

**Question 4.**

Suppose the government raises tax revenue by means of an income tax. One way of lowering the burden of the income tax on the very lowest income people is to increase the basic exemption (amount of income which is exempt from tax). Assume that for the fiscal year this exemption is 12000. Assume a marginal tax rate of 10% for an annual personal income  $y$  such that  $12000 < y < 43561$ .

a) Find the payable tax and the average tax (10 points)

$$\text{payable tax} = 0.1[y - (12000)]$$

$$\text{Average tax} = \frac{0.1[y - (12000)]}{y}$$

b) Compute the effect of an increase in the tax exemption on the average tax (12 points)

assume a tax exemption of  $x$ , the average tax is  $AT = \frac{0.1[y-x]}{y}$ , it is possible

to see that  $\frac{d(\frac{0.1(y-x)}{y})}{dx} = -\frac{0.1}{y} < 0$ , exemption reduces the AT.

c) Clearly state the definition of a progressive income tax system and show whether a tax exemption makes the tax system more progressive (12 points)

A progressive tax is when the average tax increases with the income, formally  $\frac{\partial AT}{\partial y} > 0$ . The solution of point b) shows that, at a given income  $y$ , the AT decreases in the exemption, therefore a tax exemption makes the income tax system more progressive. Lower income has a lower average tax when introducing the exemption.

**Question 5.**

Assume 3 individuals ( $i = 1; 2; 3$ ) each with the following gross benefit from the consumption of the public good,  $v_1 = 30; v_2 = 35; v_3 = 80$ . Assume the cost of the public good is 120 and is equally shared among individuals. Each consumer is asked to reveal his own net benefit from the consumption of the public good and a Clarke Groves Mechanism (CGM) is applied, that is, each individual receives the reported benefits of the other individuals and the public good is delivered if the sum of the reported net benefits is positive (or the sum of the gross benefit exceeds the total cost)

a) Find the net benefit of each individual if the public good is provided and show whether the resulting provision of the public good is optimal (12 points)

Each individual pays 40, the net benefits are  $b_1 = -10; b_2 = -5; b_3 = 40$ , since  $b_1 + b_2 + b_3 = 25 > 0$ , then the provision is optimal

b) Show that the mechanism induces truth-telling as a dominant strategy when each player reports independently his own benefit (12 points)

Since player 1 has the lowest benefit it is enough to show the existence of a dominant strategy for this player as sufficient condition for the equilibrium in which all individuals truthfully report their benefit.

Player 1's utility is:

$$U_1 = \begin{cases} -10 + r_2 + r_3 = 25 & \text{if } r_1 + 35 > 0 \\ 0 & \text{otherwise} \end{cases}$$

a false report inducing no provision (inducing  $r_1 + 35 < 0$ ) would give  $U_1 = 0$  therefore true report  $r_1$  is dominant

c) Show whether the mechanism is robust to a collusive agreement among some individuals. In particular a collusive agreement is a scheme such that instead of setting a true report non-cooperatively, individuals decide to cooperatively communicate false reports in order to maximize their payoff (10 points)

player 1 and 2 can agree on a false report like overestimating their benefits, like  $r_1 = -8; r_2 = -3$ , the condition for the provision does not change and  $r_1 + r_2 + r_3 > 0$  but player 1 and 2 increase their utility  $U_1 = -10 - 3 + 40 = 27$ ;  $U_2 = -5 - 8 + 40 = 27 > 25$ , where 25 was the utility of player 2 without collusion.