

## Two Exercises on Consumption and Investment

1: Consider a consumer who chooses consumption in period  $t$  ( $C_t$ ) for  $t = 1, 2, 3 \dots$  to maximize the Sum from  $t = 1$  to infinity of  $(C_t)^{0.5}/(1.10)^t$  subject to the budget constraint that the present discounted value of consumption is equal to initial wealth ( $K_1$ ) plus the present discounted value of wages ( $w_t$ ) where wages and consumption are discounted at a constant rate  $r$

so  $K_1 =$  the sum from 1 to infinity of  $(C_t - w_t)/(1+r)^{t-1}$

- a) If initial wealth  $K_1 = 110$  and wages are zero what is  $C_1$  if  $r = 0.10$ ?
- b) If Initial Wealth  $K_1 = 0$  and  $w_t = 10$  for every  $t$ , what is  $C_1$  if  $r = 0.10$ .

2) Consider the model of investment with no financial market imperfections presented by Romer.

- a) Write down the equations for  $\dot{K}$  and  $\dot{Q}$  -- the time derivatives of  $K$  and  $Q$  (you are not obliged to re-derive them)
- b) what is the slope of the  $\dot{K}=0$  curve ?
- c) draw a graph of  $Q$  on  $K$  showing the  $\dot{Q}=0$  curve and the  $\dot{K}=0$  curve (that is draw the phase diagram)
- d)  $K^*$  is the steady state level of capital. Write down the equation for  $K^*$  (that is find the point where  $\dot{K}=0$  and  $\dot{Q}=0$ )
- e) suppose that initial  $K$  is half of  $K^*$ . Illustrate what would happen using the phase diagram you just drew.
- f) Now imagine that initial  $k = K^*$  but that, in a surprise move, the state introduces a tax  $\tau$  on profits so from the point of view of the firm  $\pi(K)$  is replaced by  $(1 - \tau)\pi(K)$ . Look at the equations for  $\dot{Q}$  and  $\dot{K}$ . Which one is changed by the tax ?
- g) Illustrate what happens after the unexpected introduction of the tax using the phase diagram.