Practice session 6

Game Theory - MSc EEBL

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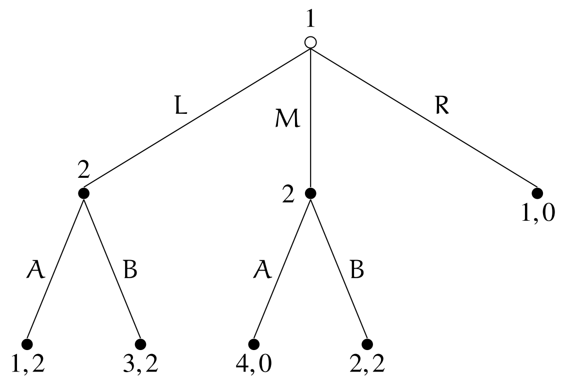
Exercise 1.

Consider the following **dynamic game of complete information**.



1. Compute the *Nash Equilibria* and the *Subgame-Perfect Nash Equilibria*.
2. Are the sets of NE and SPNE different? The same? Carefully explain why it is the case.

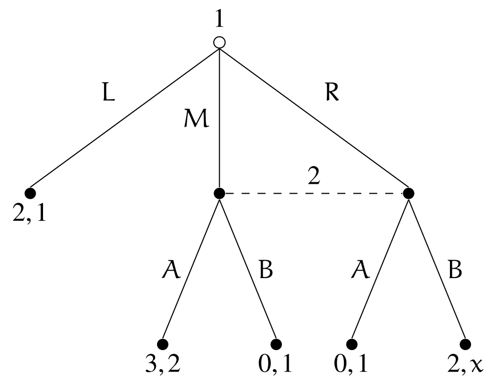
From now on, assume that **player 2 cannot distinguish** action L from action M.

3. Explain how we should modify the tree above to represent this game.
4. The *Nash Equilibrium* of this game is (L, B). Explain why the strategy of player 2 is not a couple of actions (for instance AB or BB) like it was the case in question 1.
5. Consider the strategy profile (L, B). Is it a *Perfect Bayesian Nash Equilibrium*?

6. Show that (M, B) **is not** a *Perfect Bayesian Nash Equilibrium*.

Exercise 2. *Perfect Bayesian Nash Equilibrium*

Consider the following dynamic game of incomplete information.

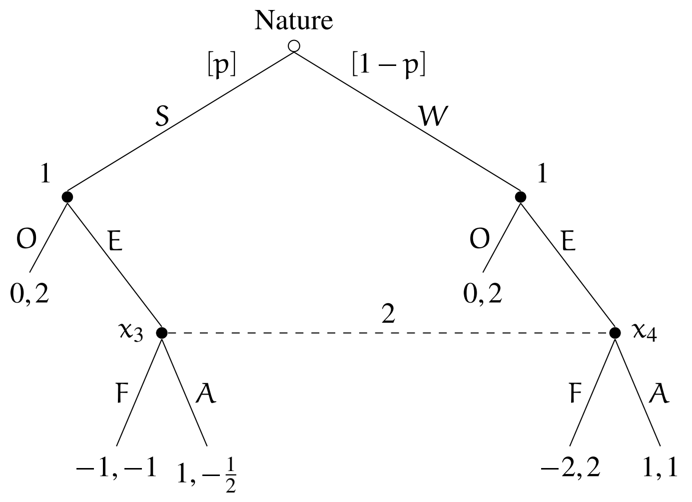


Assume that x is either equal to 0 or to 3. Let $\mu \in [0,1]$ and $1-\mu$ denote player 2's beliefs about M and R, respectively.

1. Assume for now that $x = 0$.
   (a) Can you identify a dominant strategy for player 2?
   (b) If player 1 had to choose only between M and R, what would they do?
   (c) Deduce what should be player 2's beliefs about M and R.
   (d) Find a perfect Bayesian Nash equilibrium of this game.
2. Assume for now that $x = 3$.
   (a) For which belief $\mu \in [0,1]$, player 2 prefers to play A to B?
   (b) Characterize the perfect Bayesian Nash equilibria with respect to the value of μ.

Exercise 3. ***An entry game***

Consider the following entry game.



Nature first draws the type of player 1, the entrant, where S stands from "Strong" and W for "Weak". Player 1, and only them, observes their type and decide whether to "enter the market" (E) or "stay out of the market" (O). If player 1 chooses O, the game ends. If player 1 chooses E, the game proceeds to the next period where player 2, the incumbent, must choose whether to "Fight" (F) or "Accommodate" entry (A).

Player 2 does not observe whether they are in x_3 or x_4 when they have to play. Let $\mu \in [0,1]$ denote the belief of player 2 that they are at the decision node x_3 when they reach the information set $\{\{x_3, x_4\}\}$.

1. Assume for now that $p = 0.5$ and consider the strategy profile (EE, F).

   (a) What is the belief μ consistent with this strategy profile?

   (b) Compute the expected payoffs of player 2 when choosing F and when choosing A.

   (c) From the previous question, is F a best-response to EE?

   (d) Is EE a best response to F?

   (e) Conclude about whether (EE, F) is a PBNE.

2. Assume that p can take any value in $[0,1]$ and consider the strategy profile (EE, A).

   (a) What is μ now?

   (b) Compute the expected payoffs of player 2 when playing F and then when playing A.

   (c) For which values of p, player 2 prefers A to F?

(d) Can (EE, A) be a PBNE of this game? If yes, under which conditions.

Exercise 4. ***Additional (difficult) - Rational Handicap*** **(***Zahavi, 1975 and Grafen, 1990***)**

Male peacocks grow beautiful and colorful tails. It is, however, a handicap as it makes the male more noticeable to predators. From the Darwinian evolutionary theory, it may seem surprising that male peacocks with the most noticeable tails survive the law of evolution and this question has been puzzling biologists for a long time.

One of the possible answer to this problem is to consider that peacocks are playing the following signaling game. Nature chooses the male peacock type: With probability p the male is "strong" (H) and with probability $1-p$ it is "weak" (L). The male privately observes his type and **chooses** whether to "grow a colorful tail" (T) or "not" (nT). Then, a female peacock observes the male's choice (but not his type) and must decide to "mate" (M) or not (nM).

The female's payoffs are as follows: Mating with a type H yields 2, Mating with a type L yields 0 and not mating yields 1. The male's payoff are as follows: Not growing a colorful tail yields $v > 0$, growing a colorful tail when H yields $z_H v$ and growing a colorful tail when L yields $z_L v$. We assume $z_L < z_H < 1$. Finally, a male gets an additional payoff $r > 0$ if the female decides to mate, irrespective of his type or tail choice.

1. Write the normal-form representation and represent the game in a tree.
2. Find the pooling equilibria.
3. Find the separating equilibria.
4. Discuss the term "rational handicap".