

Games, Information and Contract Theory

Problem Set 4

Instructors

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This problem set will be solved in class on Monday, October 14, 2019.

Exercise 1. (Bayesian Cournot Duopoly)

Two firms compete in a market by simultaneously setting the quantities of a (homogeneous) good to produce (q_1, q_2) . Firm 1 faces a *constant marginal cost* $c = 3$. Firm 2 faces constant marginal costs $c_H = 4$ if it is inefficient and $c_L = 2$ if it is efficient. Firm 1 does not observe Firm 2's cost, but it believes that the latter is efficient with probability $\theta = 1/2$. The two firms face the *inverse demand function* $P(Q) = 9 - Q$, where Q is the aggregate quantity produced. Payoffs are given by each firm's profits.

Answer the following questions and explain your answers in detail.

1. Describe the game as a normal-form game (N, A_i, T_i, p_i, u_i) .

Baseline - Complete Info: Firm 1 observes Firm 2's marginal cost

2. Write the maximisation problem for each firm and type.
3. Solve the maximisation problem for each firm and type, obtaining its *reaction function*. (*hint: In this case we have two pairs of reaction functions as Firm 2's type is observable*).
4. Find the *NE* of this game (equilibrium quantities for both firms) when Firm 2 is efficient and inefficient (*hint: use the two reaction functions and solve for $q_1^*(c_H), q_1^*(c_L), q_2^*(c_H), q_2^*(c_L)$*).

5. Find the payoffs $\pi_1(q_1^*(c_H), q_2^*(c_H))$, $\pi_2(q_1^*(c_H), q_2^*(c_H))$, $\pi_1(q_1^*(c_L), q_2^*(c_L))$, and $\pi_2(q_1^*(c_L), q_2^*(c_L))$ obtained by firms when they play this NE (*hint: replace equilibrium quantities in profits*).

Incomplete Info: Firm 1 does not observe Firm 2's marginal cost

6. Write the maximisation problem for each firm. (*hint: there are three maximisation problems, one involving expected profits*).
7. Solve the maximisation problem for each firm and type, obtaining its *reaction function*. (*hint: In this case we have three reaction functions as Firm 2's type is not observable*).
8. Find the NE of this game, i.e. equilibrium quantities for both Firm 1 and both types of Firm 2 (*hint: use the three reaction functions and solve for q_1^* , $q_2^*(c_H)$, $q_2^*(c_L)$*).
9. Find the payoffs $\pi_1(q_1^*, q_2^*(c_H), q_2^*(c_L))$, $\pi_2(q_1^*, q_2^*(c_H))$, and $\pi_2(q_1^*, q_2^*(c_L))$ obtained by firms when they play this NE (*hint: replace equilibrium quantities in profits*).
10. Compare the equilibrium quantities obtained in the baseline with the ones of incomplete information. Explain the effect of uncertainty on the quantities produced by Firm 1 and both types of Firm 2.