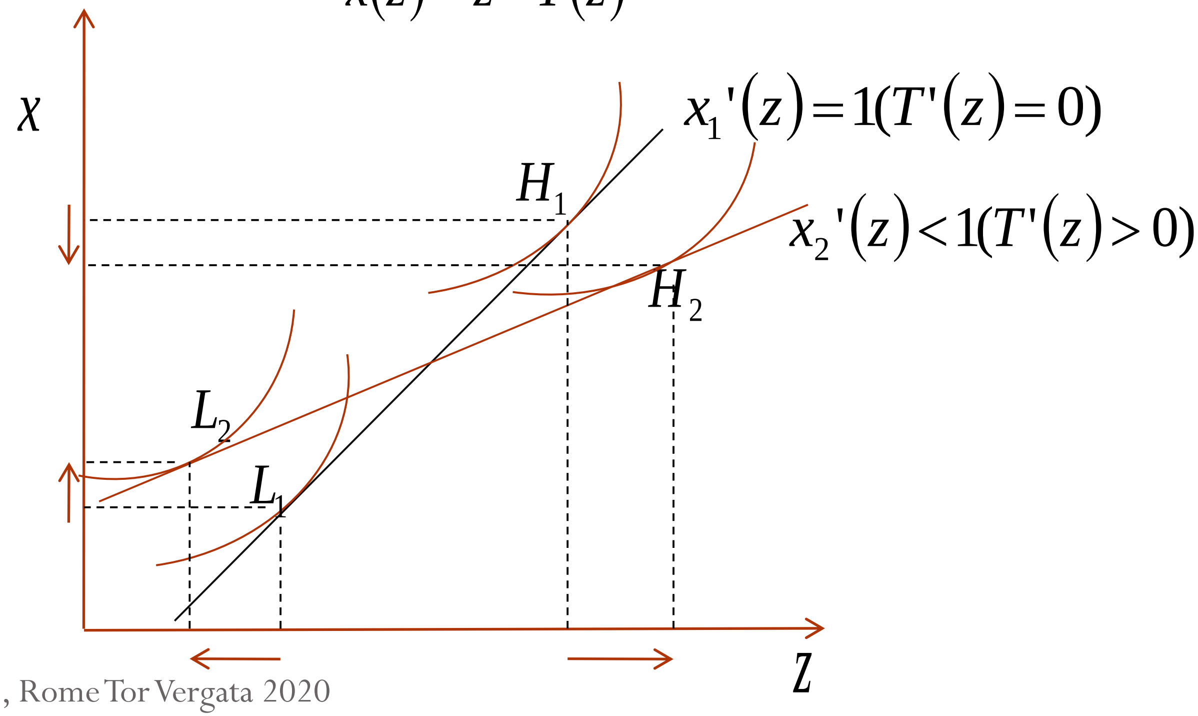
$$T'(z^*(s)) = (1 - MRS_{x,z})$$

Remark: $T'(z^*(s)) = (1 - MRS_{x,z})$ because the individual chooses his optimal bundle that is the tangency point between the indifferent curve and the non linear budget set *x(s)*

- Marginal tax rate is $T'(z^*(s)) \geq 0$
- Marginal tax is less than 100%
- Marginal tax at the **highest** and at the **lowest able** individual is zero: $T'(z^*(s)) = 0$

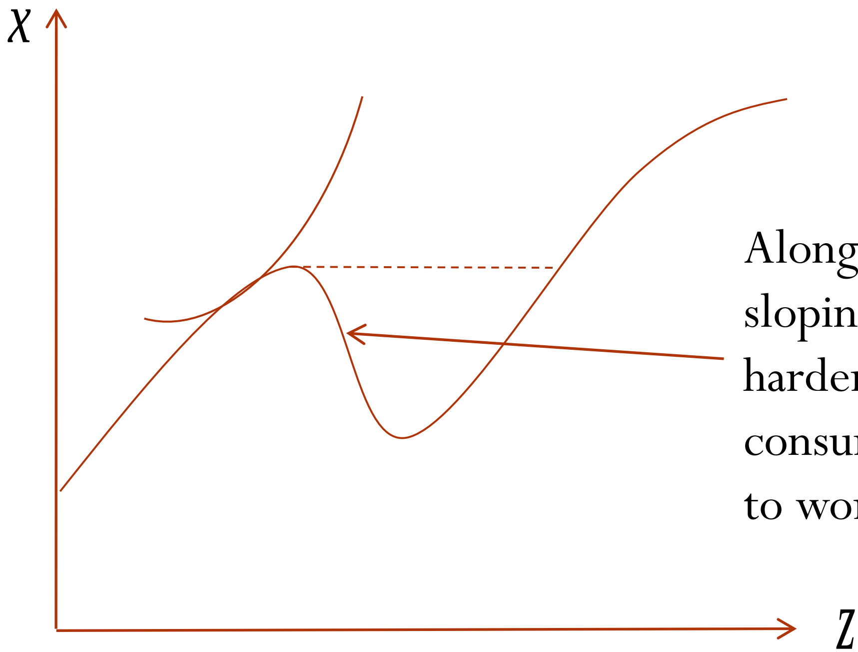
- **Positive marginal tax**. A negative marginal tax implies a subsidy. This cannot be optimal because there always exists a different positive marginal tax being welfare improving.
  - Let's assume the initial tax function such that $x'_1(z)=1 \Rightarrow (T'(z)=0)$
  - Assume a new tax function $x_2'(z)<1(T'(z)>0)$ such that extra (z) earned by the high skill is equal to the reduction for the low skill and (x) of the low skill rises by exactly the amount of the reduction for the high skill.

$$x(z)=z-T(z)$$



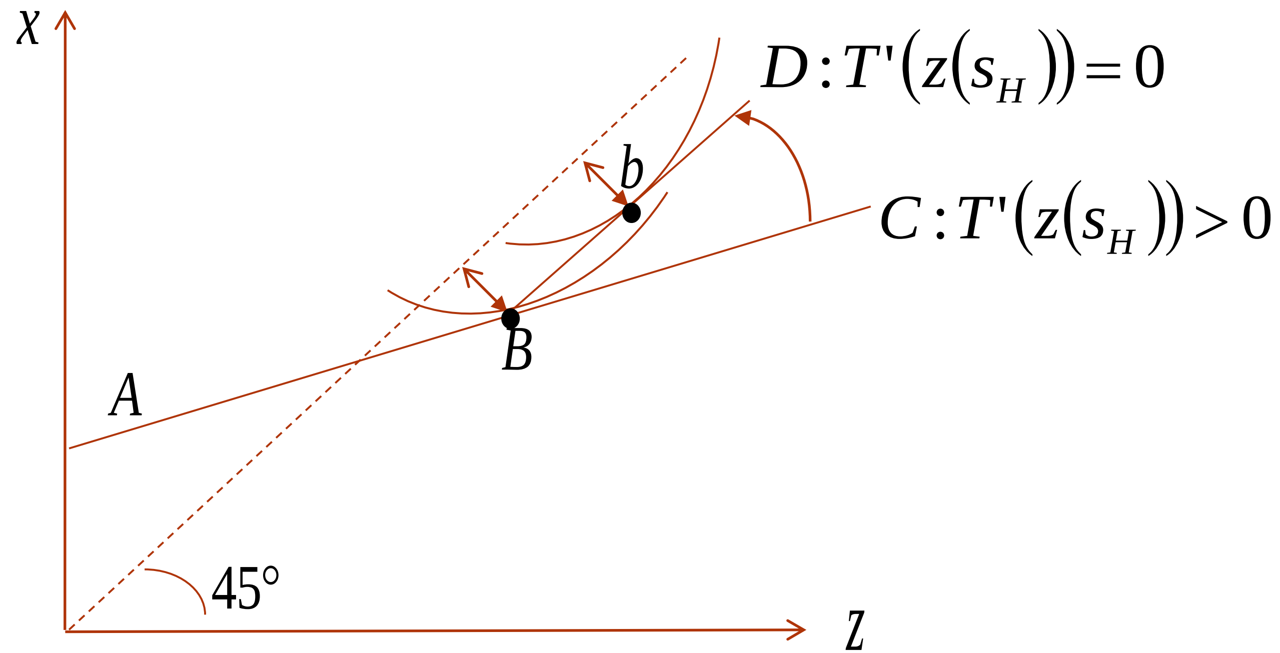
- However, we can see that $x_2'(z) < 1 (T'(z) > 0)$ is welfare improving.
- There is a net transfer of consumption to the low-skill and work effort to the high shill
- Welfare rises:
  - marginal utility of consumption for low ability is higher than for high ability $U_x(s_L) > |U_x(s_H)|$
  - The extra work is less arduous for the high-ability

- **<u>Marginal tax less than 100%</u>** (convex part of the budget line ruled out)
  - A marginal tax $T'(z)>1 \Rightarrow x'(z)<0$



Along the downward sloping part who works harder reduces his private consumption: no incentive to work harder

- **Marginal tax is zero at the individual with the highest ability** $T'(z(s_H)) = 0$
  - Assume the highest individual is at *B*



  - In *b* and *B* the tax payment is the same (vertical distance *T(z)*) but the taxpayer is better off on *b*, then *b* implies a Pareto improvement

- The same intuition holds for the lowest-ability individual: a zero marginal tax induces him to start supplying labor.

- The assumption is that as long as there exists a $T'(z)>0$ there also exists a highest *s* (and a lowest *s*) such that the ablest individual does not supply further labor (and the lowest individual does not start offering labor).

- The "zero" marginal tax induces these two individuals to supply more labor (efficiency concern about labor supply)