Practice session 4

Game Theory - MSc EEBL

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Exercise 1. *Collusion in Cournot*

Consider a *Cournot game* in which two firms, 1 and 2, produce an *homogeneous good* and interact an *infinite number of times*. Both firms have a common *discount factor* $\delta \in (0, 1)$, which is a measure of their *patience* concerning future profits. In each of the (infinite and identical) stage games, the two firms simultaneously set the quantity of the goods they produce (q_1, q_2). The *marginal cost* of production is $c = 3$. Inverse market demand in each stage is given by

$$P(Q) = 9 - Q,$$

where $Q := q_1 + q_2$.

1. Find the *Nash equilibrium* quantity and profits of the *stage game*.

2. What are the maximum profits that can be achieved by the two firms if they cooperate?

   Consider now the following grim trigger strategy: "*In stage* t*, produce* $q_i = \frac{Q^m}{2}$ *if firm* j *has produced* $q_j = \frac{Q^m}{2}$ *in all previous periods; otherwise, produce the Cournot quantity forever*".

3. Consider first a subgame starting at period t such that at least one firm has played something different from half the monopoly quantity in the past. Assume that firm j plays the trigger strategy and show that firm i has no incentive to deviate from the trigger strategy.

4. Consider a subgame starting at period t such that both firms have always cooperated in the past, that is, they have chosen $q_i = q_j = \frac{Q^m}{2}$ for all periods from 0 to $t - 1$.

(a) Assume that firm j plays the trigger strategy. Find the profit of firm i when it sticks to the trigger strategy.

(b) Still assume that firm j plays the trigger strategy but now assume firm i deviates from the trigger strategy. Find q_i^d, the deviation quantity that maximizes firm i's profit when it deviates.

(c) Finally, compute the profit that firm i can obtained by deviating.

(d) Compare the two profits (trigger strategy and deviation) and deduce a condition on δ.

Exercise 2. *Repeated Prisoner's Dilemma*

Consider the following static Prisoner's dilemma.

		Player 2: Cooperate	Player 2: Defect
Player 1	Cooperate	(1,1)	(-1,2)
	Defect	(2,-1)	(0,0)

1. Find all Nash equilibria of the static game.

2. Assume now that the game is repeated T times, where $T < \infty$. Find the subgame perfect Nash equilibrium (*Hint: as there is a finite number of repetitions, you can use backward induction*).

3. Assume now that the game is repeated an infinite number of periods. Furthermore, let $\delta \in (0, 1)$ be the discount factor common to both players.

   (a) Consider the *non-forgiving trigger strategy* for both players. What does it mean in this particular game?

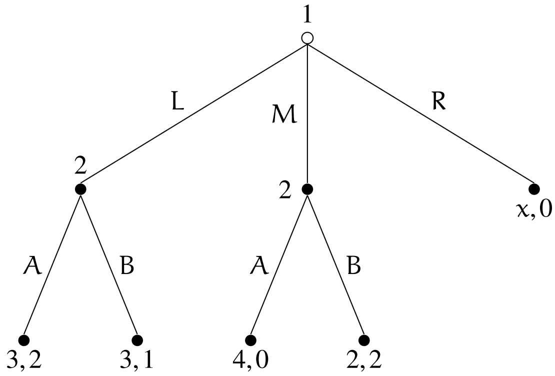
   (b) Consider any subgame of the game starting at period t. Among all possible history of play, there is only two relevant cases: (i) No one has played D in the past, and (ii) someone has played D in the past. In each case, compute the payoff a player obtains when playing D and when playing C.

   (c) Using the previous payoff, find conditions on δ such that the *non-forgiving trigger strategy* is a best response in each case (i) and (ii).

   (d) When those conditions are met, what does it mean?

Exercise 3. *Simple characterization of SPNE in a dynamic game*

Consider the following **dynamic game of complete information**.



Throughout the analysis, we will consider two cases for the value of x. Either $x = 1$ or $x = 5$. It will be specified which value of x you have to consider to answer each question.

For the following questions (1, 2, 3 and 4) let $x = 1$.

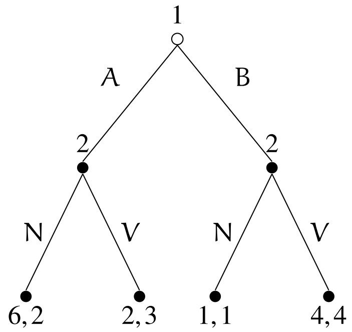
1. Write the payoff matrix of this game and find all Nash equilibria.
2. What is player 2's best response after L? After M?
3. Would your answer to question 2 be different if $x = 5$? Justify.
4. Find the subgame perfect Nash equilibrium.

   Now assume that $x = 5$ for questions 5 and 6.

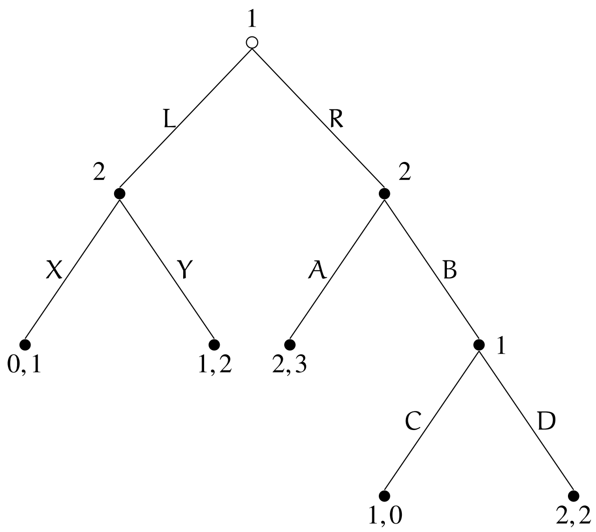
5. Find the subgame perfect Nash equilibrium.
6. Explain carefully why the SPNE of question 5 differs from the one you found in question 4.
7. If x can take any positive value ($x \in \mathbb{R}_+$), what is the minimal value of x such that the SPNE is that of question 5 and not that of question 4?

Exercise 4. *Additional dynamic games*

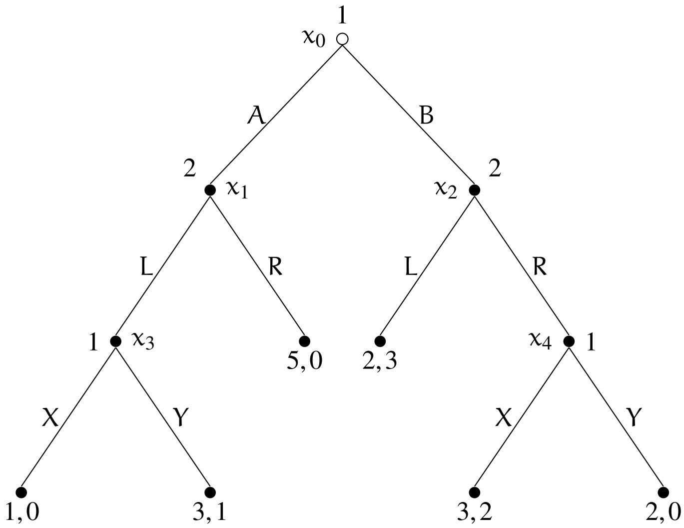
1. Find the Nash equilibria and the SPNE in the following game.



2. Find the SPNE of the following game.



3. Find the SPNE of the following game.


1
x_0
A
B
2
x_1
x_2
2
L
R
L
R
1
x_3
5, 0
2, 3
x_4
1
X
Y
X
Y
1, 0
3, 1
3, 2
2, 0