# Practice session 5

Game Theory - MSc EEBL

Guillaume Pommey
guillaume.pommey@uniroma2.eu

Simone Senesi
simone.senesi@students.uniroma2.eu

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## Exercise 1. *A simple static game of incomplete information*

In this exercise, the goal is to determine whether some strategy profiles are Bayesian Nash equilibria or not. See Exercise 2 for the method to find *all* Bayesian Nash equilibria in a similar game.

Consider the following *static game of incomplete information*.

| | L | R |
|---|---|---|
| U | $2,2$ | $1,-3$ |
| D | $-3,1$ | $0,0$ |

prob. $1/2$

| | L | R |
|---|---|---|
| U | $0,0$ | $1,-3$ |
| D | $-3,1$ | $2,2$ |

prob. $1/2$

We further assume that player 1 is fully informed about which of these two games is played whereas player 2 only knows that each game has the same probability of occurring.

1. Write the normal-form representation of this game.
2. For this question only, assume that player 1 **always** play U, that is, player 1's strategy is UU.
   (a) What is player 2's expected payoff when playing L? When playing R? Deduce player 2's best response to UU.
   (b) Show that if player 2 plays L, then UU is actually a best response for player 1.
   (c) Conclude whether $(\mathsf{UU}, \mathsf{L})$ is a *Bayesian Nash equilibrium*.
3. Show that $(\mathsf{UD}, \mathsf{R})$ is not a Bayesian Nash equilibrium of this game.
4. Assume now that player 2 believes that the game on the left is played with probability $\alpha \in [0,1]$.

(a) Find a threshold value $\hat{\alpha}$, such that for all $\alpha \leqslant \hat{\alpha}$, R is a best response to UD for player 2.

(b) Is $(\mathsf{UD}, \mathsf{R})$ is a Bayesian Nash equilibrium for $\alpha \leqslant \hat{\alpha}$?

## Exercise 2. *Hiring Decisions*

Consider a Firm (F) and a Worker ($W$). If $W$ has a *high ability* ($\mathsf{T}_W = \text{high}$), they would like to *Work* when they are hired; if instead $W$ has a *low ability* ($\mathsf{T}_W = \text{low}$), they would rather *Shirk*.

F wants to *Hire* $W$ if the latter is willing to *Work* and *Not Hire* them otherwise. $W$ knows their ability level while F does not. However, F *believes* that $W$'s ability is high with probability $\mathbb{P}(\mathsf{T}_W = \text{high}) = \frac{2}{3}$ and low with probability $\mathbb{P}(\mathsf{T}_W = \text{low}) = \frac{1}{3}$. The payoffs (different for each type of $W$) are as follows:

| | Work | Shirk |
|---|---|---|
| Hire | $1,2$ | $0,1$ |
| Don't | $0,0$ | $0,0$ |

$\mathsf{T}_W = \text{High}$

| | Work | Shirk |
|---|---|---|
| Hire | $1,1$ | $-1,2$ |
| Don't | $0,0$ | $0,0$ |

$\mathsf{T}_W = \text{Low}$

1. Describe the game as a normal-form game.
2. Write the strategy profile for each player.
3. Write the game as an extensive-form game.
4. Find all (pure-strategy) Bayesian Nash equilibria of the game.

## Exercise 3. *Matching technologies*

Consider two firms: Firm 1 is producing a video game console and firm 2 is producing a video game. Each firm can choose to *cooperate* (C) or *not cooperate* (N). For instance, firm 1 can cooperate by providing better development tools and firm 2 can cooperate by better optimizing their game to run on firm 1's console.

Each firm's product can be either of type *High* (H) or *Low* (L). Payoffs are as follows.

| | C | N |
|---|---|---|
| C | $3,3$ | $-2,0$ |
| N | $0,-2$ | $0,0$ |

$\mathsf{G}_1 : \{\mathsf{H},\mathsf{H}\}$

| | C | N |
|---|---|---|
| C | $-1,-1$ | $-2,0$ |
| N | $0,-2$ | $0,0$ |

$\mathsf{G}_2 : \{\mathsf{L},\mathsf{L}\}$ or $\{\mathsf{H},\mathsf{L}\}$ or $\{\mathsf{L},\mathsf{H}\}$

In words, firms enjoy cooperation only when they both have high types. Each firm only knows their own type. They both think that the probability that the other firm is of type $H$ is $1/2$. Let $p_1(H) = p_2(H) = 1/2$ denote this belief.

1. (a) What is the probability that firms are in configuration $G_1$? Same question for $G_2$.
   (b) Now assume firm 1 knows it is of type $H$. From its point of view, what is the probability that it faces configuration $G_1$?
2. What is a strategy for firm $i = 1,2$ in this game?
3. Assume firm 2 decides to cooperate for each of its type, that is, firm 2 plays $CN$.
   (a) What is the best response of firm 1 when it is of type $H$?
   (b) What is the best response of firm 1 when it is of type $L$?
   (c) Deduce a Bayesian Nash equilibrium of this game.