The Vickrey-Clarke-Groves Mechanism (VCM):

Why is it important?

- *Provision of Public Good*. It allows to achieve an *ex post Pareto efficient* allocation.
- *Applied to Auction Theory* (*second-price auction*) induces the bidder, under asymmetric information, to truthfully bid their own value (types).

Main intuition:

- ❑ In a contest of *N* individuals choosing their strategy to maximize their own expected payoff, anyone should be charged by an amount equal to the *externality* his strategy induces on the others.

- ❑ *N* individuals *internalize* this externality and an *ex post Pareto efficient* allocation is enforced.

The Vickrey-Clarke-Groves Mechanism (a general approach):
Assume two (i=1,2) individuals.

Each individual simultaneously reports his type (r) (i.e. benefit/loss) from the provision of a specific good (public good, auctioned private good). The GVT sets the level of good according to a rule that make this provision (i.e. whether to provide or not) conditional on (a combination of) these reports

In addition, <u>each individual</u> *i* is assessed a cost c_i^{VCG} for the delivery of the good that is equal to the net effect he induces on the other individual

$$c_1^{VCG}(r) = \underbrace{v_2(0, r_2)}_{\text{2'utility when player 1doesn't play}} - \underbrace{v_2(r_1, r_2)}_{\text{2's utility when player 1 plays}}.$$

Where $c_i^{VCG}(r)$ are called VCG cost functions.

Comments:

- Each individual *i* pays the externality his presence (action) induces on the other individuals.

- When maximizing his own payoff by choosing his strategy, each individual internalizes (through the VCG cost) the externality imposed on the others, then an ex post Pareto efficient allocation arises

Example (Auction): The designer (the auctioneer) runs an auction to award a single good among *2* bidders with type t_i (here the value for the good)
Each individual utility is assumed $v_i(t_i) = t_i$

The simple *mechanism* here is:

Auction: the good is awarded to the highest bid $b_i = t_i$ (the highest reported type t_i), that is:

If $b_1 = t_1 \geq b_2 = t_2$ then *1* wins

If $b_1 = t_1 < b_2 = t_2$ then *2* wins

In particular, a **first price auction** (assume 1 bids more and wins)

$c_1(t_1, t_2) = b_1 > b_2 = t_2$

$c_2(t_1, t_2) = 0; b_2 = t_2 < t_1$

$\Rightarrow$ Inefficient because none bids his real type.

Introducing the VCG in the auction and compute the payment for the winner (player 1)

$$v_2(0, b_2) = t_2$$

bidder' 2 (looser) utility $\Rightarrow v_2(0, b_2) - v_2(b_1, b_2) = t_2$

$$v_2(b_1, b_2) = 0$$

This auction becomes a mechanism (VCG) in which each bidder bids <u>his real type</u> and pays the <u>second highest bid</u> (not his bid) given by:

$$c_1^{VCG}(t) = v_2(0, b_2) - v_2(b_1, b_2) = v_2 = b_2 = t_2$$

Second Price Auction

The VCG is truth-telling **DOMINANT STRATEGY**: any individual truthfully bids his type. Then VCG becomes the so called *Second price* auction.

Short digression about (**auction theory**) first and second price auction

First price auction, no incentive to truthfully bid his/her own real type .

- Assume two bidders, whose types are independently drawn from a (common knowledge) uniform distribution on $[0,1]$
- Assume 2 is truthfully revealing his type, is it optimal for 1 doing so?

$$\max_{\hat{\theta}_1}\left(\theta_1-\hat{\theta}_1\right) prob\left(\theta_2 \le \hat{\theta}_1\right)$$

$$F\left(\theta_2 \le \hat{\theta}_1\right)=\frac{\hat{\theta}_1-0}{1-0}$$

$$\max_{\hat{\theta}_1}\left(\theta_1-\hat{\theta}_1\right)\hat{\theta}_1$$

$$\hat{\theta}_1=\frac{\theta_1}{2} \quad \ldots\ldots\ \text{NO}$$

- The same for player 2, then in the Bayesian equilibrium each equilibrium bid is:

$$b_i\left(\theta_i\right)=\frac{\theta_i}{2}$$

Second price auction, truthfully bidding the real type is *weakly dominant strategy*

- Assume 2 announces his real type and announces $\hat{\theta}_2 \leq \theta_1$

 If player 1 bids his type, he gets:

$$\begin{cases} \left(\theta_1 - \hat{\theta}_2\right) \geq 0 & \hat{\theta}_2 \leq b_1 \\ 0 & \hat{\theta}_2 > b_1 \end{cases}$$

So if $\hat{\theta}_2 \leq \theta_1$ truth telling is weakly best for 1(it increase the probability of victory)

- Assume 2 announces $\hat{\theta}_2 > \theta_1$, then 1 gets:

$$\begin{cases} 0 & b_1 = \theta_1 \ ; b_1 < \theta_1 \\ \left(\theta_1 - \hat{\theta}_2\right) < 0 & b_1 > \hat{\theta}_2 \end{cases}$$

truth telling is optimal regardless 2's announcement (again, a higher probability of victory)

⇒ truth telling is weakly dominant for 1.

Application of VCG to the provision of public good

Brief introduction

Players (consumers) may have incentives to make false statement (understatement and overstatement) when the provision of public good is <u>conditional</u> to the <u>revelation</u> of their benefits.

False Understatement

Public good can be provided *G=1* or not *G=0*

The cost of providing G is $C = 1$

Gross true benefits (types) for both players from the provision are $v^1 = v^2 = 1$

$v^1 + v^2 = 2 > C = 1$ providing G is socially beneficial

Player *h's* strategy: report on his benefit: $r^h = \{0, v^h\} = \{0,1\}$

Mechanism:

- G is provided if the sum of the announced valuations is at least equal to the cost

$$\begin{cases} G = 1, & r^1 + r^2 \geq C = 1 \\ G = 0, & \text{otherwise} \end{cases}$$

- Each player bears the cost of the provision proportionally to his report

$c^h = 1$ If $r^h = 1$ and $r^{h'} = 0$

$c^h = 1/2$ If $r^h = 1$ and $r^{h'} = 1$

$c^h = 0$ If $r^h = 0$ and $r^{h'} = 0$ or $r^{h'} = 1$

Incentive to understate his evaluation to let the other player to bear all the cost

Net benefit (payoff from such a mechanism)

$U^h = v^h - c^h$ if $r^1 + r^2 \geq C = 1$

$U^h = 0$ otherwise

	Player 2: $r^2=0$	Player 2: $r^2=1$
Player 1: $r^1=0$	0, 0	1, 0
Player 1: $r^1=1$	0, 1	1/2, 1/2

Since *r=0* is the weak dominant strategy for both players the only equilibrium in weak dominant strategies is:

Player 1 plays r=0, player 2 plays r=0, G is not provided, equilibrium payoffs are $U^h = 0$

Inefficiency= public good is not provided in equilibrium despite the social benefit is higher than the social cost

This result holds only if you use the concept of equilibrium in *dominant strategies* (what we need for the application of VCM). But…there exists a Nash equilibrium in which G is provided

		Player 2 $r^2=0$	Player 2 $r^2=1$
Player 1	$r^1=0$	0 (Player 1), 0 (Player 2)	1 (Player 1), 0 (Player 2)
Player 1	$r^1=1$	0 (Player 1), 1 (Player 2)	1/2 (Player 1), 1/2 (Player 2)

We have three Nash equilibria, in two of them G is provided:

$\{r^1=1, r^2=0\}$ and $\{r^1=0, r^2=1\}$

If instead we assume $G=1$ if $r^1+r^2>C=1$ then G=0 in both Nash and only the dominant strategy equilibrium survives

Vickery-Clarke-Groves Mechanism

In this mechanism the inventive to make a false announcement disappears

- The cost of the project is known (length of road, public monument)
- The allocation of this cost among consumers is known
- Each consumer knows its private benefit from the provision of G and the cost they have to pay in case of provision

Government only cares about the net benefit from the provision for each consumer

(whether their evaluations of *G* exceeds their contribution to the cost)

- Gross true benefits: $v^1 = v^2 = 1$
- Player *h's* strategy: report on his ***net*** benefit: r^h

Mechanism:

- *Decision rule: G=1* (provision) if $r^1 + r^2 \geq 0$
- In case of no provision each consumer's payoff is zero
- If *G* is provided each consumer receives a side-payment equal to the reported benefit of the other consumer

Consumer 1 receives the payoff $v^1 + r^2$

Consumer 2 receives the payoff $v^2 + r^1$

- Each player ***internalizes*** the effect of his action (report) on the others
- The incentive to behave opportunistically and make the other worse off disappears
- The report (and also the presence) of one individual may reduce or increase the utility of the other (i.e. a public good is (un)likely to be provided)

Practical application

- Assume that the true net benefits (types) and the reports can take value of either **1** or **-1**

G is not provided if both report $r^h = -1$

G is provided when **at least one** consumer reports $r^h = 1$

Player 1		Player 2: $r^2 = -1$	Player 2: $r^2 = 1$
Player 1	$r^1 = -1$	0 0	$v^2 - 1$ $v^1 + 1$
	$r^1 = 1$	$v^2 + 1$ $v^1 - 1$	$v^2 + 1$ $v^1 + 1$

To show whether there is no incentive to misreport we consider player 1 (the same arguments hold for player 2).

We find that the dominant strategy for player 1 is truth-telling under both

$$v^1 = \{1, -1\}$$

- Assume $v^1 = -1$: is truth-telling the best action for player 1?

		Player 2	
		$r^2 = -1$	$r^2 = 1$
Player 1	$r^1 = -1$	0, 0	0, $v^2 - 1$
	$r^1 = 1$	-2, $v^2 + 1$	0, $v^2 + 1$

Yes, because *0*>-2

- Assume $v^1 = 1$: is truth-telling the weakly DS for player 1?

		Player 2	
		$r^2 = -1$	$r^2 = 1$
Player 1	$r^1 = -1$	0 0	$v^2 - 1$ 2
	$r^1 = 1$	$v^2 + 1$ 0	$v^2 + 1$ 2

Yes, because consumer 1 is indifferent between truth and no truth-telling, however no incentive to misreport.

- Assume the case $v^1 = v^2 = -1$: is truth-telling an equilibrium?

Player 1 \ Player 2	$r^2 = -1$	$r^2 = 1$
$r^1 = -1$	0, 0	0, -2
$r^1 = 1$	-2, 0	0, 0

Yes: truth report is a weak dominant strategy for both consumers then the equilibrium strategy is:

$$r^1 = -1; r^2 = -1$$

Main drawbacks of the VCG Mechanism:

- Enforcing truth-telling is costly: when *G* is provided and $v^1 = v^2 = 1$ the total side payment (2) is equal to the total benefit
- Information rents are necessary to induce consumers to reveal their information.
- Collecting this further resource for *side payments* may rise another issue for inefficiency
- A ***Clarke Tax*** can reduce the use of side transfers (but it cannot eliminate them at all): side transfers are given only if the report of a player changes the social decision (only if the player is *pivotal*)
- Extracting information can never be done for free in several applications of mechanism design (Auction Theory, Regulation).