Practice session 2

Game Theory - MSc EEBL

Guillaume Pommey
guillaume.pommey@uniroma2.eu

Uliana Sulakshina
sulakshina@gmail.com

September 26, 2024

Exercise 1. *Cournot Duopoly*

Two firms, i and j, compete in a market by simultaneously setting their quantities (q_i, q_j) of an homogeneous good. Each firm faces a *constant marginal cost* $c_k \in \mathbb{R}_+$, where $k = i, j$. The two firms face the *inverse demand function* $P(Q) = a - bQ$, where $Q := q_i + q_j$ is the aggregate quantity and $(a, b) \in \mathbb{R}^2_+$ are demand parameters. Payoffs are given by each firm's profits.

1. Describe the game as a normal-form game (Players, Strategies, Payoffs).

2. Write the maximization problem for each firm.

3. Solve the maximization problem for each firm, obtaining its *reaction function*. (*Hint: Take the first derivative of profits with respect to each firm's quantity, taking the quantity produced by the other as given).*

4. Find the *NE* of the game, i.e., equilibrium quantities for both firms (*Hint: Use the two reaction functions and solve for* $q_i^\star$).

5. Comment on the equilibrium quantities when a, c_1 and c_2 vary.

6. Find the payoffs $\pi_i(q_i^\star, q_j^\star)$ for $i = 1, 2$ obtained by firms when they play this *NE* (*Hint: Replace equilibrium quantities in profits*).

7. Now suppose that both firms also face a *fixed cost* $F = 1$. Does this affect the equilibrium quantity? Does this change equilibrium payoffs? Explain.

Exercise 2. *Setting a Standard*

A new type of consumer product is about to be introduced in a market in which two firms are active (for example, a video game). The two firms own *competing technologies* (for example, two game consoles) that can be used to run this product, and would like their technology to be the *standard* in the market.

Each firm would prefer its technology to be used exclusively to run the product, as this would increase its sales. In particular, each firm has a payoff of zero if no standard is set (both firms use their own technology). If only one firm's technology is adopted as a standard, that firm gets a payoff of 2, and the other gets 1. Finally, if both firms employ the other firm's technology, they both get a payoff of 0.

1. Fill the payoff matrix using the available information and write the game as a normal-form game.
2. Is there any strictly *dominated strategy* for the two players?
3. Find the *pure-strategy Nash equilibria (NE)* of this game, if any.
4. Find the *mixed-strategy NE* of this game, if any.
5. Now, suppose that firm 1 has a *superior technology*, that is, the latter gets a payoff of 3 when it manages to set the standard. Does this affect the *pure and mixed-strategy NE*? Explain (*Hint: payoffs do not change for firm 2*).

Exercise 3. *Rock paper scissors*

Pat and Carl meet to play the famous game *Rock paper scissors*. According to this game, both players simultaneously choose between rock, paper, or scissors. Not surprisingly, rock beats scissors, scissors beat paper, and paper beats rock. If a player wins, they get 1 Euro from the other player. If they loose, they pay 1 Euro to the other player. If both players choose the same action, then they both get nothing.

1. Fill the *payoff matrix* using the available information and write the game as a *normal-form* game.
2. Is there any strictly *dominated strategy* for players?
3. Find the *pure-strategy Nash equilibria (NE)* of this game, if any.
4. Find the *mixed-strategy* NE of this game, if any.

Exercise 4. *Bertrand duopoly with differentiated products* (*Optional*)

Two firms compete in a market by simultaneously setting the prices, (p_i, p_j), of a differentiated good. Each firm faces a *constant marginal cost* $c = 2$. The demand for each firm's good is $q_i(p_i, p_j) = 6 - p_i + bp_j$ and $q_j(p_i, p_j) = 6 - p_j + bp_i$, where b is a parameter capturing product differentiation. Assume that $b \in [0, 1]$. Payoffs are given by each firm's profits.

1. Describe the game as a normal-form game (Players, Strategies, Payoffs).
2. Write the maximization problem for each firm.
3. Solve the maximization problem for each firm, obtaining its *reaction function*. (*Hint: Take the first derivative of profits with respect to each firm's price, taking the price set by the other as given).*
4. Find the *NE* of the game, i.e., equilibrium quantities for both firms (*Hint: Use the two reaction functions and solve for* $p_i^\star$).
5. How do prices vary with b? Explain.
6. Find the payoffs ($\pi_i(p_i^\star, p_j^\star)$) obtained by firms when they play this *NE* (*Hint: Replace equilibrium prices in profits*).

Exercise 5. *Cournot Duopoly: Numerical application* (*Additional*)

Two firms compete in a market by simultaneously setting the quantity of a homogeneous good to produce. Firm 1 faces a *constant marginal cost* $c_1 = 1$, while Firm 2 faces a *constant marginal cost* $c_2 = 4$. There is no *fixed cost* of production. *Inverse market demand* is $P(Q) = 25 - 3Q$, where Q is aggregate quantity. Payoffs are given by each firm's profits.

1. Describe the game as a normal-form game.
2. Write the maximization problem for each firm.
3. Find the Nash Equilibrium of the game.
4. Find the payoffs that firms obtain when they play the Nash equilibrium.
5. Suppose each firm faces the same fixed cost $F = 10$. How is the equilibrium affected?