The Overlapping Generations Model (OLG)

In the OLG model, agents live for two periods. When young they work and divide their labour income between consumption and savings. When old they consume their savings. As the name of the model implies, generations overlap so in each period there are young agents saving and old agents consuming.

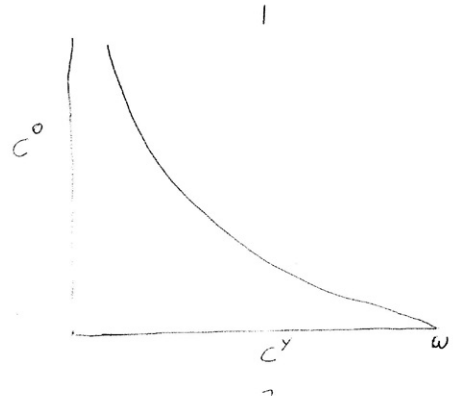
This model allows us to consider two issues, the consequences of finite time horizons and the consequences of difference between agents.

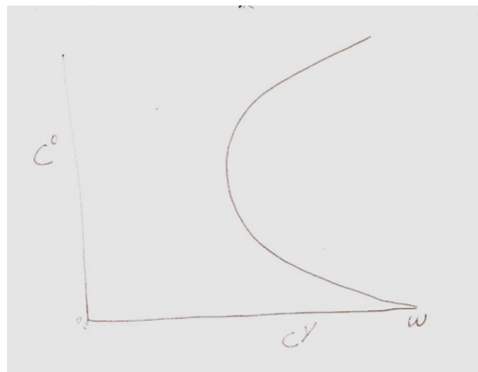
We begin with the agent's consumption saving decision. Consumers born in generation t get labour income w_t and consume c_t^y when young and $c_t^o = (1+r_{t+1})(w_t-c_t^y)$ when old.

Consumers maximize $U(c_t^y,c_t^o)$. For example, they might maximize $\log(c_t^y) + (1/(1+d))\log(c_t^o)$ or slightly more generally $u(c_t^y) + (1/(1+d))u(c_t^o)$ for some function u. We assume that they choose c_t^y rationally so

1) $u'(c_t^y) - ((1+r_{t+1})/(1+d))u'(c_t^o) = 0$

For a fixed wage w_t, different interest rates r_{t+1} can cause a pattern of consumption like that shown in figure one or like that shown in figure two. Where $x = (c^y(w_t,r_{t+1}),c^o(w_t,r_{t+1}))$. An increase in r_{t+1} must cause an increase in c_t^o but may cause c_t^y to increase or to decrease.





In the simplest type of OLG model there are no long lived assets. This means that there is no way to save so $1+r_{t+1} = 0$ (see Samuelson 1958 or Cass and Yaari 1968). In this case people starve when they are old. This illustrates one unusual feature of OLG models; the market outcome may be very bad. In fact the market outcome may be Pareto inferior to some other feasible outcome.

The grim world described above can clearly be improved if the young give some fraction of their income to the old each period. Everyone can be made better off by such a social security system.

Indeed, the population grows by a factor 1+n each generation, everyone can get 1+n units of consumption good when old for each unit of consumption good given away when young.

The optimal steady state is achieved when each of the young contributes x units to the scheme and each of the old get x(1+n) units and

2) $u'(w_t-x) - ((1+n)/(1+d))u'(x(1+n)) = 0$.

This is what a social planner who cared only about the steady state level of happiness would require people to do.

Such a desirable outcome can be achieved without a social security system. Let's say that there is one unit of intrinsically worthless asset, say pieces of paper with pictures printed on them (money). If these cannot be counterfeited, they may be valuable in equilibrium in an OLG model. Each generation of young may buy the money of the old because they know that the new young will buy the money form them when they are old.

If p_t is the amount of money required to buy one unit of goods at time t then the return to holding money is p_t/p_{t+1}, for each unit of consumption good given up at t one gets p_t/p_{t+1} units of good when old. In other words $1+r_{t+1} = p_t/p_{t+1}$.

In the case of w_t fixed at w, total production in time t is $N_t w$ where N_t is the number of young born at t. Since the good can not be stored, total consumption equals total production so

2) $c_{t-1}^{o}N_{t-1} + c_t^{y}N_t = wN_t$

This means that the value of the money $1/p_t$ is given by

3) $1/p_t = N_{t-1}c_{t-1}^{o} = N_t(w - c_t^{y})$

A little algebra tells us that

4) $c_t^{y} = w - (1/(1+n))c_{t-1}^{o}$.

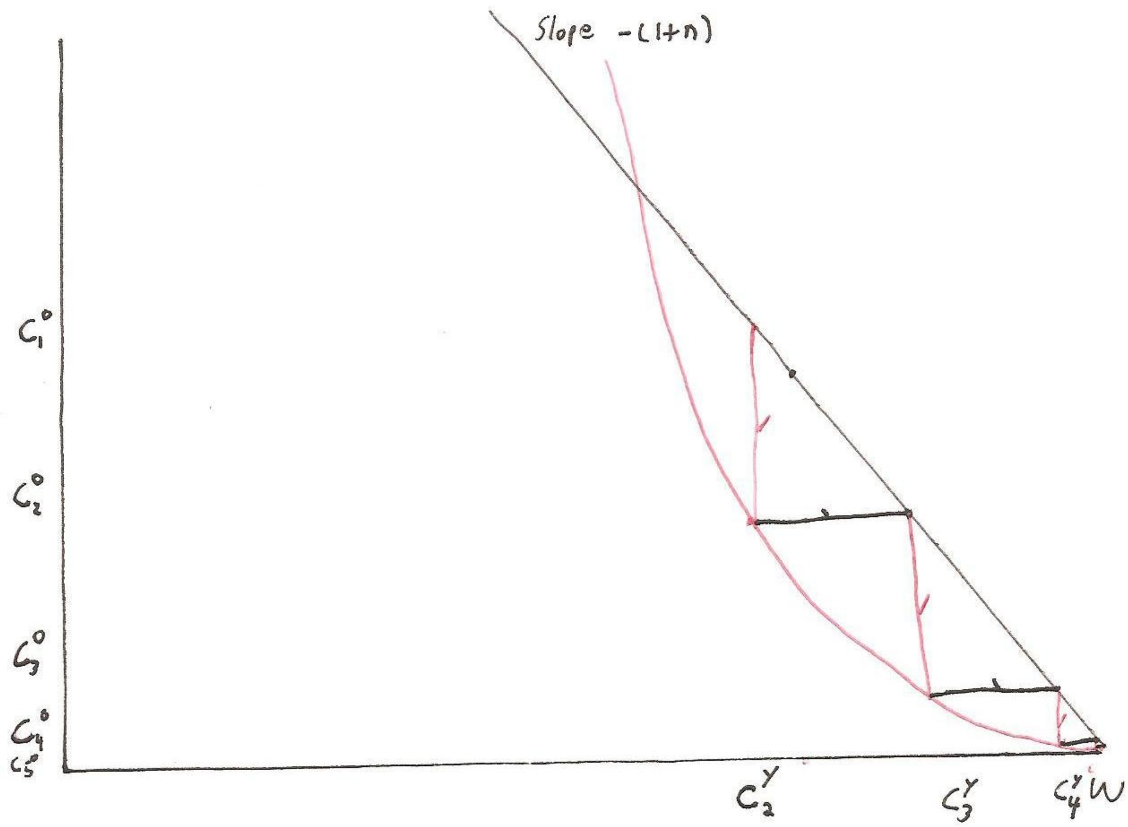
This and the first order condition of the young

5) $u'(c_t^{y}) - p_t/((1+d)p_{t+1})u'(c_t^{o}) = 0$

mean that we can calculate the equilibrium p_{t+1} from p_t. This calculation can be done graphically where c_t^y is calcualted from c_{t-1}^o by drawing the horizontal line to the graph of 4) the line through (w,0) with slope -1+n. Then $c_t^o = 1/(N_t p_{t+1}$ can be calculated using equation 5 by drawing the vertical line to the offer curve, the graph of equation 5.

The optimal steady state defined by 2 can always be an equilibrium of the overlapping generations economy with money. The value of the money at t has to equal xN_t for x as defined in equation 2. This means that money has to become more and more valuable as the price of the good declines by a factor 1/(1+n) each period. This is an equilibrium no matter what the shape of the offer curve.

The optimal steady state is <u>not</u> the only possible equilibrium of the overlapping generations economy with money. Figure 3 shows an inflationary equilibrium in which the stock of money becomes less an less valuable compared to wN_t. This implies that those unfortunate enough to be born at a high t starve when they are old which is very sad.


Slope $-(1+n)$
c_1^o
c_2^o
c_3^o
c_4^o
c_5^o
c_2^y
c_3^y
c_4^y
W

In contrast if the offer curve is as in figure 4 a deterministic cycle is possible in which e.g. even numbered generations consume a large amount both when young and when old, but odd numbered generations consume little when young and when old which is less sad but not fair.

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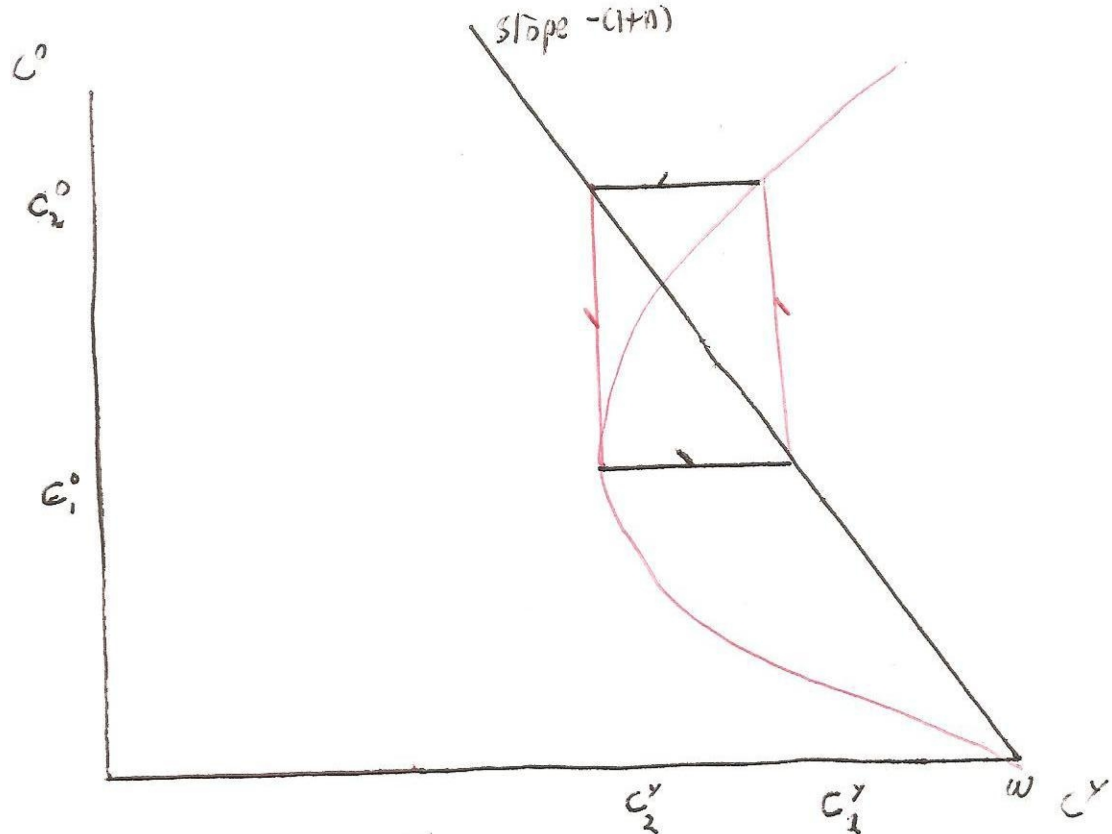

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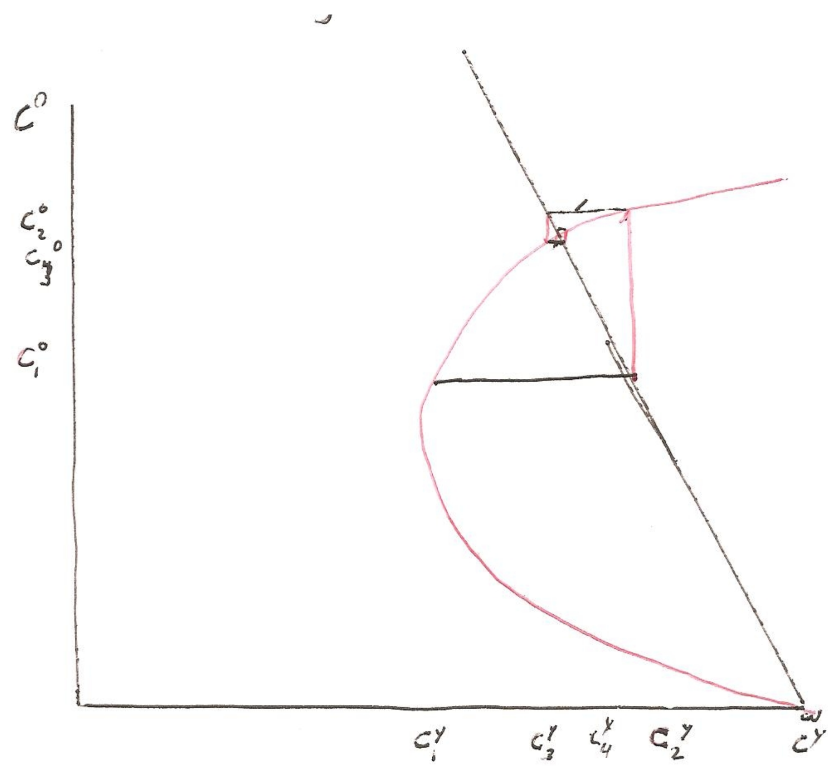
As figure 5 shows the optimal steady state can be a stable equilibrium so the economy ends up there even if it started somewhere with p different from the optimal p but not to far from it.



These figures also illustrate another odd feature of overlapping generations economies, the equilibrium outcome is not unique. Equations 4 and 5 allow us to calculate p_2, p_3 &c for many different values of p_1. This means that knowledge of the tastes and the technology does not enable us to say if the market outcome will be Pareto optimal or very very bad.

In the case of the offer curve in figure 5 there are also equilibria in which something truly strange happens. In period 3, the Walrasian auctioneer flips a coin. If it comes up heads (testa so t) then p_3 is high so the old have low consumption c_2^o is low and c_3^y is high and vice versa if the coin comes up tails (croce so c). When 2nd generation agents are young, they know this will happen and they choose consumption under uncertainty. The in each case (testa or croce) there is a series of prices which makes markets clear. Thus the equilibrium can be stochastic even if tastes and technology are not. In figure 6 "etc". is et cetera, that is I claim that I could show the continuation of the testa equilibrium (if I could draw better).