Games, Information and Contract Theory
Problem Set 2

Instructors

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This problem set will be solved in class on Monday, September 30, 2019.

Exercise 1. (Setting a Standard)

A new type of consumer product is about to be introduced in a market in which two firms are active (for example, a video game). The two firms own *competing technologies* (for example, two game consoles) that can be used to run this product, and would like their technology to be the *standard* in the market. In other words, each firm would prefer its technology to be used exclusively to run the product, as this would increase its sales. In particular, each firm has a payoff of zero if no standard is set (both firms use their own technology). If only one firm's technology is adopted as a standard, that firm gets a payoff of 2, and the other gets 1. Finally, if both firms employ the other firm's technology, they get a payoff of zero.

		Firm 2	
		Own Tech.	Comp Tech.
Firm 1	Own Tech.	(,)	(,)
	Comp Tech.	(,)	(,)

Answer the following questions and explain your answers in detail.

1. Fill the *payoff matrix* using the available information.
2. Write the game as a *normal-form* game.
3. Detect any strictly *dominated strategy* for both players.

4. Find the *pure-strategy Nash equilibria (NE)* of this game, if any (*hint: find the players' best response to their opponent's strategy*).

5. Find the *mixed-strategy* NE of this game, if any.

6. Now, suppose that firm 1 has a *superior technology.* In other words, the latter gets a payoff of 3 when it manages to set the standard. Does this affect the *mixed-strategy NE*? Explain (*Hint: payoffs do not change for firm 2*).

Exercise 2. (Rock-Paper-Scissors)

Pat and Carl meet to play the famous game *rock-paper-scissors.* According to this game, both players simultaneously choose between rock, paper, or scissors. Not surprisingly, rock beats scissors, scissors beat paper, and paper beats rock. If a player wins, she gets 1 Euro from the other player. If she looses, she pays 1 Euro to the other player. If both players choose the same action, then they both get nothing.

		Carl		
		Rock	Paper	Scissors
	Rock	(,)	(,)	(,)
Pat	Paper	(,)	(,)	(,)
	Scissors	(,)	(,)	(,)

Answer the following questions and explain your answers in detail.

1. Fill the *payoff matrix* using the available information.

2. Write the game as a *normal-form* game.

3. Detect any strictly *dominated strategy* for both players.

4. Find the *pure-strategy Nash equilibria (NE)* of this game (*hint: find the players' best response to their opponent's strategy*), if any.

5. Find the *mixed-strategy* NE of this game, if any (*hint: define x_1, x_2 the probabilities of playing respectively Rock and Paper for Pat, and y_1, y_2 the probabilities of playing respectively Rock and Paper for Carl. Do not forget that probability of playing a strategy for each player must sum up to 1. Remember that if we want Pat (or Carl) to play a mixed strategy, she (he) must be indifferent between her (his) pure strategies when the other player is randomising over his (her) pure strategies*).

Exercise 3. (Entry Game)

Consider the following dynamic game of complete and perfect information.

An incumbent monopolist (I) faces the threat of entry by a challenger firm (C) in the market in which it operates. In the first stage, C decides whether to *invest* or *not invest* in a technology that increases its efficiency. In the second stage, C decides whether to *enter* (*IN*) or *stay out* (OUT) of the market. In the third stage, after observing the actions by C, I decides to react aggressively to entry (*fight*) or not (*accommodate*). Payoffs for C and I are:

- (3,1) if C invests, enters the market, and I accommodates;
- (1,0) if C invests, enters the market, and I fights;
- (-1,2) if C invests and stays out of the market;
- (2,1) if C does not invest, enters the market, and I accommodates;
- (0,0) if C does not invest, enters the market, and I fights;
- (0,2) if C does not invest and stays out of the market.

Answer the following questions and explain your answers in detail.

1. Write the game as an *extensive-form* game (*Hint: draw the game tree with payoffs first. Then write separately the set of players $N = \{\}$, the set of strategies available to each player $S_i = \{\}$ (remember the definition of strategy in dynamic games), the set of decision nodes $X_i = \{\}$ and of information sets $I_i = \{\}$ for each player, the root of the game $r = \{\}$, and the terminal nodes $TN = \{\}$*).
2. Identify all the *subgames* which compose this game.
3. Write the game in the corresponding *normal-form* and detect any *pure-strategy NE.* (*hint: use the strategies detected in point (1)*).
4. Find the *backwards-induction* outcome and *subgame-perfect NE* of this game. Verify that this is an equilibrium of each *subgame.*

Exercise 4. (Stackelberg Duopoly)

Two firms compete in a market by sequentially setting the quantities of a (homogeneous) good to produce (q_i, q_j). The game has *two stages*:

1. Firm i sets quantity q_i;
2. Firm j observes q_i, and then sets q_j.

Each firm faces a *constant marginal cost* $c = 3$. The two firms face the *inverse demand function* $P(Q) = 9 - Q$, where Q is the aggregate quantity produced. Payoffs are given by each firm's profits, determined after both quantities have been chosen.

Answer the following questions and explain your answers in detail.

1. Describe the game as a normal-form game (*Players, Strategies, Payoffs*).
2. Write the maximisation problem for each firm.
3. Find equilibrium quantities ($q_i^\star$,$q_j^\star$) using *backwards-induction* (*Starting from the second stage, get firm j's reaction function to q_i. Then, go back to the first stage and plug j's reaction function in the maximisation problem for firm i*).
4. Find the *SPNE* of this game (equilibrium quantities for both firms).
5. Find the payoffs ($\pi_i(q_i^\star, q_j^\star)$ and $\pi_j(q_i^\star, q_j^\star)$) obtained by firms when they play this *SPNE* (*hint: replace equilibrium quantities in the profit functions*).
6. Compare Firm i's profits in *Stackelberg* with *Cournot* profits.
7. Compare overall quantities and profits in *Stackelberg* and *Cournot.* Which is larger, and why? Explain.