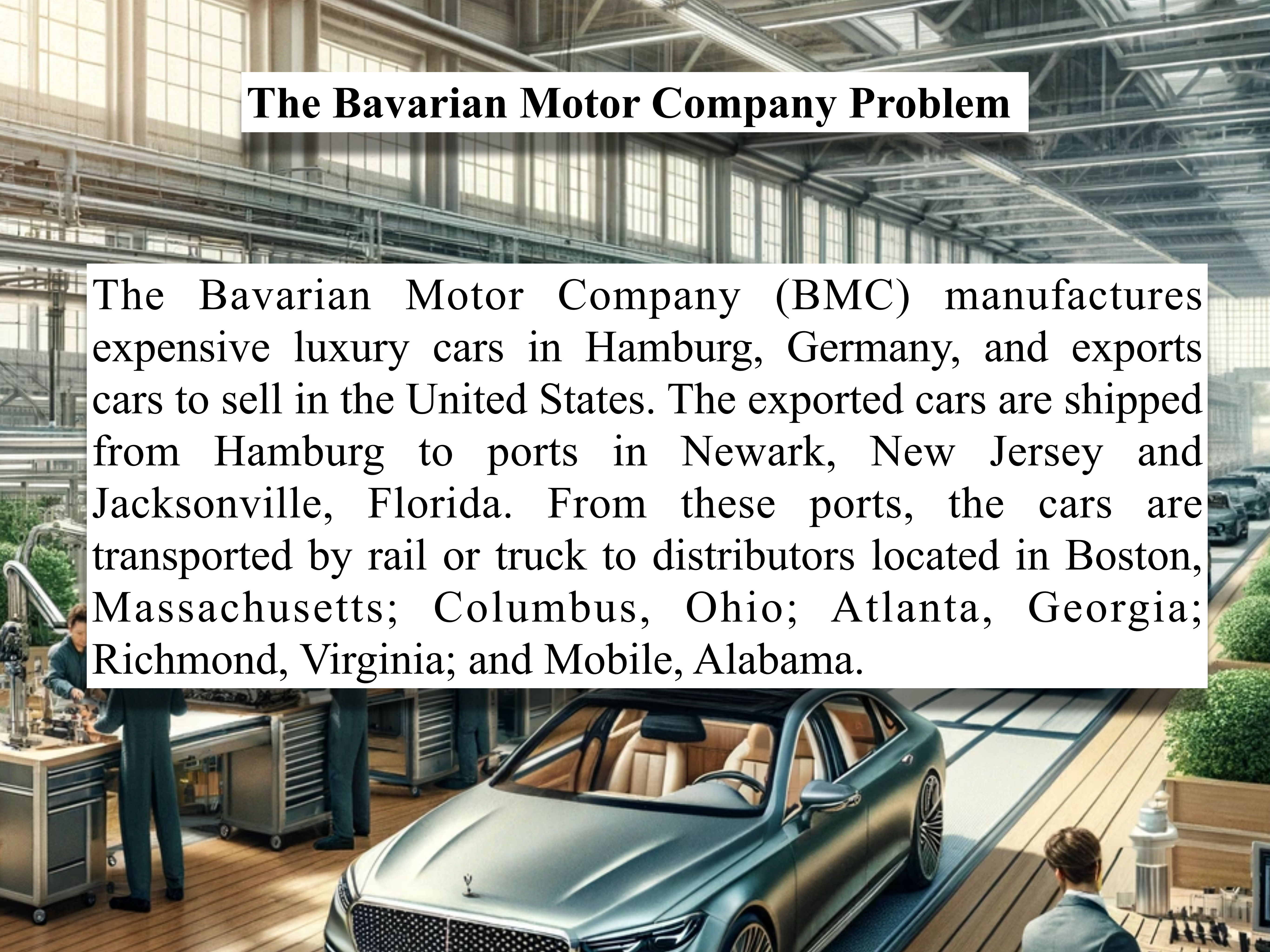
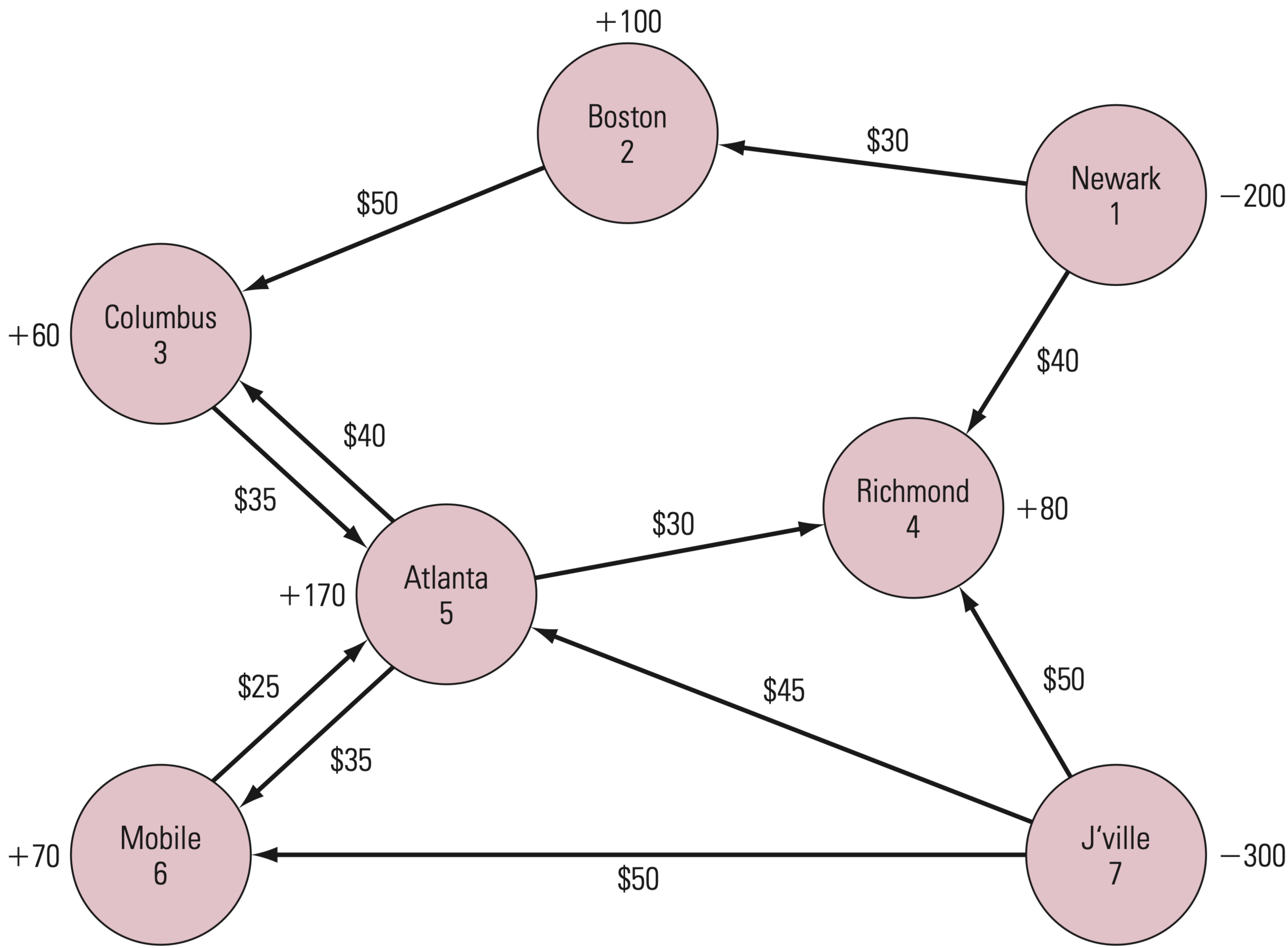
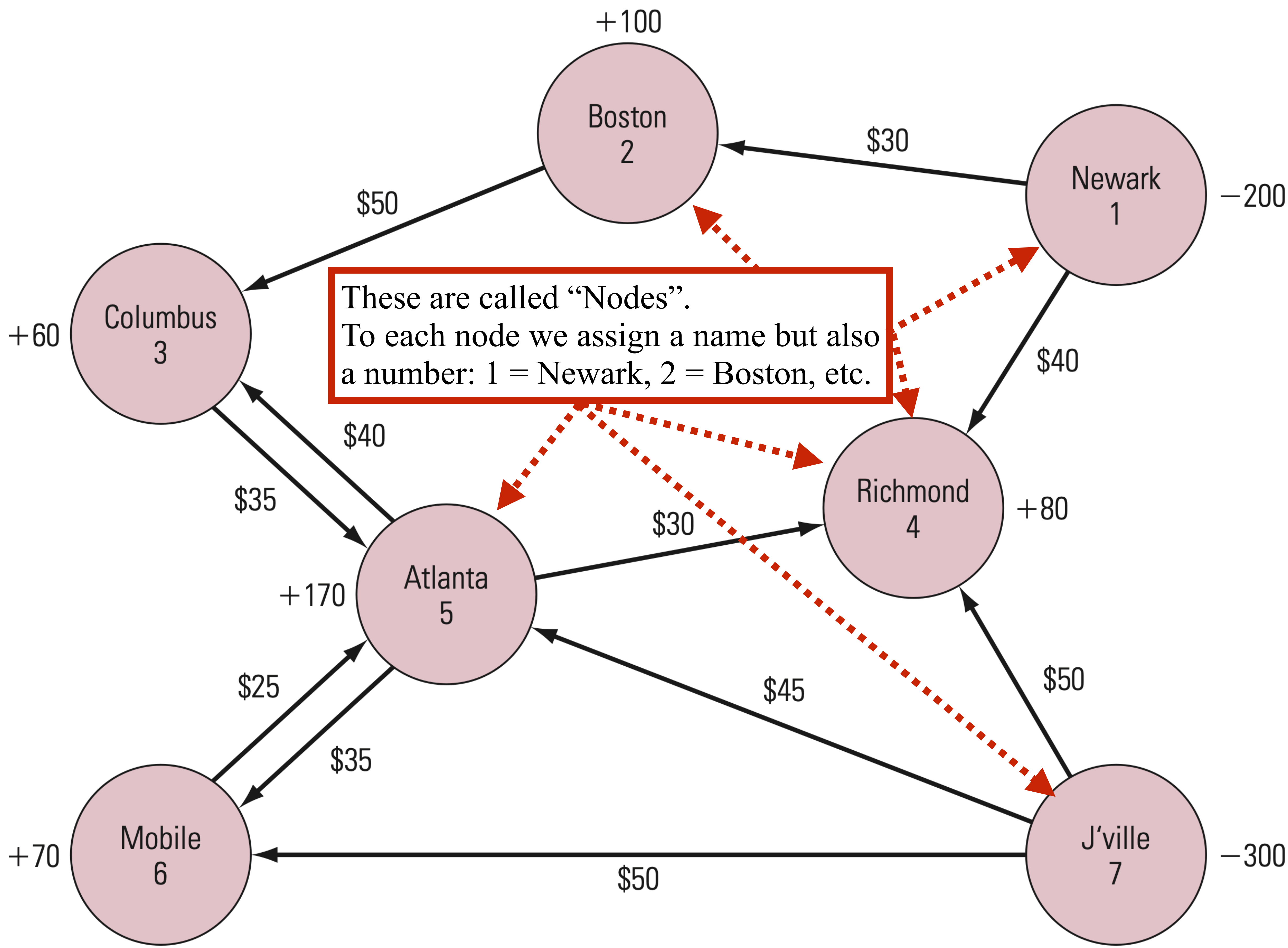
NETWORK FLOWS
with
EXCEL SOLVER

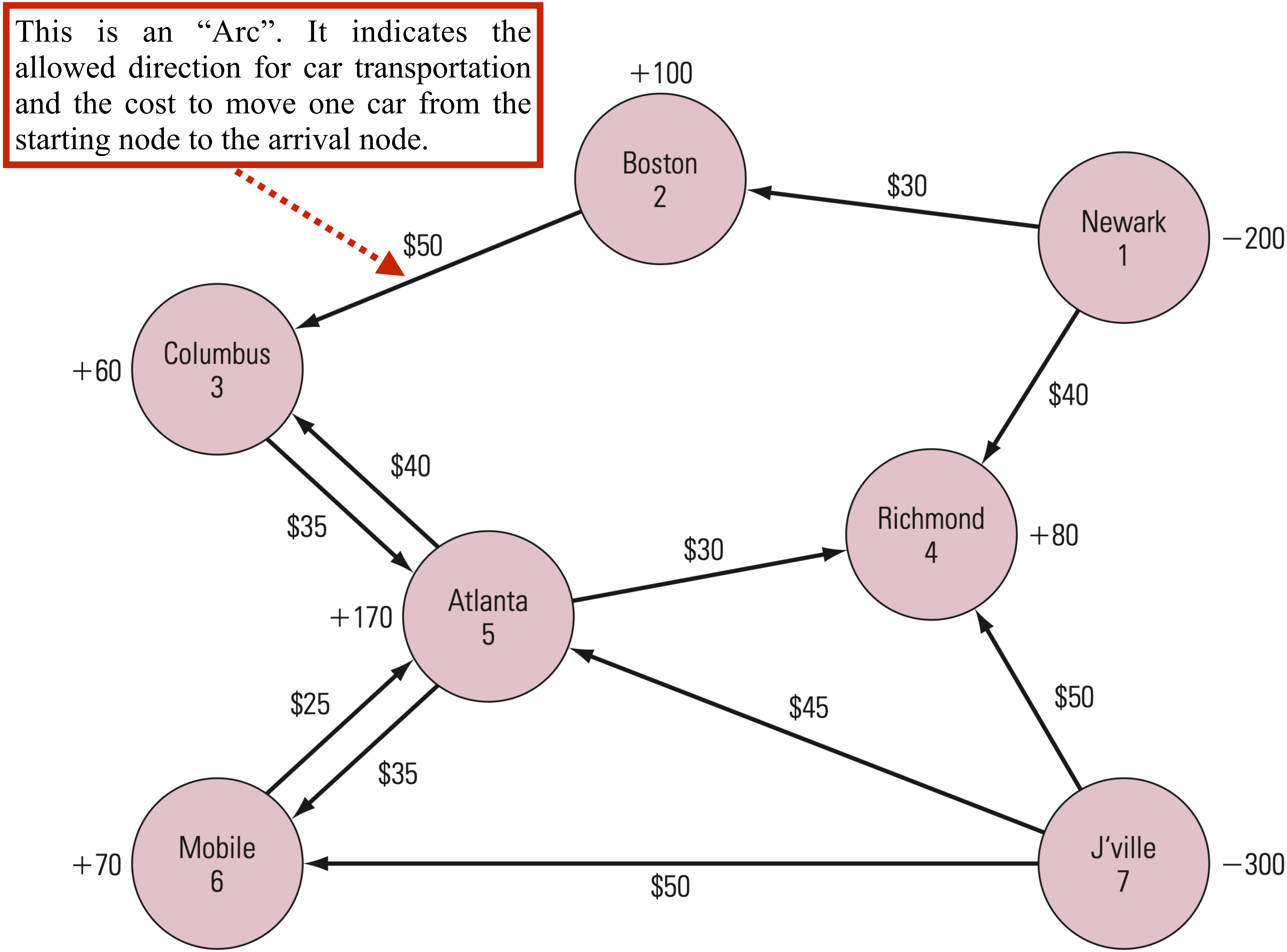


The Bavarian Motor Company Problem

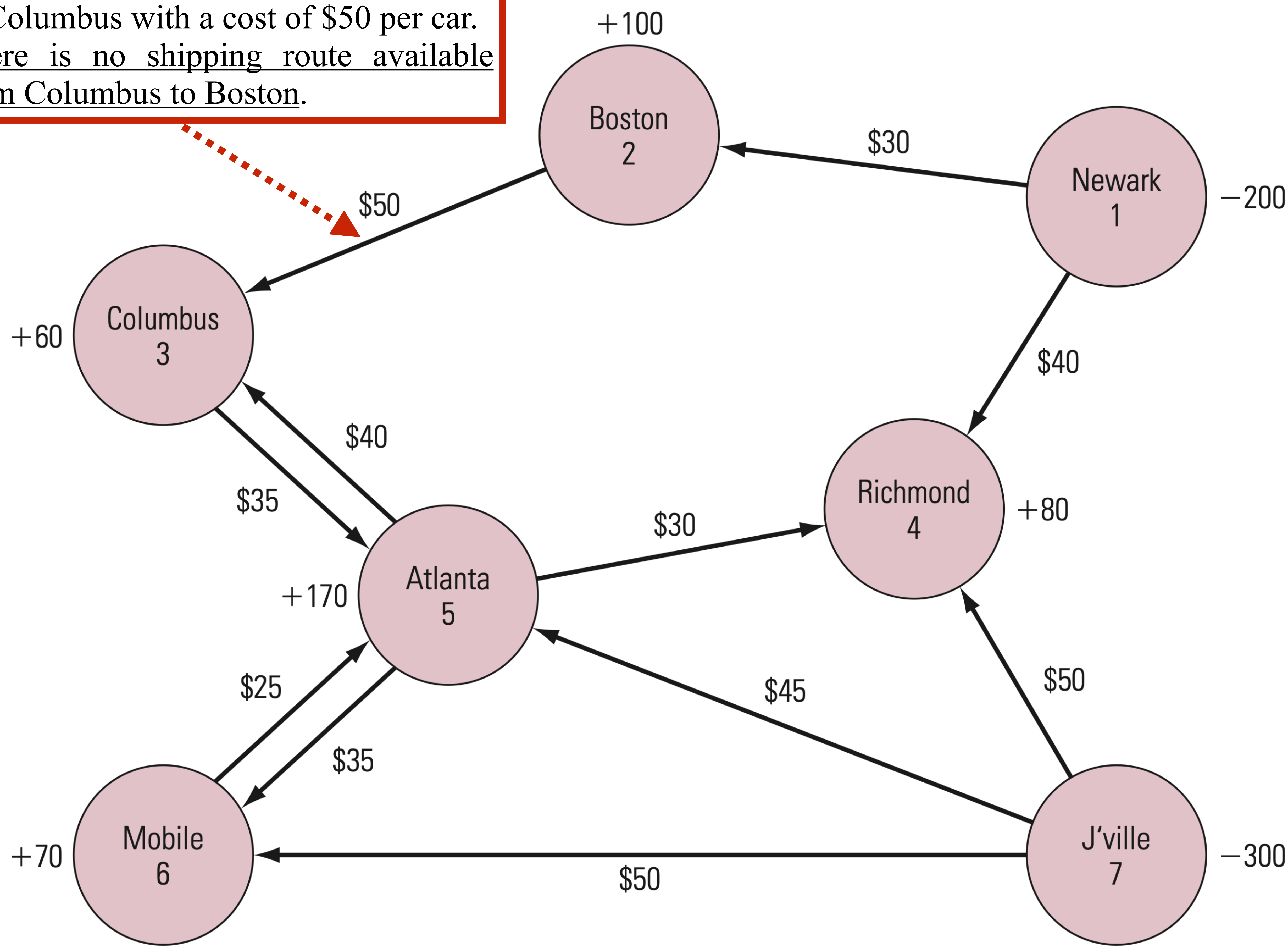
The Bavarian Motor Company (BMC) manufactures expensive luxury cars in Hamburg, Germany, and exports cars to sell in the United States. The exported cars are shipped from Hamburg to ports in Newark, New Jersey and Jacksonville, Florida. From these ports, the cars are transported by rail or truck to distributors located in Boston, Massachusetts; Columbus, Ohio; Atlanta, Georgia; Richmond, Virginia; and Mobile, Alabama.

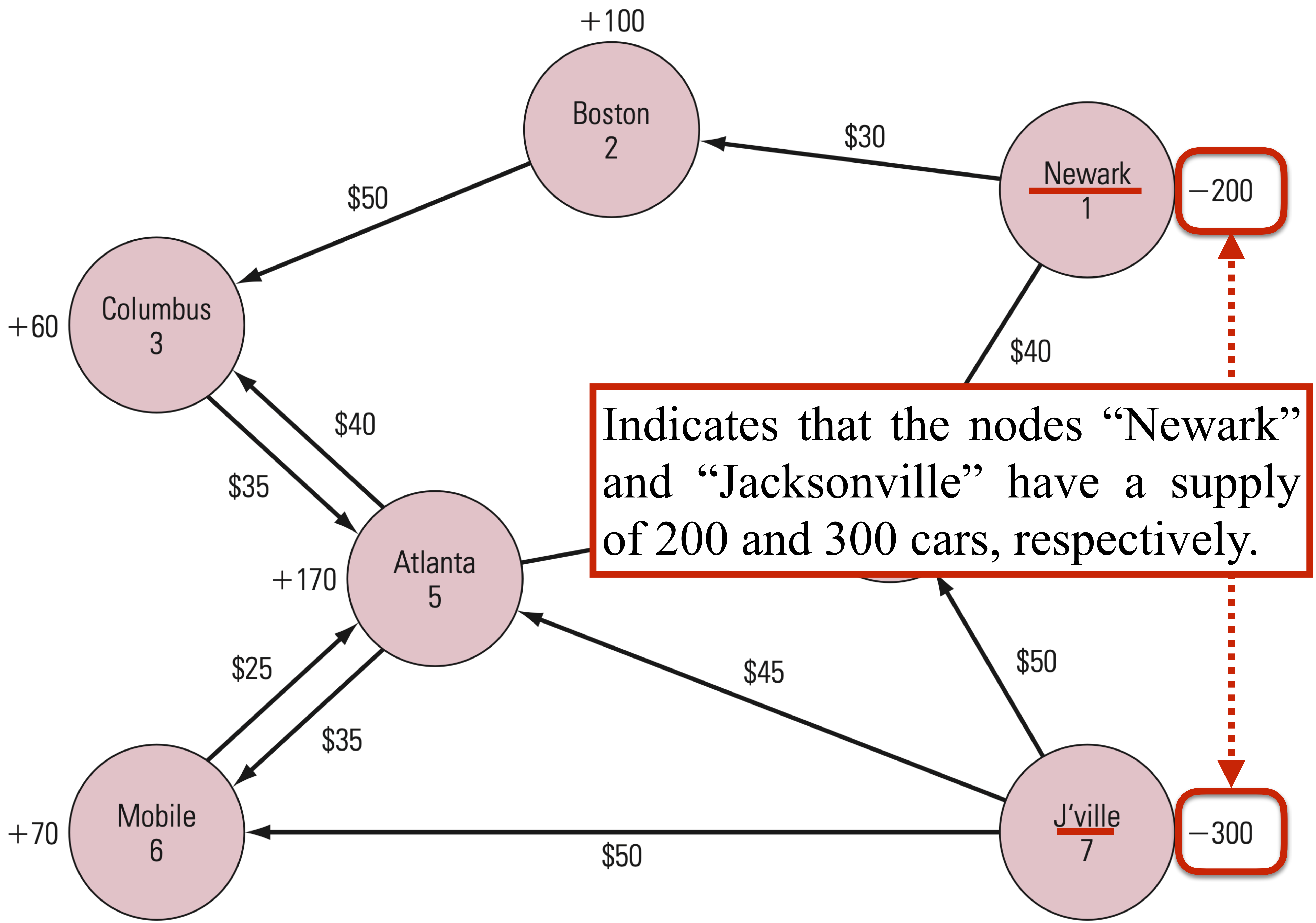

+100
Boston
2
$30
Newark
1
−200
$50
Columbus
3
+60
$40
$40
Richmond
4
+80
$35
$30
Atlanta
5
+170
$25
$45
$50
$35
Mobile
6
+70
J'ville
7
−300
$50

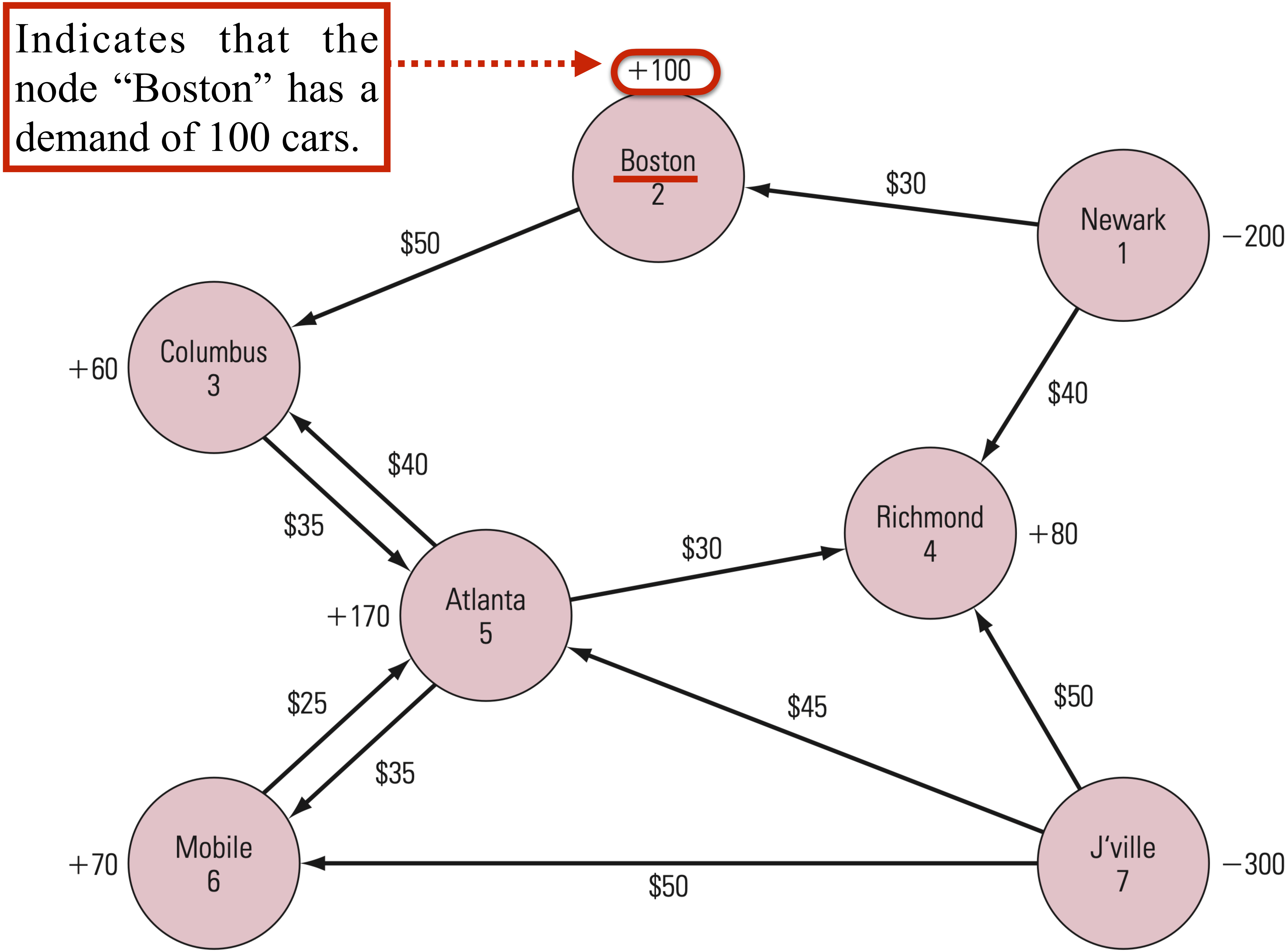

+100
Boston
2
$30
Newark
1
−200
$50
Columbus
3
+60
These are called “Nodes”.
To each node we assign a name but also
a number: 1 = Newark, 2 = Boston, etc.
$40
$40
$35
Richmond
4
+80
$30
Atlanta
5
+170
$25
$45
$50
$35
Mobile
6
+70
J’ville
7
−300
$50


This is an "Arc". It indicates the allowed direction for car transportation and the cost to move one car from the starting node to the arrival node.
+100
Boston
2
$30
Newark
1
−200
$50
+60
Columbus
3
$40
$40
Richmond
4
+80
$35
$30
+170
Atlanta
5
$25
$45
$50
$35
+70
Mobile
6
J'ville
7
−300
$50

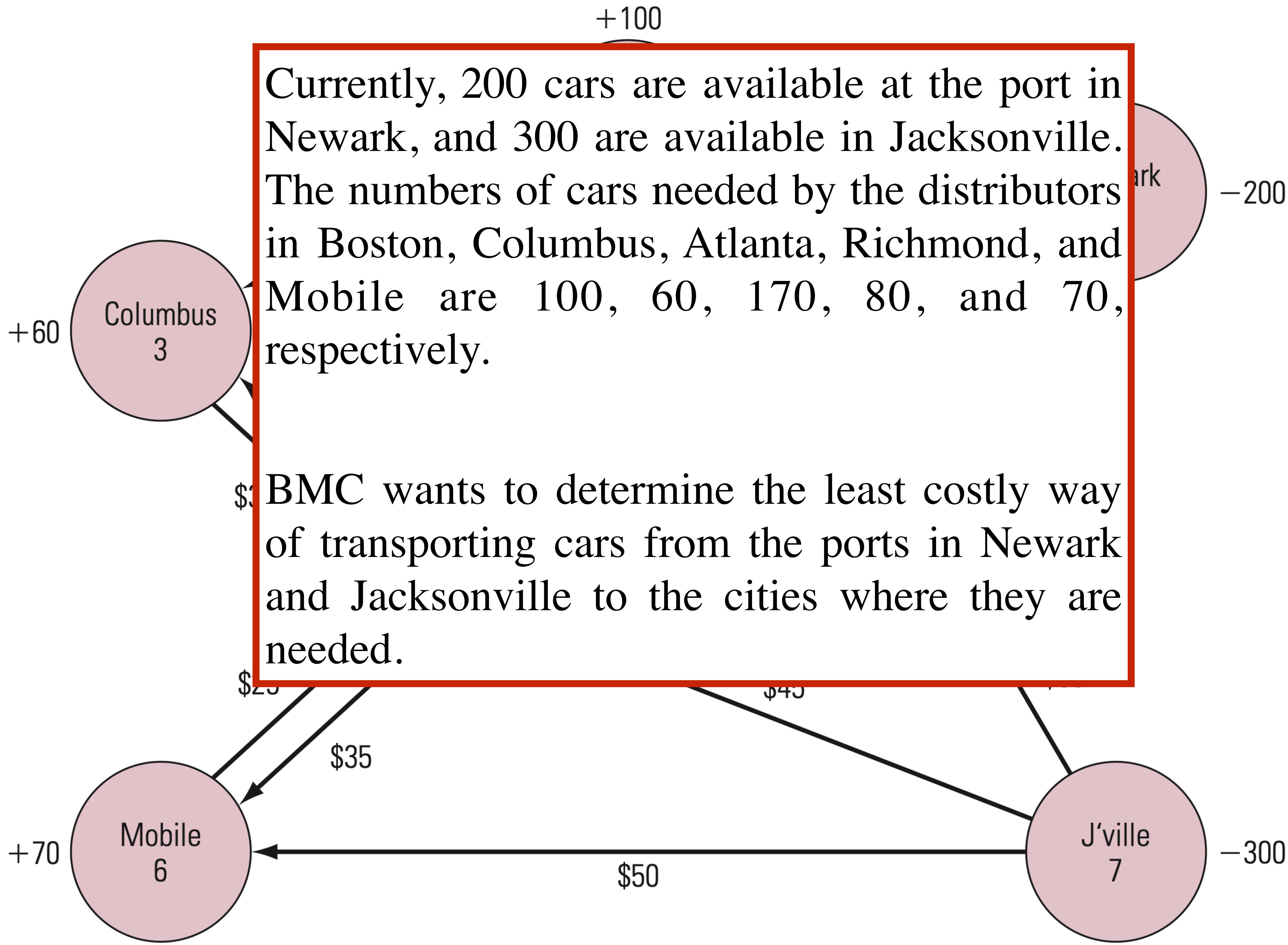
There is a shipping route from Boston to Columbus with a cost of $50 per car. There is no shipping route available from Columbus to Boston.




+100
Boston
2
$30
Newark
1
−200
$50
Columbus
3
+60
$40
$40
Indicates that the nodes "Newark" and "Jacksonville" have a supply of 200 and 300 cars, respectively.
$35
Atlanta
5
+170
$25
$45
$50
$35
Mobile
6
+70
$50
J'ville
7
−300



Indicates that the node "Boston" has a demand of 100 cars.
+100
Boston
2
Newark
1
−200
$30
$50
Columbus
3
+60
$40
$40
$35
Richmond
4
+80
$30
Atlanta
5
+170
$25
$35
$45
$50
Mobile
6
+70
J'ville
7
−300
$50



Currently, 200 cars are available at the port in Newark, and 300 are available in Jacksonville. The numbers of cars needed by the distributors in Boston, Columbus, Atlanta, Richmond, and Mobile are 100, 60, 170, 80, and 70, respectively.

BMC wants to determine the least costly way of transporting cars from the ports in Newark and Jacksonville to the cities where they are needed.

Prepare the template

The car shipment problem

Ship	From	To	Unit cost

Total cost	

Nodes	Net Flow	Supply/Demand

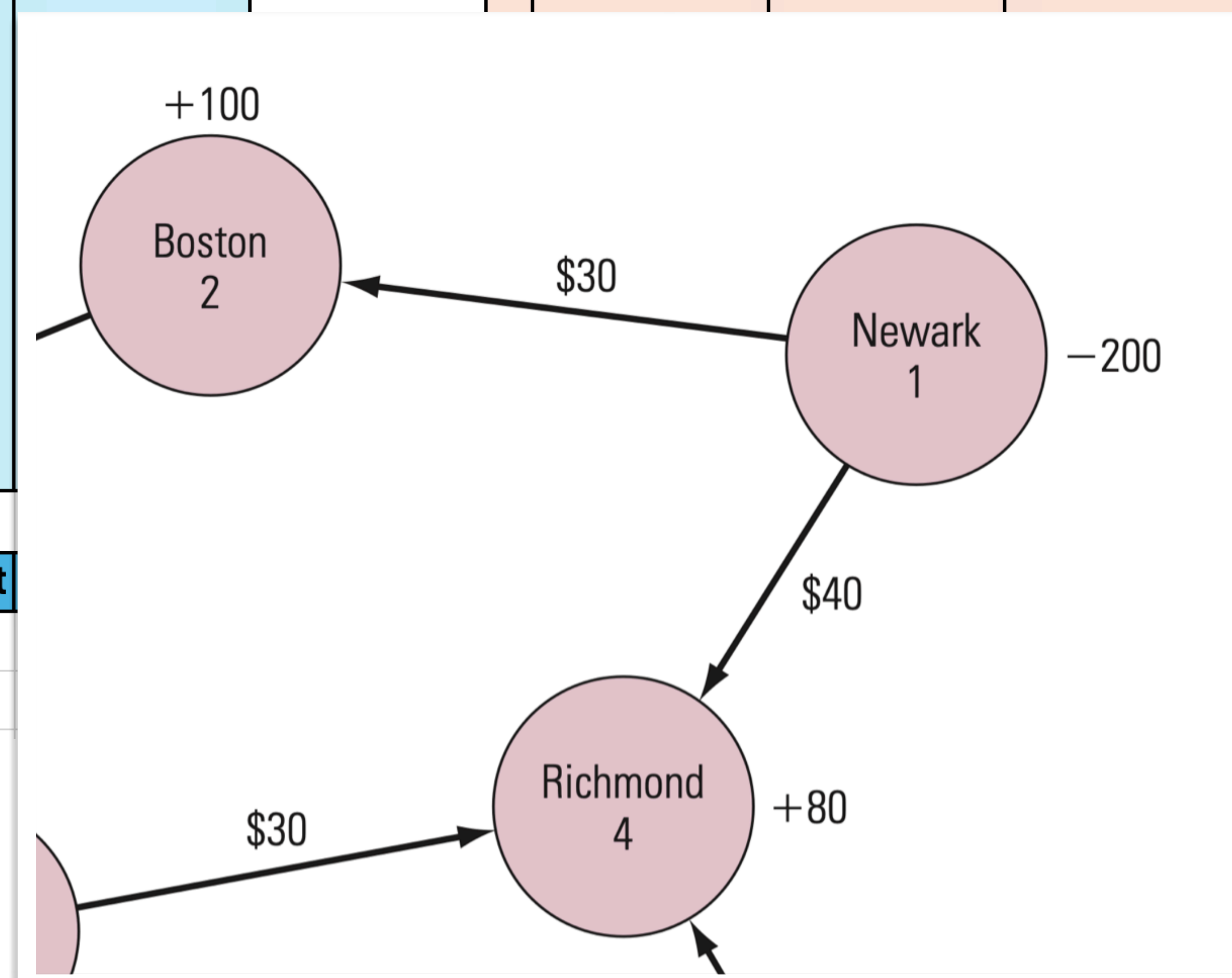
Newark (node number 1) can ship only to Boston (node 2) and Richmond (node 4) at the cost of $30 and $40, respectively

The car shipment problem

Ship	From		To		Unit cost
	1	Newark	2	Boston	$30.00
	1	Newark	4	Richmond	$40.00

Total cost

Nodes	Net Flow	Supply/Demand



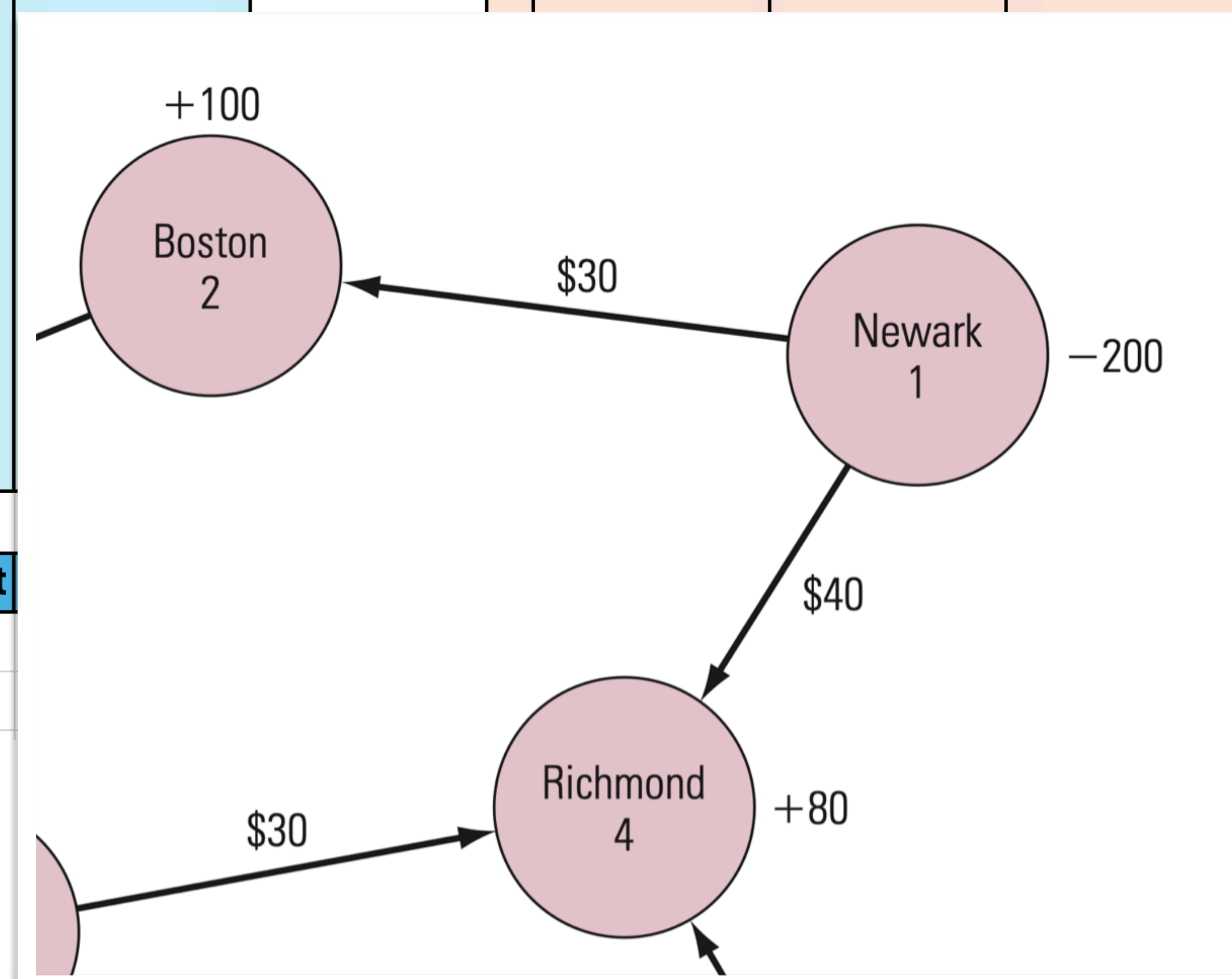
The cells from B4 to B14 contain the decision variables: how many cars to ship from the starting node to the arrival node.

The car shipment problem

Ship	From		To		Unit cost
	1	Newark	2	Boston	$30.00
	1	Newark	4	Richmond	$40.00

Nodes		Net Flow	Supply/Demand

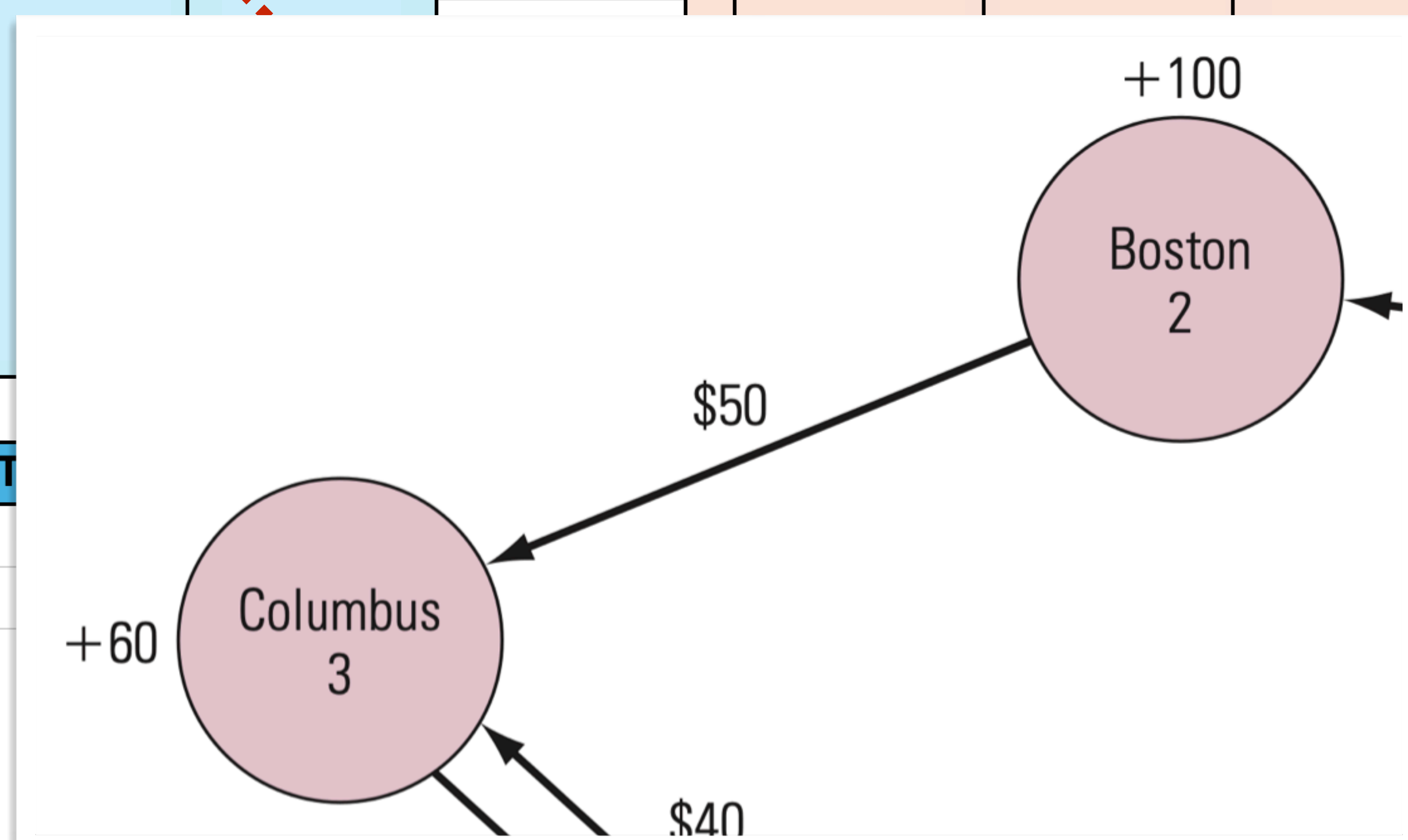
Total cost



The car shipment problem

Ship	From		To		Unit cost
	1	Newark	2	Boston	$30.00
	1	Newark	4	Richmond	$40.00
	2	Boston	3	Columbus	$50.00
	3	Columbus	5	Atlanta	$35.00

Nodes	Net Flow	Supply/Demand

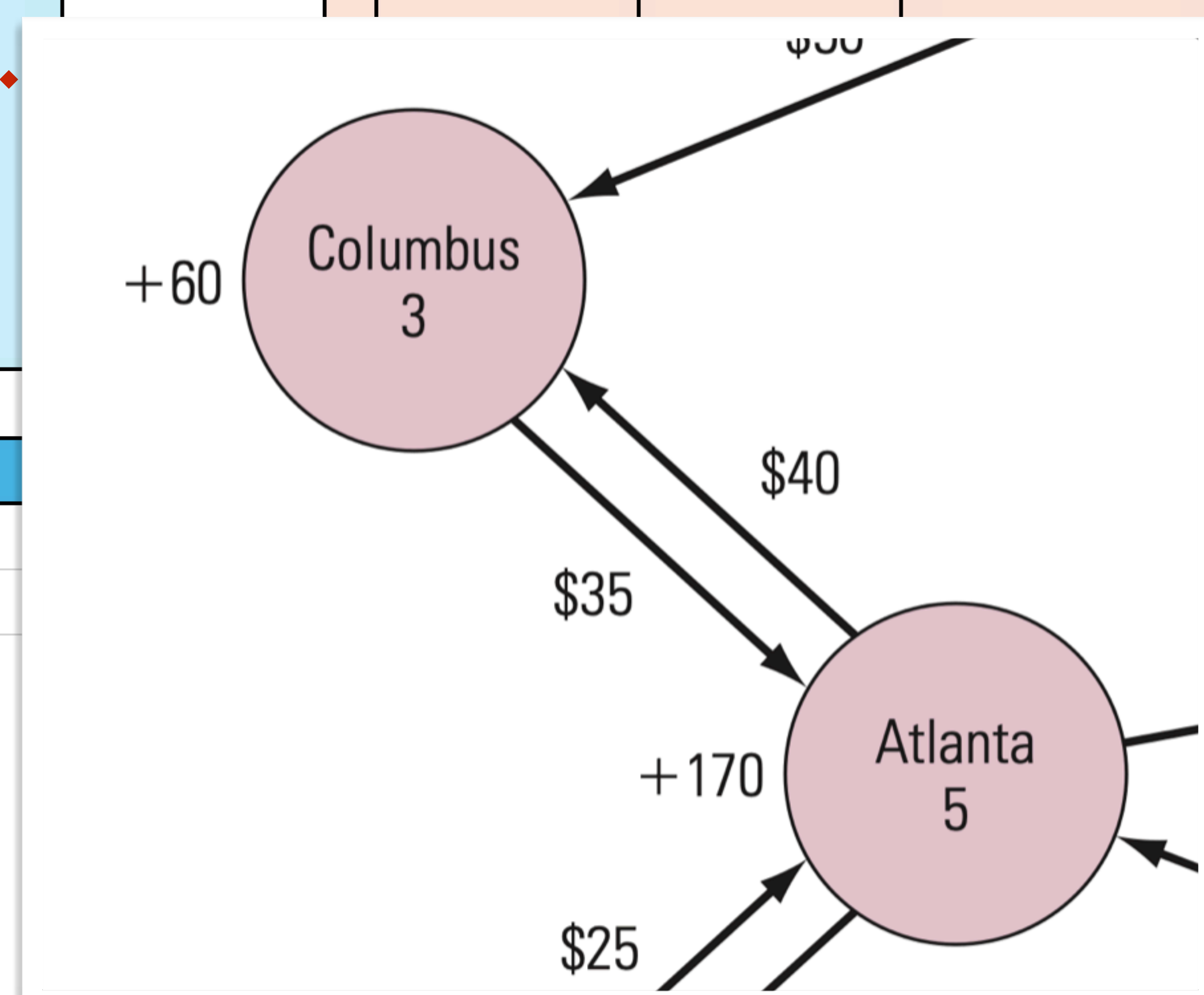


The car shipment problem

Ship	From		To		Unit cost
	1	Newark	2	Boston	$30.00
	1	Newark	4	Richmond	$40.00
	2	Boston	3	Columbus	$50.00
	3	Columbus	5	Atlanta	$35.00

Nodes	Net Flow	Supply/Demand

Total cost	



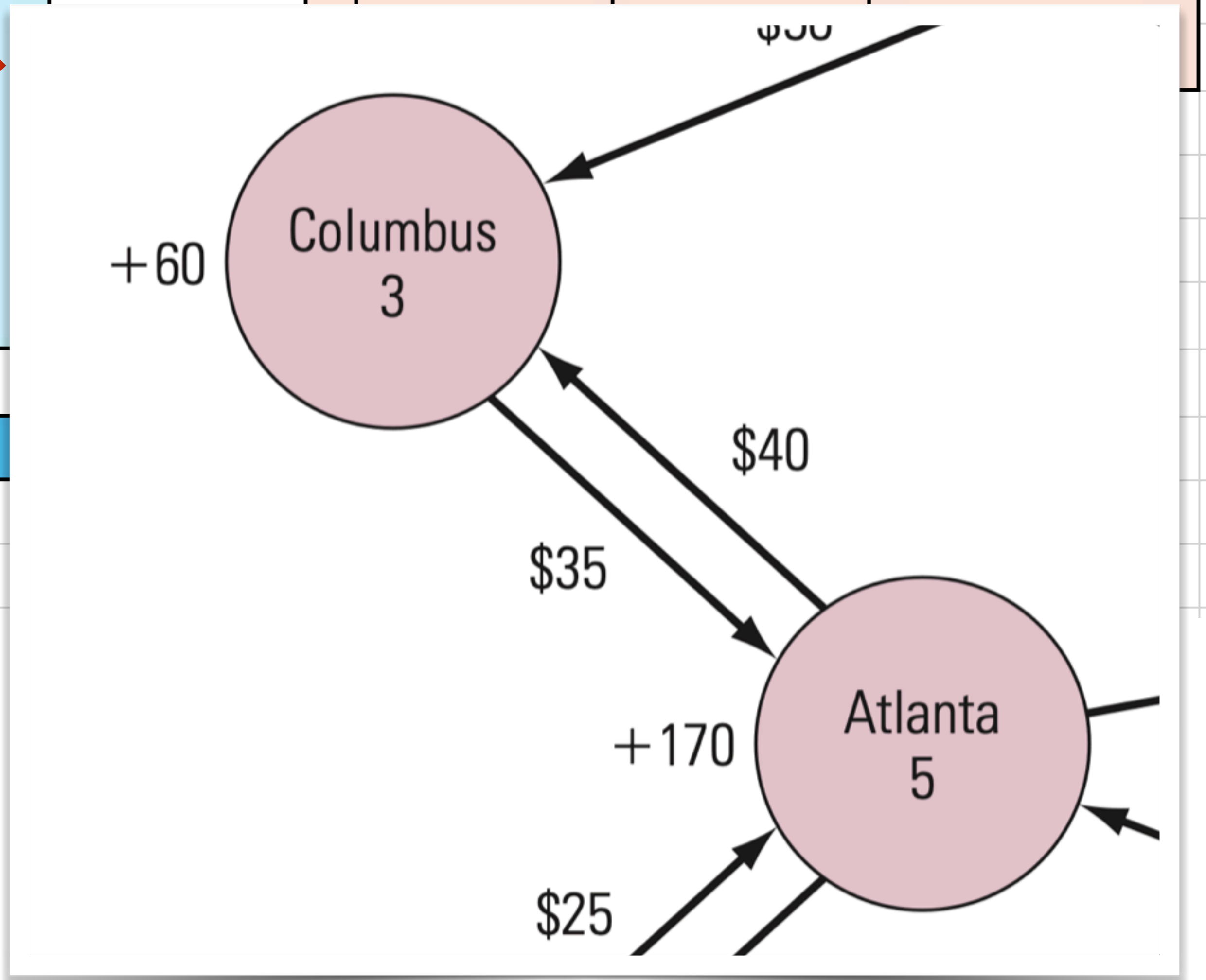
The car shipment problem

Ship	From		To		Unit cost
	1	Newark	2	Boston	$30.00
	1	Newark	4	Richmond	$40.00
	2	Boston	3	Columbus	$50.00
	3	Columbus	5	Atlanta	$35.00

Etc.. etc..

Total cost	

	Nodes	Net Flow	Supply/Demand



The car shipment problem

Ship	From		To		Unit cost
	1	Newark	2	Boston	$30.00
	1	Newark	4	Richmond	$40.00
	2	Boston	3	Columbus	$50.00
	3	Columbus	5	Atlanta	$35.00
	5	Atlanta	3	Columbus	$40.00
	5	Atlanta	4	Richmond	$30.00
	5	Atlanta	6	Mobile	$35.00
	6	Mobile	5	Atlanta	$25.00
	7	Jacksonville	4	Richmond	$50.00
	7	Jacksonville	5	Atlanta	$45.00
	7	Jacksonville	6	Mobile	$50.00

Total cost	

Nodes		Net Flow	Supply/Demand

=SUMPRODUCT(B4:B14,G4:G14)

The car shipment problem

Ship		From		To	Unit cost
	1	Newark	2	Boston	$30.00
	1	Newark	4	Richmond	$40.00
	2	Boston	3	Columbus	$50.00
	3	Columbus	5	Atlanta	$35.00
	5	Atlanta	3	Columbus	$40.00
	5	Atlanta	4	Richmond	$30.00
	5	Atlanta	6	Mobile	$35.00
	6	Mobile	5	Atlanta	$25.00
	7	Jacksonville	4	Richmond	$50.00
	7	Jacksonville	5	Atlanta	$45.00
	7	Jacksonville	6	Mobile	$50.00

Total cost	

	Nodes	Net Flow	Supply/Demand
1	Newark		-200
2	Boston		100
3	Columbus		60
4	Richmond		80
5	Atlanta		170
6	Mobile		70
7	Jacksonville		-300

Report here, for each node, the supply/demand as provided by the problem.

The car shipment problem

Ship	From		To		Unit cost
	1	Newark	2	Boston	$30.00
	1	Newark	4	Richmond	$40.00
	2	Boston	3	Columbus	$50.00
	3	Columbus	5	Atlanta	$35.00
	5	Atlanta	3	Columbus	$40.00
	5	Atlanta	4	Richmond	$30.00
	5	Atlanta	6	Mobile	$35.00
	6	Mobile	5	Atlanta	$25.00
	7	Jacksonville	4	Richmond	$50.00
	7	Jacksonville	5	Atlanta	$45.00
	7	Jacksonville	6	Mobile	$50.00

Total cost

Nodes		Net Flow	Supply/Demand
1	Newark		-200
2	Boston		100
3	Columbus		60
4	Richmond		80
5	Atlanta		170
6	Mobile		70
7	Jacksonville		-300

To solve the problem, we also have to compute, for each choice of the decision variables, the net flow in each node!

How to compute net flows and how to relate them to the constraints?

For Minimum Cost Network Flow Problems Where:	Apply This Balance-of-Flow Rule at Each Node:
Total Supply > Total Demand	Inflow − Outflow ≥ Supply or Demand
Total Supply = Total Demand	Inflow − Outflow = Supply or Demand
Total Supply < Total Demand	Inflow − Outflow ≤ Supply or Demand

How to compute net flows and how to relate them to the constraints?

For Minimum Cost Network Flow Problems Where:	Apply This Balance-of-Flow Rule at Each Node:
Total Supply > Total Demand	Inflow − Outflow ≥ Supply or Demand
Total Supply = Total Demand	Inflow − Outflow = Supply or Demand
Total Supply < Total Demand	Inflow − Outflow ≤ Supply or Demand

This is the present case. Supply = 500 cars. Demand = 480 cars.

How to compute net flows and how to relate them to the constraints?

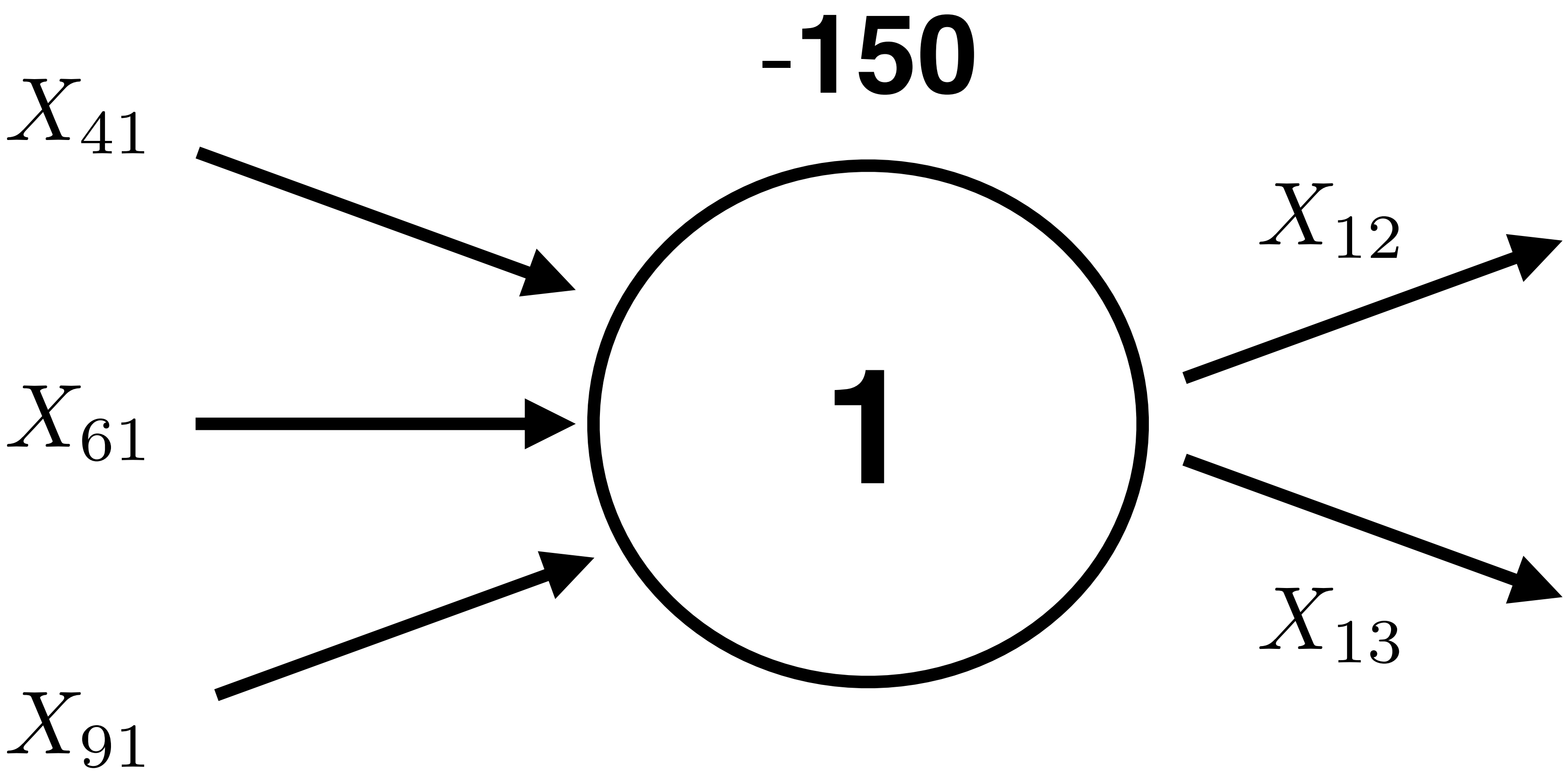
For Minimum Cost Network Flow Problems Where:	Apply This Balance-of-Flow Rule at Each Node:
Total Supply > Total Demand	Inflow − Outflow ≥ Supply or Demand
Total Supply = Total Demand	Inflow − Outflow = Supply or Demand
Total Supply < Total Demand	Inflow − Outflow ≤ Supply or Demand

In this case, you are not going to satisfy the demand, obviously!

You are going to determine the least costly way of distributing the available supply, knowing that it is impossible to satisfy all the demand.

How to compute net flows: an example.

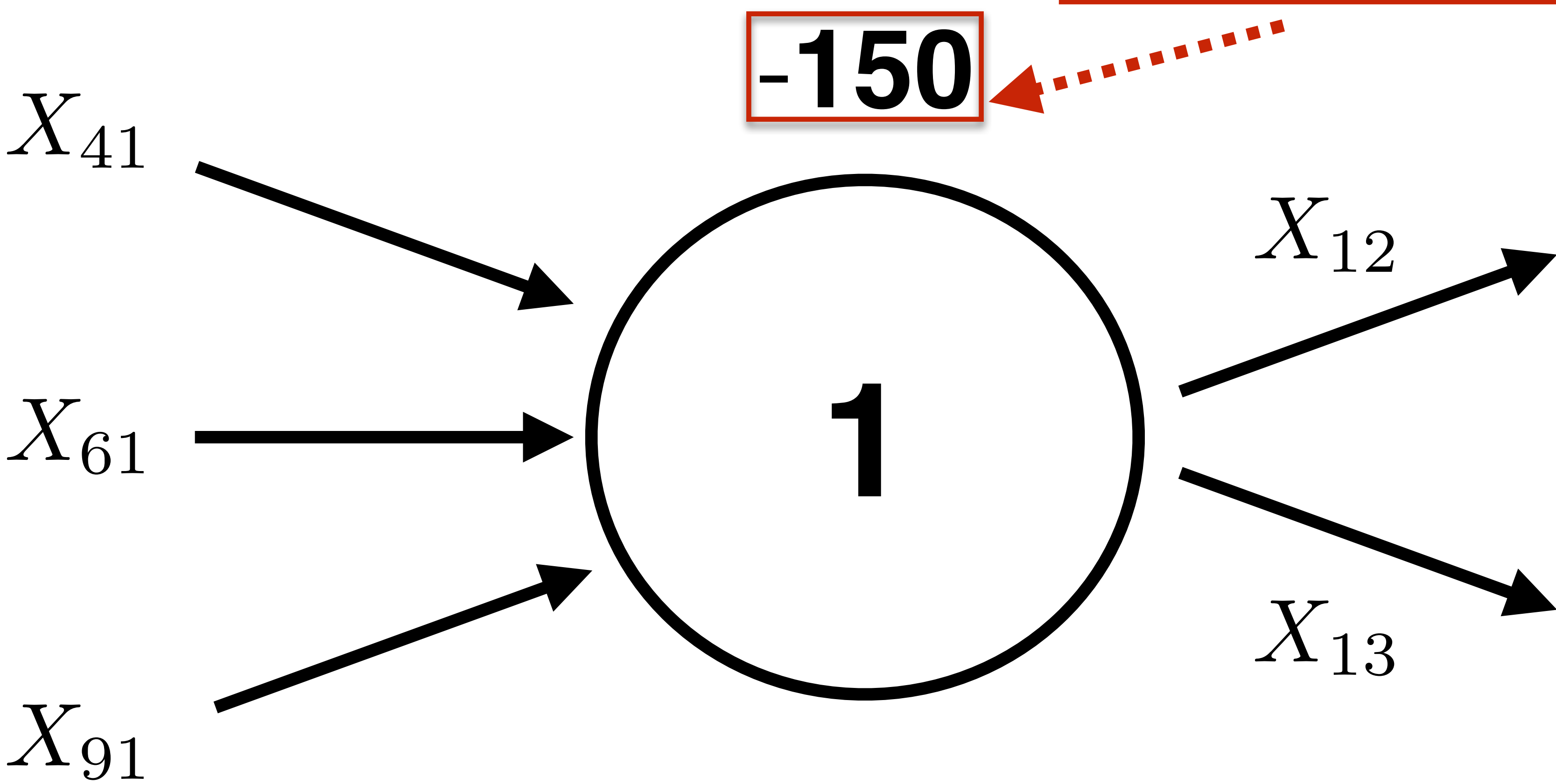
$$X_{ij} \doteq$$ the number of items shipped (or flowing) from node i to node j



How to compute net flows: an example.

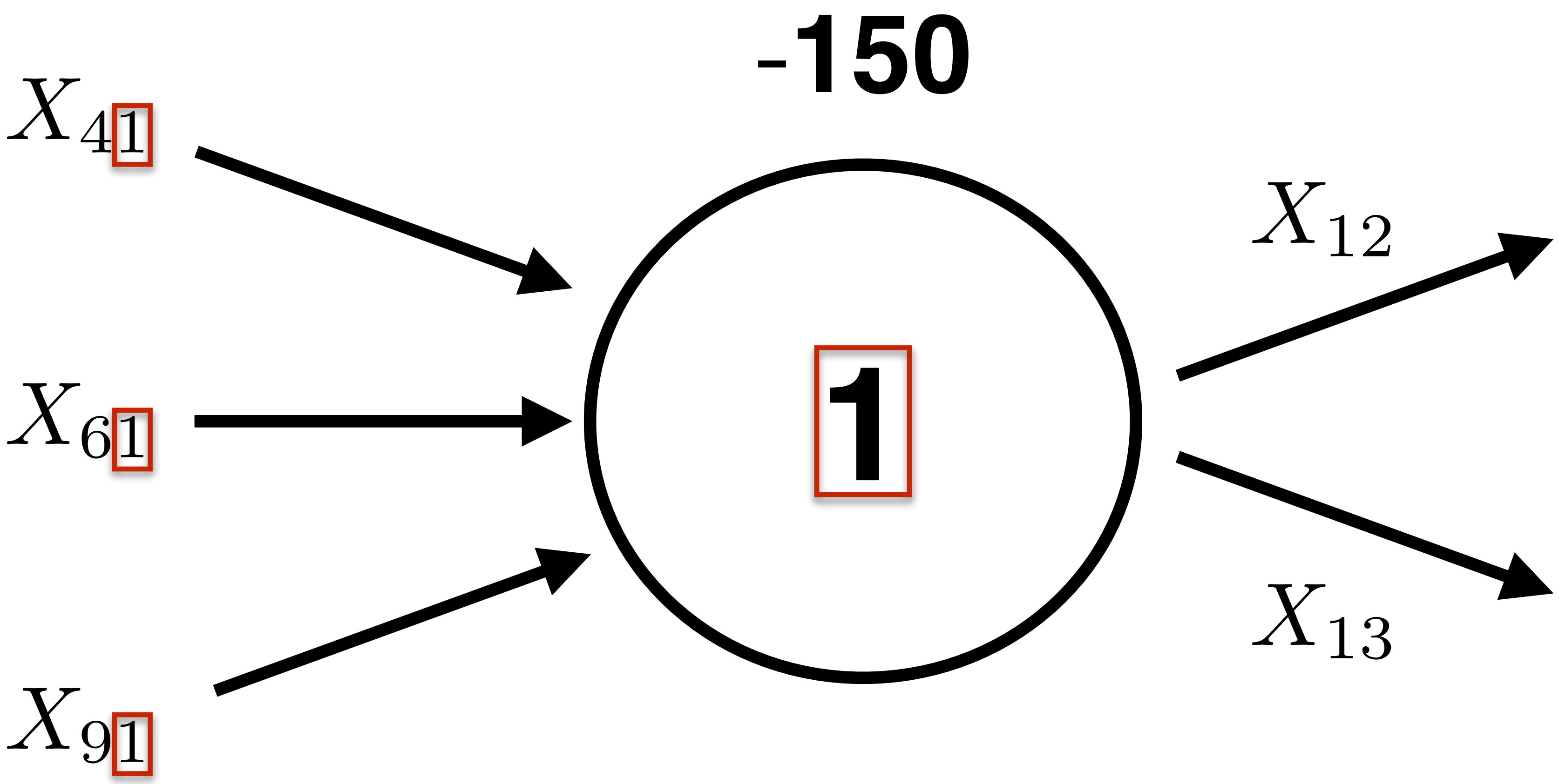
$X_{ij} \doteq$ the number of items shipped (or flowing) from node i to node j

The node number 1 has, in this example, a supply of -150 units.



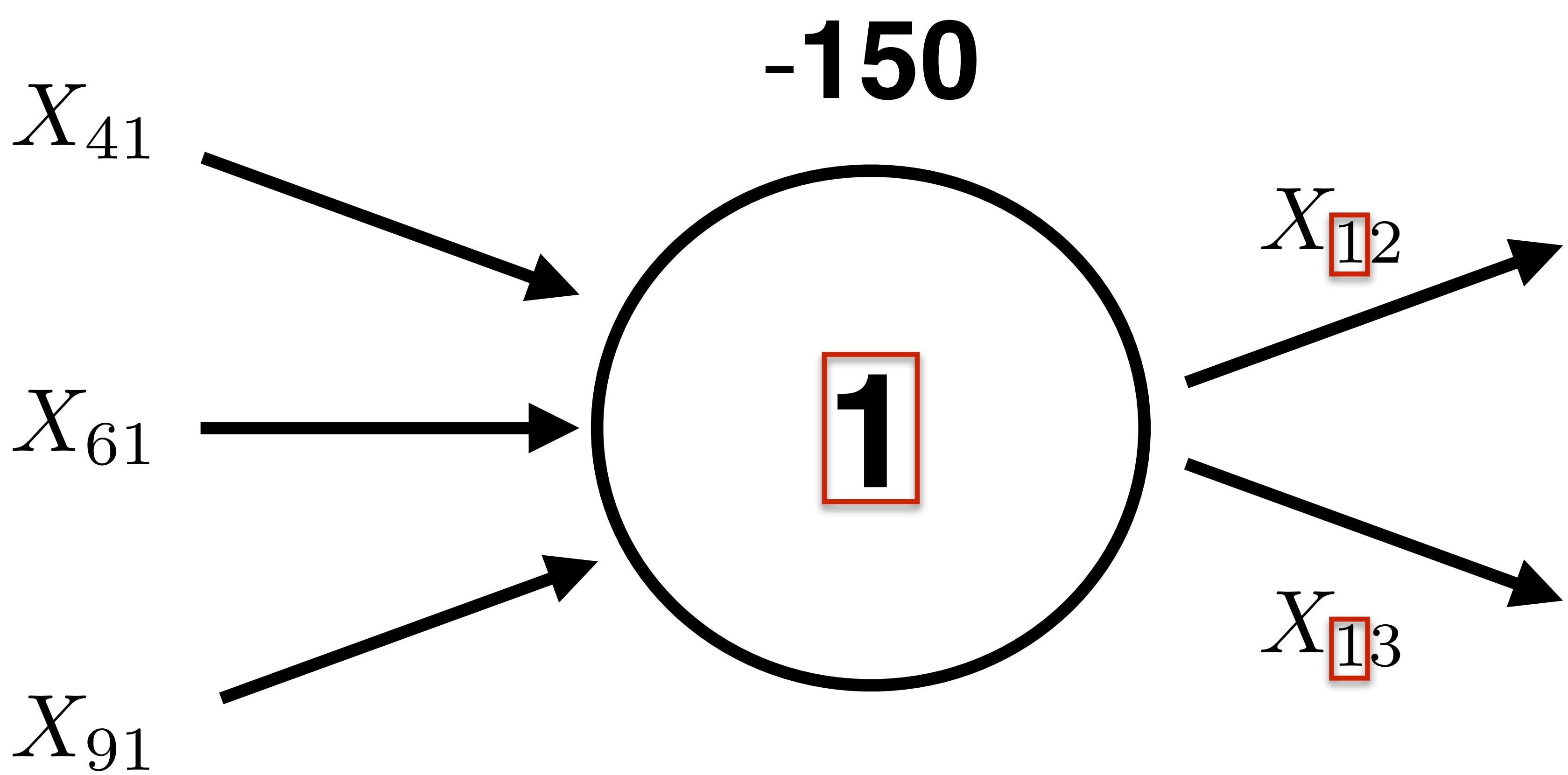
How to compute net flows: an example.

$$X_{ij} \doteq$$ the number of items shipped (or flowing) from node i to node j



How to compute net flows: an example.

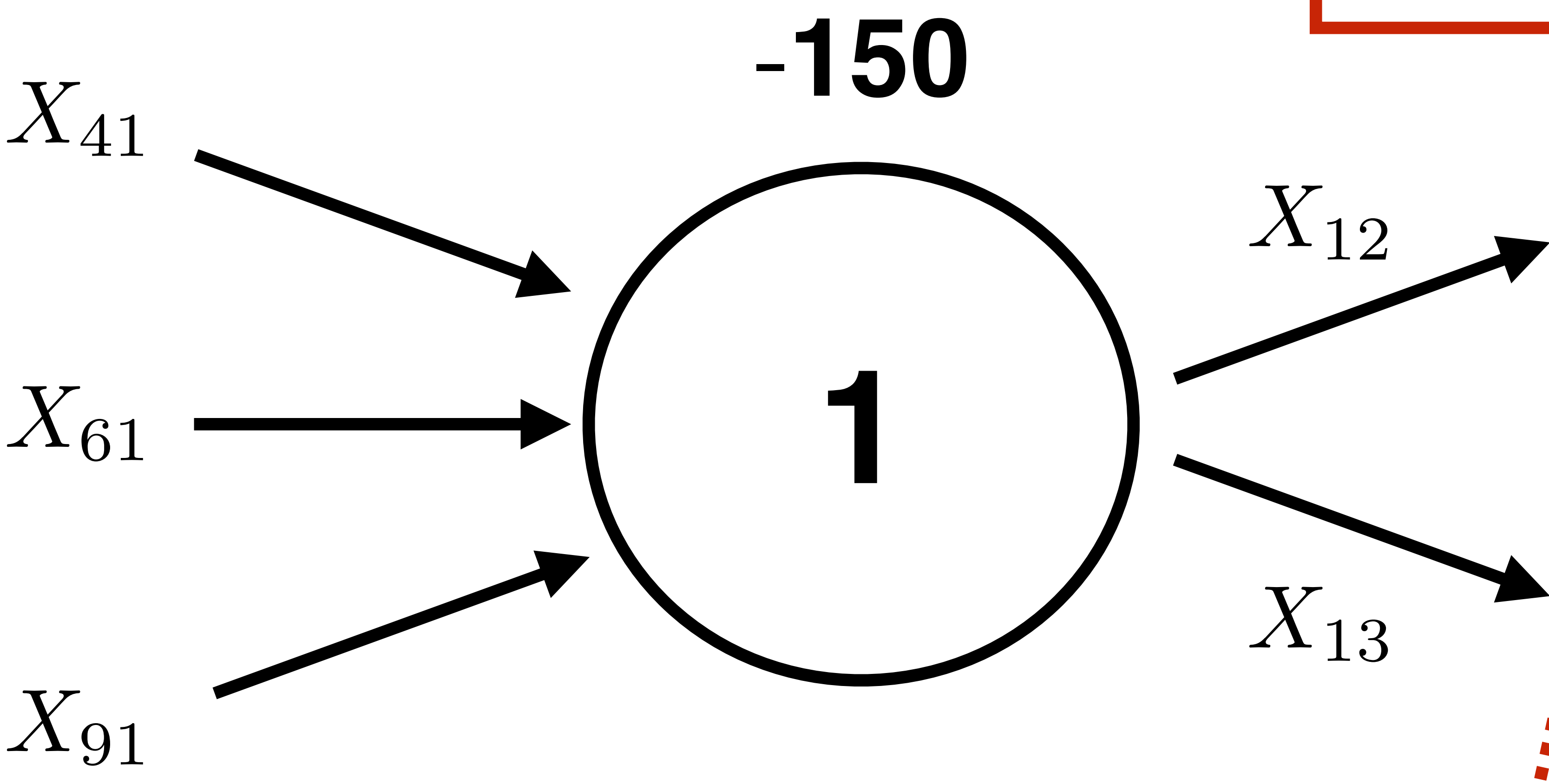
$$X_{ij} \doteq$$ the number of items shipped (or flowing) from node i to node j



How to compute net flows: an example.

$$X_{ij} \doteq$$ the number of items shipped (or flowing) from node i to node j

Depending on total demand vs. total supply.



$$X_{41} + X_{61} + X_{91} - X_{12} - X_{13} \lesseqgtr -150$$

The car shipment problem

Ship	From		To		Unit cost
	1	Newark	2	Boston	$30.00
	1	Newark	4	Richmond	$40.00
	2	Boston	3	Columbus	$50.00
	3	Columbus	5	Atlanta	$35.00
	5	Atlanta	3	Columbus	$40.00
	5	Atlanta	4	Richmond	$30.00
	5	Atlanta	6	Mobile	$35.00
	6	Mobile	5	Atlanta	$25.00
	7	Jacksonville	4	Richmond	$50.00
	7	Jacksonville	5	Atlanta	$45.00
	7	Jacksonville	6	Mobile	$50.00

Total cost	

Nodes		Net Flow	Supply/Demand
1	Newark		-200
2	Boston		100
3	Columbus		60
4	Richmond		80
5	Atlanta		170
6	Mobile		70
7	Jacksonville		-300

Let's start with cell K4.

We combine both relative and absolute cell reference.

The car shipment problem

Ship	From		To		Unit cost
	1	Newark	2	Boston	$30.00
	1	Newark	4	Richmond	$40.00
	2	Boston	3	Columbus	$50.00
	3	Columbus	5	Atlanta	$35.00
	5	Atlanta	3	Columbus	$40.00
	5	Atlanta	4	Richmond	$30.00
	5	Atlanta	6	Mobile	$35.00
	6	Mobile	5	Atlanta	$25.00
	7	Jacksonville	4	Richmond	$50.00
	7	Jacksonville	5	Atlanta	$45.00
	7	Jacksonville	6	Mobile	$50.00

Total cost	

Nodes		Net Flow	Supply/Demand
1	Newark		-200
2	Boston		100
3	Columbus		60
4	Richmond		80
5	Atlanta		170
6	Mobile		70
7	Jacksonville		-300

=SUMIF($E$4:$E$14,I4,$B$4:$B$14)-SUMIF($C$4:$C$14,I4,$B$4:$B$14)

Run from the cell E4 to E14 (absolute reference).

The car shipment problem

Ship	From		To		Unit cost
	1	Newark	2	Boston	$30.00
	1	Newark	4	Richmond	$40.00
	2	Boston	3	Columbus	$50.00
	3	Columbus	5	Atlanta	$35.00
	5	Atlanta	3	Columbus	$40.00
	5	Atlanta	4	Richmond	$30.00
	5	Atlanta	6	Mobile	$35.00
	6	Mobile	5	Atlanta	$25.00
	7	Jacksonville	4	Richmond	$50.00
	7	Jacksonville	5	Atlanta	$45.00
	7	Jacksonville	6	Mobile	$50.00

Total cost	

Nodes		Net Flow	Supply/Demand
1	Newark		-200
2	Boston		100
3	Columbus		60
4	Richmond		80
5	Atlanta		170
6	Mobile		70
7	Jacksonville		-300

=SUMIF($E$4:$E$14,I4,$B$4:$B$14)-SUMIF($C$4:$C$14,I4,$B$4:$B$14)

If a match with the cell I4 (relative reference) is found …

The car shipment problem

Ship	From		To		Unit cost
	1	Newark	2	Boston	$30.00
	1	Newark	4	Richmond	$40.00
	2	Boston	3	Columbus	$50.00
	3	Columbus	5	Atlanta	$35.00
	5	Atlanta	3	Columbus	$40.00
	5	Atlanta	4	Richmond	$30.00
	5	Atlanta	6	Mobile	$35.00
	6	Mobile	5	Atlanta	$25.00
	7	Jacksonville	4	Richmond	$50.00
	7	Jacksonville	5	Atlanta	$45.00
	7	Jacksonville	6	Mobile	$50.00

Total cost	

Nodes		Net Flow	Supply/Demand
1	Newark		-200
2	Boston		100
3	Columbus		60
4	Richmond		80
5	Atlanta		170
6	Mobile		70
7	Jacksonville		-300

=SUMIF($E$4:$E$14,I4,$B$4:$B$14)-SUMIF($C$4:$C$14,I4,$B$4:$B$14)

… sum up all the corresponding cells from B4 to B14 (absolute reference).

The car shipment problem

Ship	From		To		Unit cost
	1	Newark	2	Boston	$30.00
	1	Newark	4	Richmond	$40.00
	2	Boston	3	Columbus	$50.00
	3	Columbus	5	Atlanta	$35.00
	5	Atlanta	3	Columbus	$40.00
	5	Atlanta	4	Richmond	$30.00
	5	Atlanta	6	Mobile	$35.00
	6	Mobile	5	Atlanta	$25.00
	7	Jacksonville	4	Richmond	$50.00
	7	Jacksonville	5	Atlanta	$45.00
	7	Jacksonville	6	Mobile	$50.00

Nodes		Net Flow	Supply/Demand
1	Newark		-200
2	Boston		100
3	Columbus		60
4	Richmond		80
5	Atlanta		170
6	Mobile		70
7	Jacksonville		-300

Total cost

=SUMIF($E$4:$E$14,I4,$B$4:$B$14)-SUMIF($C$4:$C$14,I4,$B$4:$B$14)

The car shipment problem

Ship	From		To		Unit cost
	1	Newark	2	Boston	$30.00
	1	Newark	4	Richmond	$40.00
	2	Boston	3	Columbus	$50.00
	3	Columbus	5	Atlanta	$35.00
	5	Atlanta	3	Columbus	$40.00
	5	Atlanta	4	Richmond	$30.00
	5	Atlanta	6	Mobile	$35.00
	6	Mobile	5	Atlanta	$25.00
	7	Jacksonville	4	Richmond	$50.00
	7	Jacksonville	5	Atlanta	$45.00
	7	Jacksonville	6	Mobile	$50.00

Total cost	

Nodes		Net Flow	Supply/Demand
1	Newark		-200
2	Boston		100
3	Columbus		60
4	Richmond		80
5	Atlanta		170
6	Mobile		70
7	Jacksonville		-300

=SUMIF($E$4:$E$14,I4,$B$4:$B$14)-SUMIF($C$4:$C$14,I4,$B$4:$B$14)

This is just the sum of the INFLOWS.

The car shipment problem

Ship	From		To		Unit cost
	1	Newark	2	Boston	$30.00
	1	Newark	4	Richmond	$40.00
	2	Boston	3	Columbus	$50.00
	3	Columbus	5	Atlanta	$35.00
	5	Atlanta	3	Columbus	$40.00
	5	Atlanta	4	Richmond	$30.00
	5	Atlanta	6	Mobile	$35.00
	6	Mobile	5	Atlanta	$25.00
	7	Jacksonville	4	Richmond	$50.00
	7	Jacksonville	5	Atlanta	$45.00
	7	Jacksonville	6	Mobile	$50.00

Total cost	

Nodes		Net Flow	Supply/Demand
1	Newark		-200
2	Boston		100
3	Columbus		60
4	Richmond		80
5	Atlanta		170
6	Mobile		70
7	Jacksonville		-300

=SUMIF($E$4:$E$14,I4,$B$4:$B$14)-SUMIF($C$4:$C$14,I4,$B$4:$B$14)

For the same reason, this is the sum of the outflows.

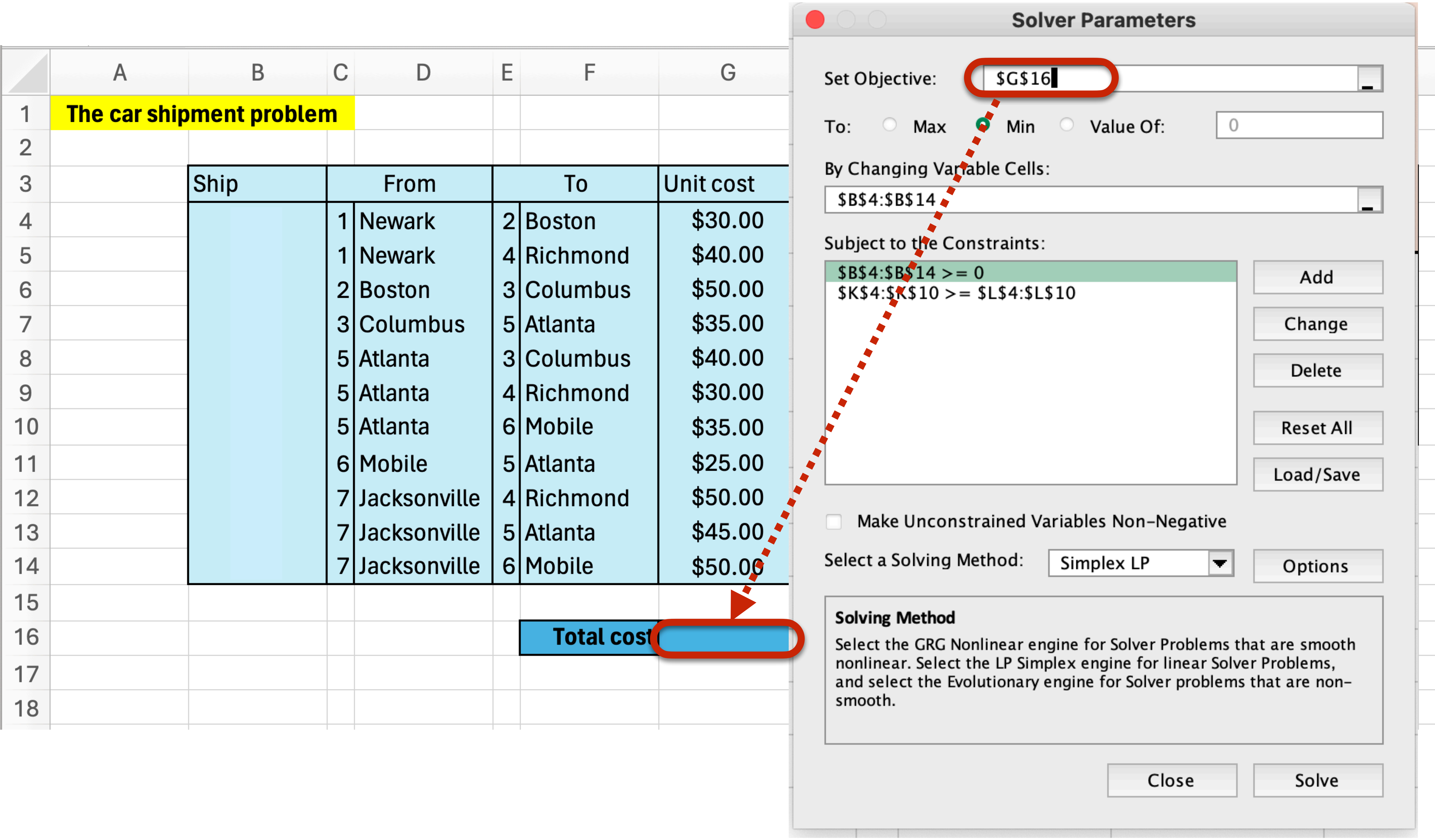
The car shipment problem

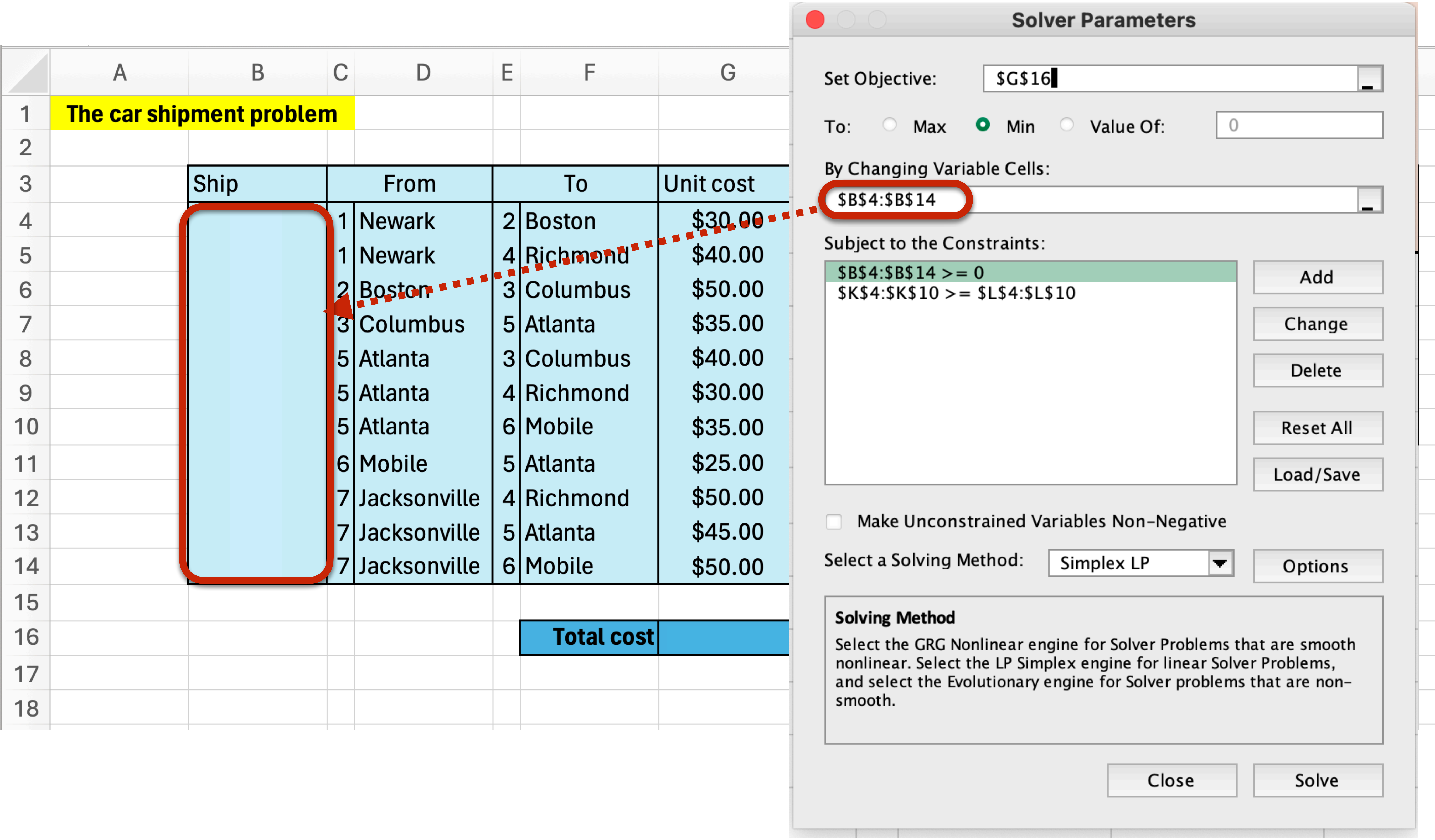
Ship	From		To		Unit cost
	1	Newark	2	Boston	$30.00
	1	Newark	4	Richmond	$40.00
	2	Boston	3	Columbus	$50.00
	3	Columbus	5	Atlanta	$35.00
	5	Atlanta	3	Columbus	$40.00
	5	Atlanta	4	Richmond	$30.00
	5	Atlanta	6	Mobile	$35.00
	6	Mobile	5	Atlanta	$25.00
	7	Jacksonville	4	Richmond	$50.00
	7	Jacksonville	5	Atlanta	$45.00
	7	Jacksonville	6	Mobile	$50.00

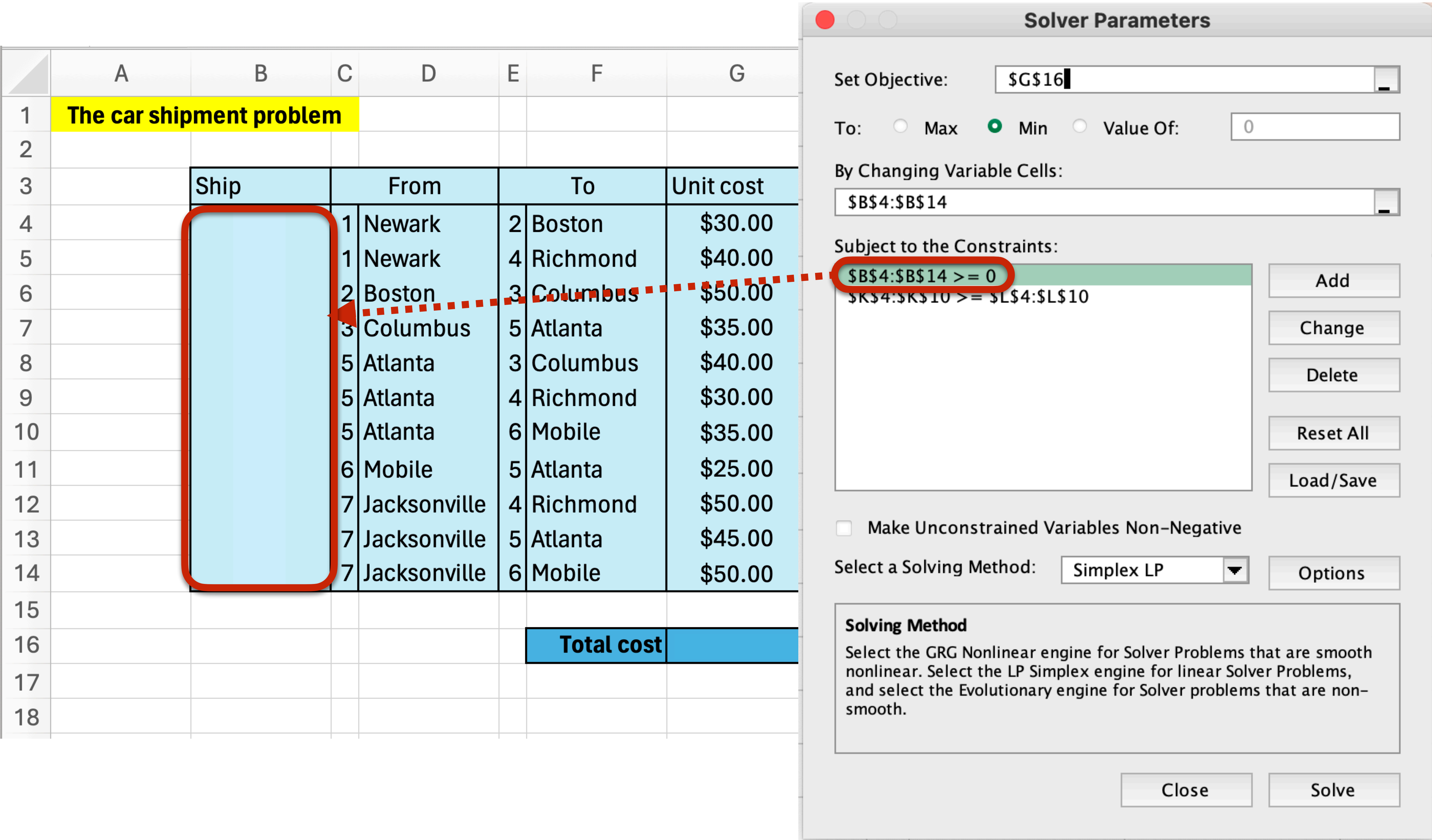
Total cost	

Nodes		Net Flow	Supply/Demand
1	Newark		-200
2	Boston		100
3	Columbus		60
4	Richmond		80
5	Atlanta		170
6	Mobile		70
7	Jacksonville		-300

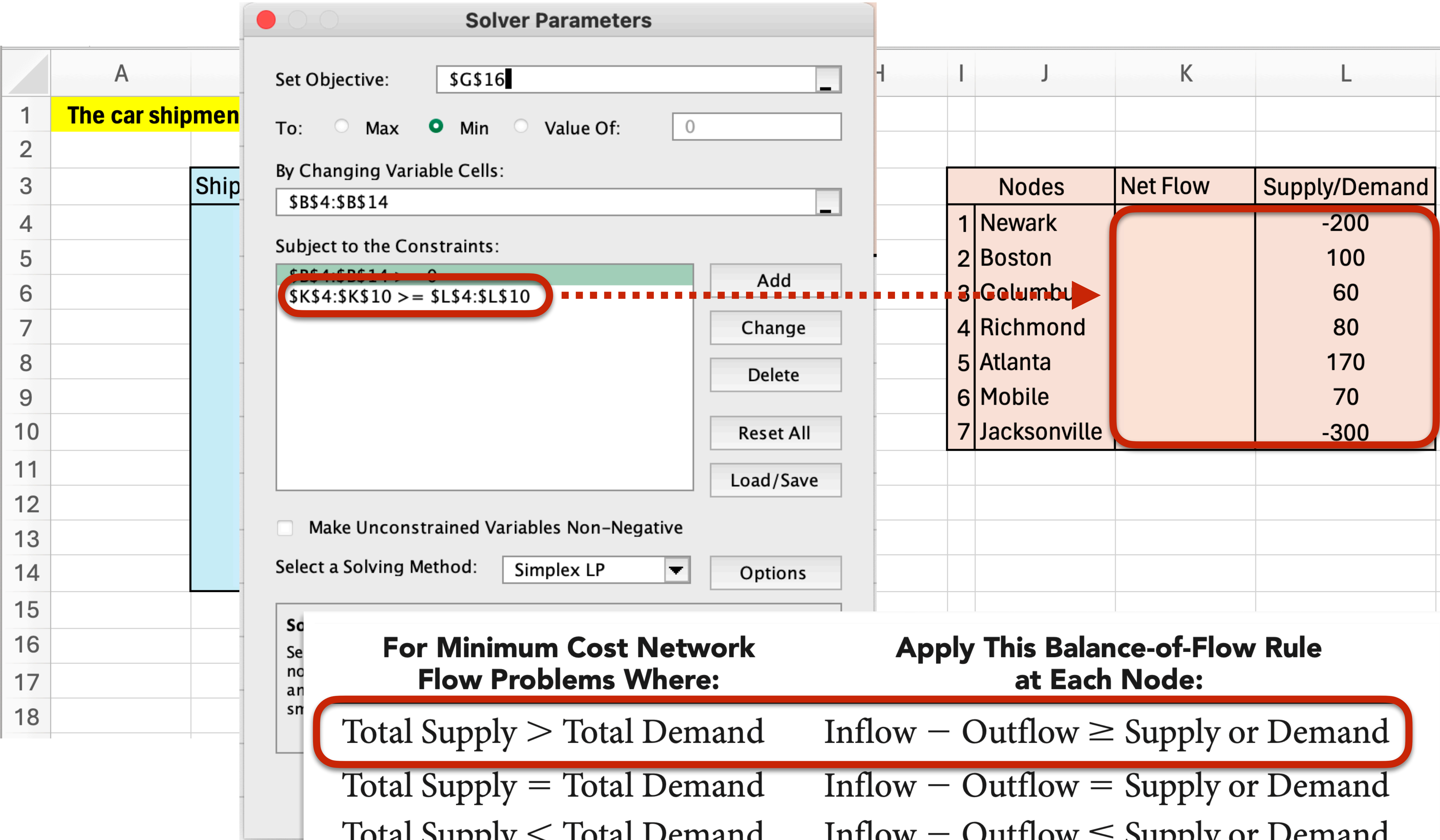
Hand-fill! Thanks to relative and absolute reference we can safely use the hand-fill feature (and save time!)


The car shipment problem
Ship
From
To
Unit cost
1 Newark
2 Boston
$30.00
1 Newark
4 Richmond
$40.00
2 Boston
3 Columbus
$50.00
3 Columbus
5 Atlanta
$35.00
5 Atlanta
3 Columbus
$40.00
5 Atlanta
4 Richmond
$30.00
5 Atlanta
6 Mobile
$35.00
6 Mobile
5 Atlanta
$25.00
7 Jacksonville
4 Richmond
$50.00
7 Jacksonville
5 Atlanta
$45.00
7 Jacksonville
6 Mobile
$50.00
Total cost
Solver Parameters
Set Objective:
$G$16
To:
Max
Min
Value Of:
0
By Changing Variable Cells:
$B$4:$B$14
Subject to the Constraints:
$B$4:$B$14 >= 0
$K$4:$K$10 >= $L$4:$L$10
Add
Change
Delete
Reset All
Load/Save
Make Unconstrained Variables Non-Negative
Select a Solving Method:
Simplex LP
Options
Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.
Close
Solve

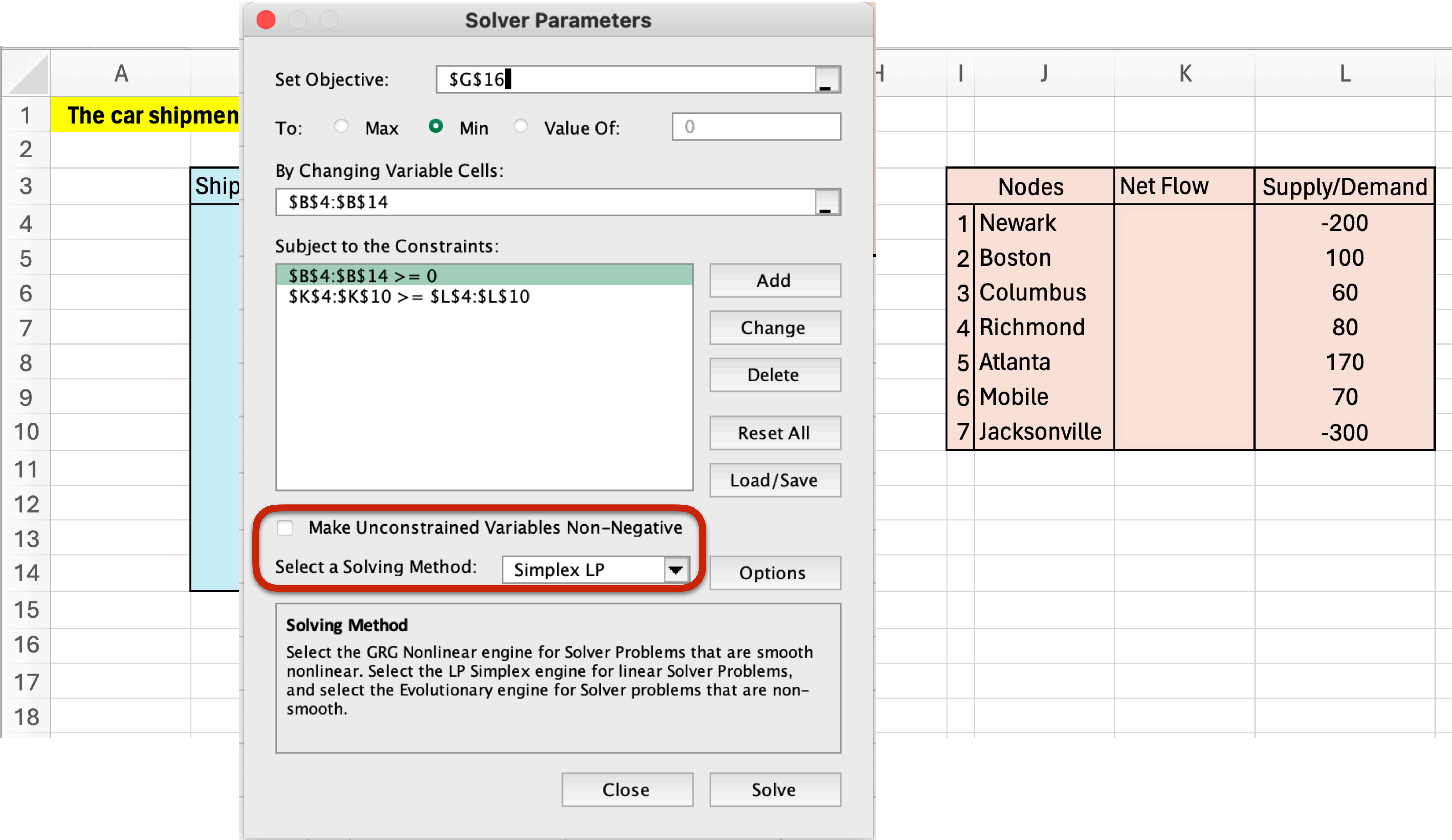

The car shipment problem
Ship
From
To
Unit cost
1 Newark 2 Boston $30.00
1 Newark 4 Richmond $40.00
2 Boston 3 Columbus $50.00
3 Columbus 5 Atlanta $35.00
5 Atlanta 3 Columbus $40.00
5 Atlanta 4 Richmond $30.00
5 Atlanta 6 Mobile $35.00
6 Mobile 5 Atlanta $25.00
7 Jacksonville 4 Richmond $50.00
7 Jacksonville 5 Atlanta $45.00
7 Jacksonville 6 Mobile $50.00
Total cost
Solver Parameters
Set Objective: $G$16
To: Max Min Value Of: 0
By Changing Variable Cells:
$B$4:$B$14
Subject to the Constraints:
$B$4:$B$14 >= 0
$K$4:$K$10 >= $L$4:$L$10
Add
Change
Delete
Reset All
Load/Save
Make Unconstrained Variables Non-Negative
Select a Solving Method: Simplex LP
Options
Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.
Close
Solve



The car shipment problem
Ship
From
To
Unit cost
1 Newark
2 Boston
$30.00
1 Newark
4 Richmond
$40.00
2 Boston
3 Columbus
$50.00
3 Columbus
5 Atlanta
$35.00
5 Atlanta
3 Columbus
$40.00
5 Atlanta
4 Richmond
$30.00
5 Atlanta
6 Mobile
$35.00
6 Mobile
5 Atlanta
$25.00
7 Jacksonville
4 Richmond
$50.00
7 Jacksonville
5 Atlanta
$45.00
7 Jacksonville
6 Mobile
$50.00
Total cost
Solver Parameters
Set Objective: $G$16
To: Max Min Value Of: 0
By Changing Variable Cells:
$B$4:$B$14
Subject to the Constraints:
$B$4:$B$14 >= 0
$K$4:$K$10 >= $L$4:$L$10
Add
Change
Delete
Reset All
Load/Save
Make Unconstrained Variables Non-Negative
Select a Solving Method: Simplex LP
Options
Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.
Close
Solve



For Minimum Cost Network Flow Problems Where:	Apply This Balance-of-Flow Rule at Each Node:
Total Supply > Total Demand	Inflow − Outflow ≥ Supply or Demand
Total Supply = Total Demand	Inflow − Outflow = Supply or Demand
Total Supply < Total Demand	Inflow − Outflow ≤ Supply or Demand


The car shipmen
Ship
Solver Parameters
Set Objective: $G$16
To: Max Min Value Of: 0
By Changing Variable Cells:
$B$4:$B$14
Subject to the Constraints:
$B$4:$B$14 >= 0
$K$4:$K$10 >= $L$4:$L$10
Add
Change
Delete
Reset All
Load/Save
Make Unconstrained Variables Non-Negative
Select a Solving Method: Simplex LP
Options
Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.
Close
Solve
Nodes
Net Flow
Supply/Demand
1 Newark -200
2 Boston 100
3 Columbus 60
4 Richmond 80
5 Atlanta 170
6 Mobile 70
7 Jacksonville -300

The optimal solution.

The car shipment problem

Ship	From		To		Unit cost
120	1	Newark	2	Boston	$30.00
80	1	Newark	4	Richmond	$40.00
20	2	Boston	3	Columbus	$50.00
0	3	Columbus	5	Atlanta	$35.00
40	5	Atlanta	3	Columbus	$40.00
0	5	Atlanta	4	Richmond	$30.00
0	5	Atlanta	6	Mobile	$35.00
0	6	Mobile	5	Atlanta	$25.00
0	7	Jacksonville	4	Richmond	$50.00
210	7	Jacksonville	5	Atlanta	$45.00
70	7	Jacksonville	6	Mobile	$50.00
				Total cost	$22,350.00

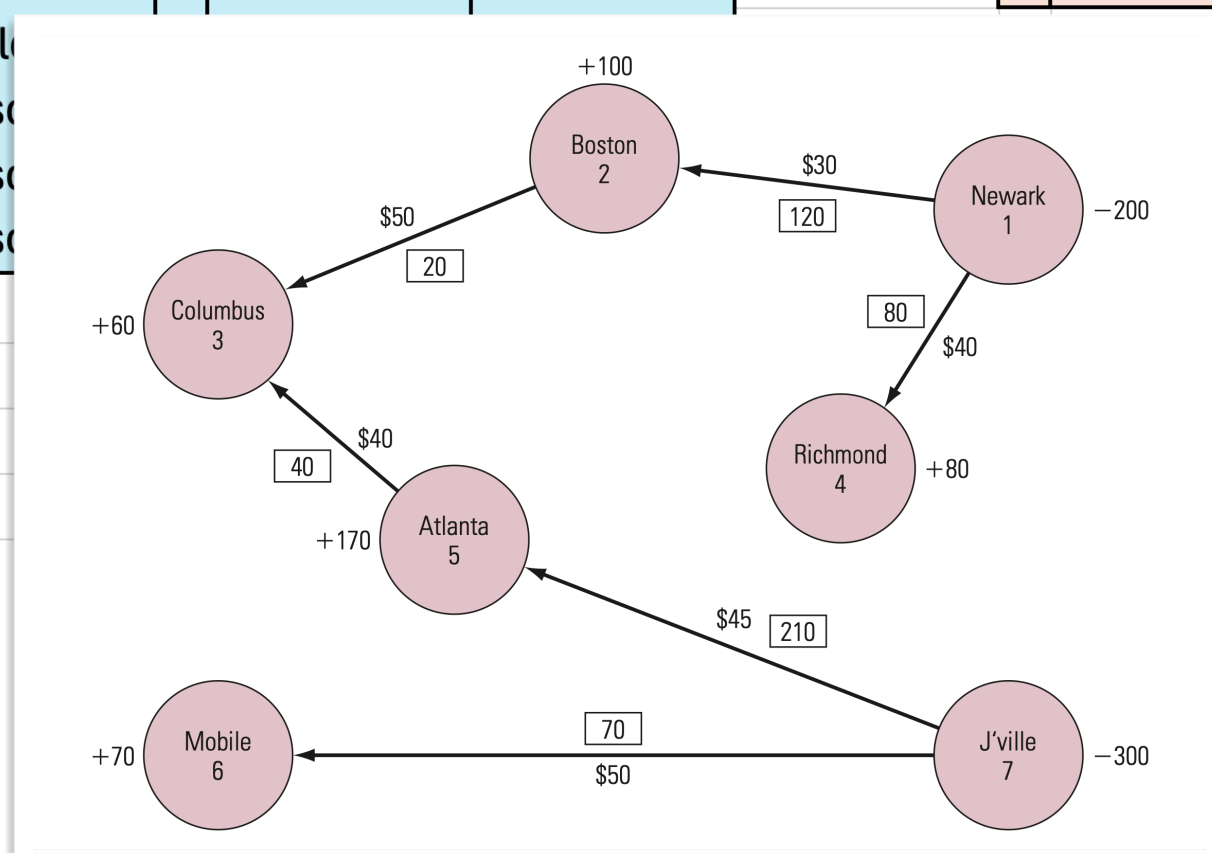
Nodes		Net Flow	Supply/Demand
1	Newark	-200	-200
2	Boston	100	100
3	Columbus	60	60
4	Richmond	80	80
5	Atlanta	170	170
6	Mobile	70	70
7	Jacksonville	-280	-300

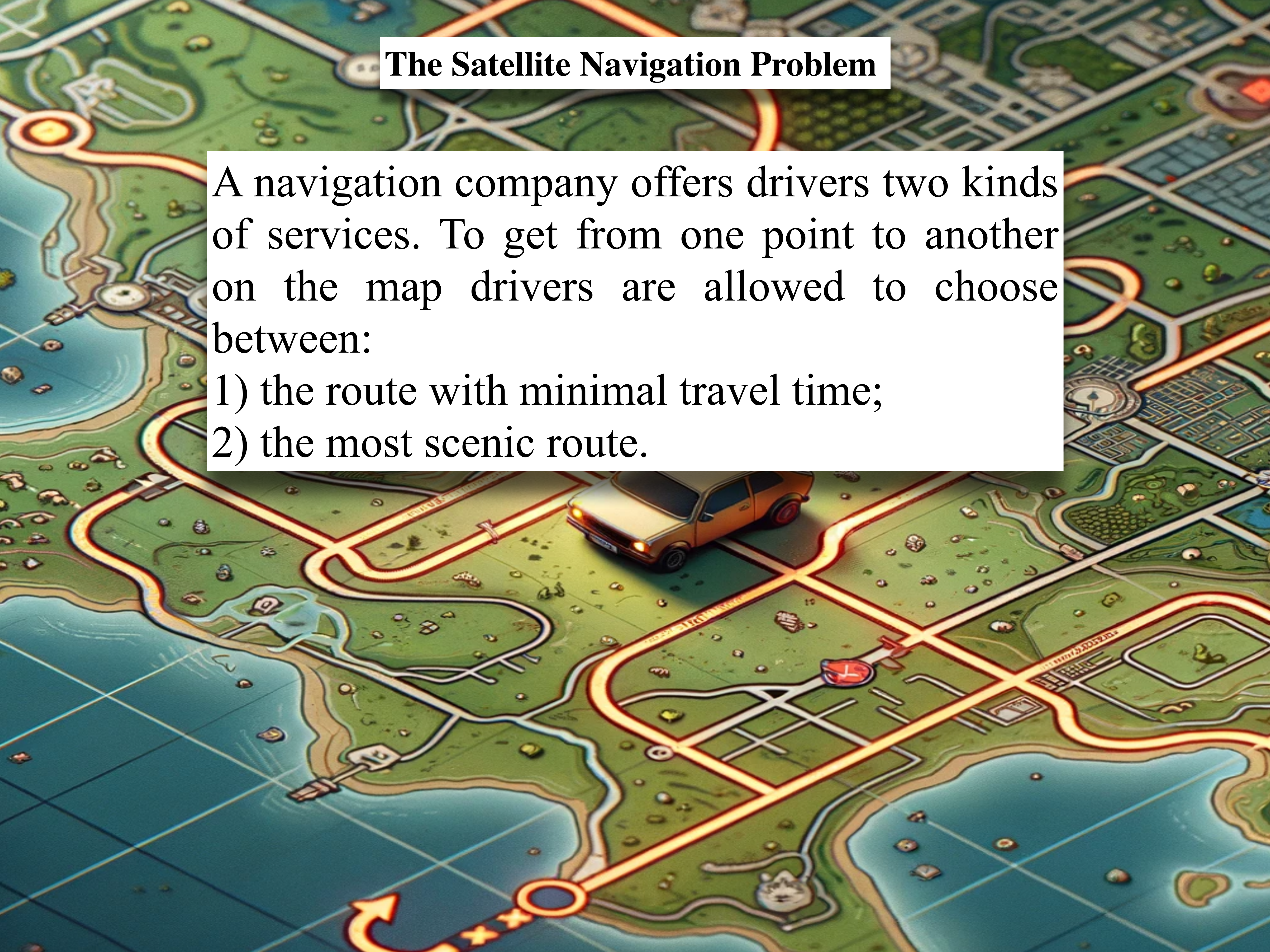
The optimal solution.

The car shipment problem

Ship	From		To		Unit cost
120	1	Newark	2	Boston	$30.00
80	1	Newark	4	Richmond	$40.00
20	2	Boston	3	Columbus	$50.00
0	3	Columbus	5	Atlanta	$35.00
40	5	Atlanta	3	Columbus	$40.00
0	5	Atlanta	4	Richmond	$30.00
0	5	Atlanta	6	Mobile	$35.00
0	6	Mobil			
0	7	Jacks			
210	7	Jacks			
70	7	Jacks			

Nodes		Net Flow	Supply/Demand
1	Newark	-200	-200
2	Boston	100	100
3	Columbus	60	60
4	Richmond	80	80
5	Atlanta	170	170
6	Mobile	70	70
7	Jacksonville	-280	-300

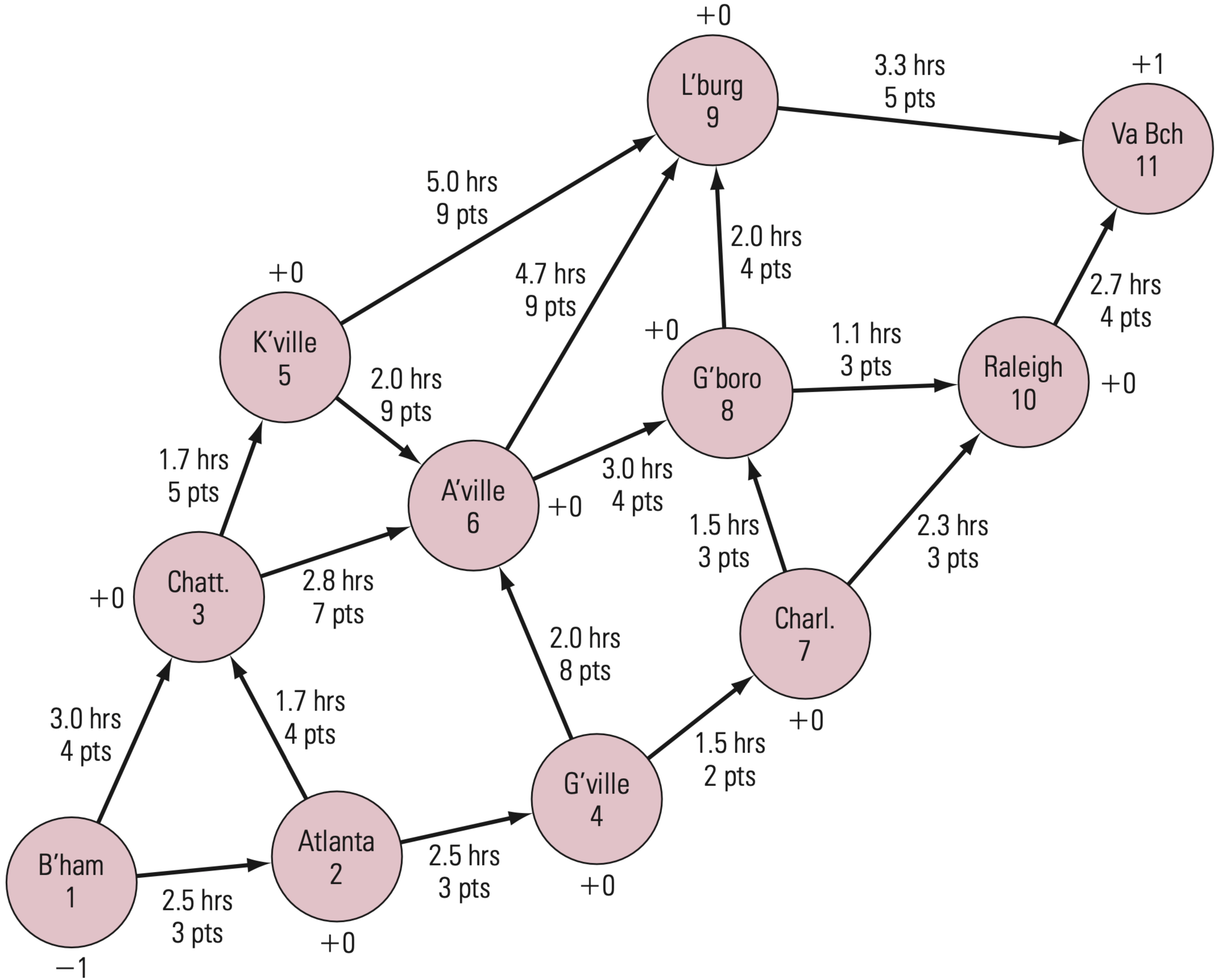




The Satellite Navigation Problem

A navigation company offers drivers two kinds of services. To get from one point to another on the map drivers are allowed to choose between:

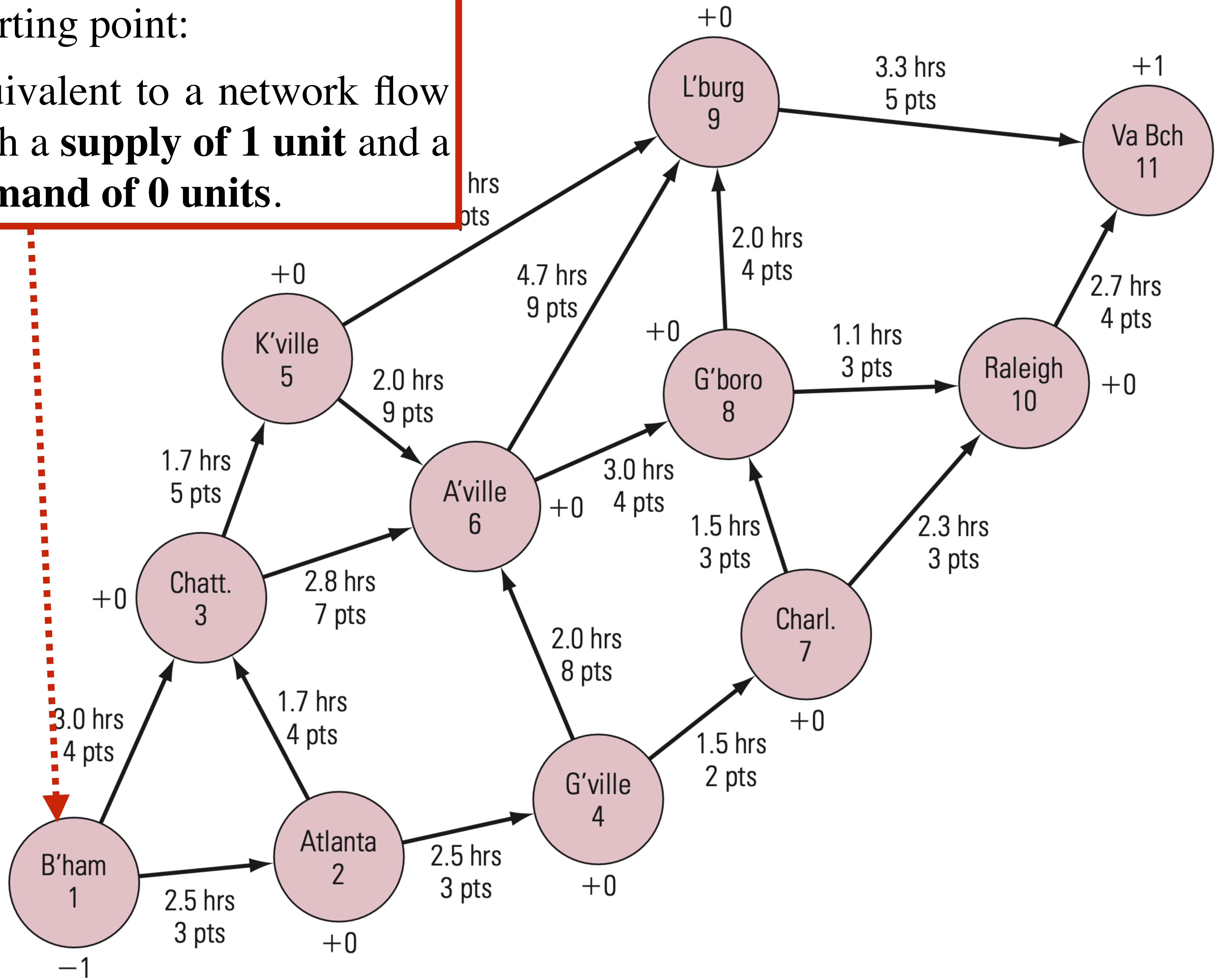
1) the route with minimal travel time;
2) the most scenic route.



L'burg
9
+0
Va Bch
11
+1
3.3 hrs
5 pts
K'ville
5
+0
5.0 hrs
9 pts
4.7 hrs
9 pts
2.0 hrs
4 pts
G'boro
8
+0
1.1 hrs
3 pts
Raleigh
10
+0
2.7 hrs
4 pts
2.0 hrs
9 pts
A'ville
6
+0
3.0 hrs
4 pts
1.7 hrs
5 pts
Chatt.
3
+0
2.8 hrs
7 pts
1.5 hrs
3 pts
2.3 hrs
3 pts
Charl.
7
+0
2.0 hrs
8 pts
3.0 hrs
4 pts
1.7 hrs
4 pts
G'ville
4
+0
1.5 hrs
2 pts
B'ham
1
−1
2.5 hrs
3 pts
Atlanta
2
+0
2.5 hrs
3 pts

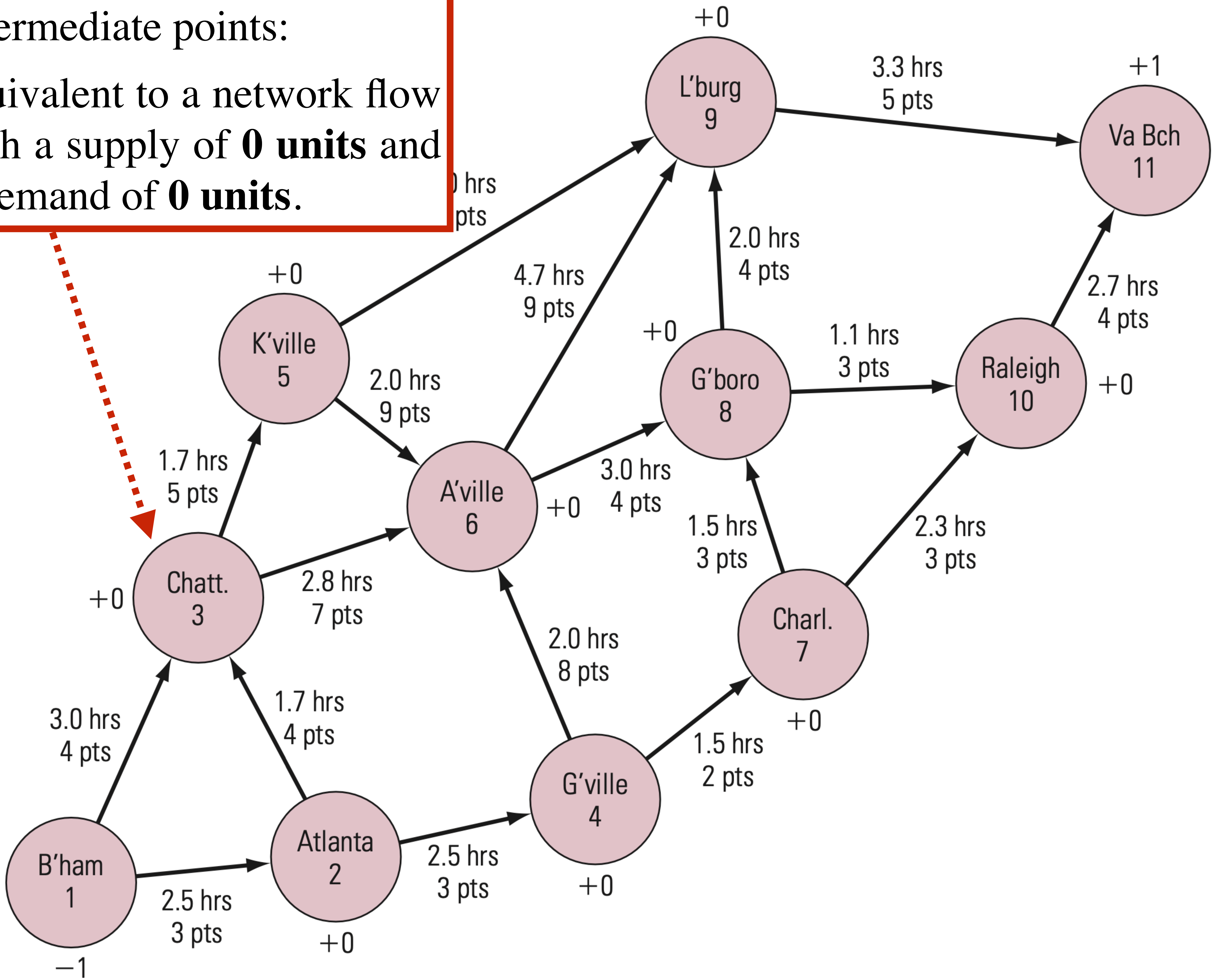
Starting point:

equivalent to a network flow with a **supply of 1 unit** and a **demand of 0 units**.



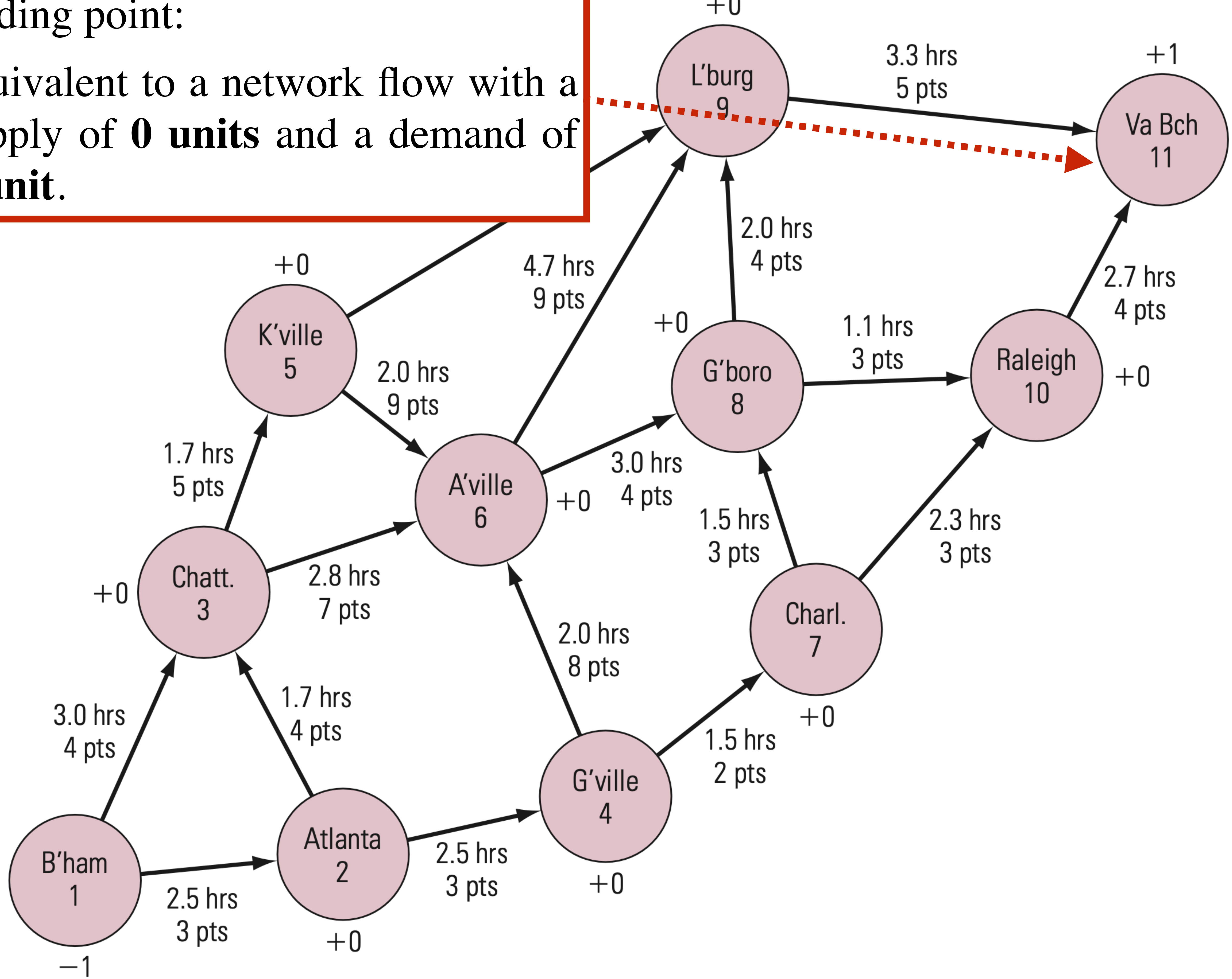
Intermediate points:

equivalent to a network flow with a supply of **0 units** and a demand of **0 units**.



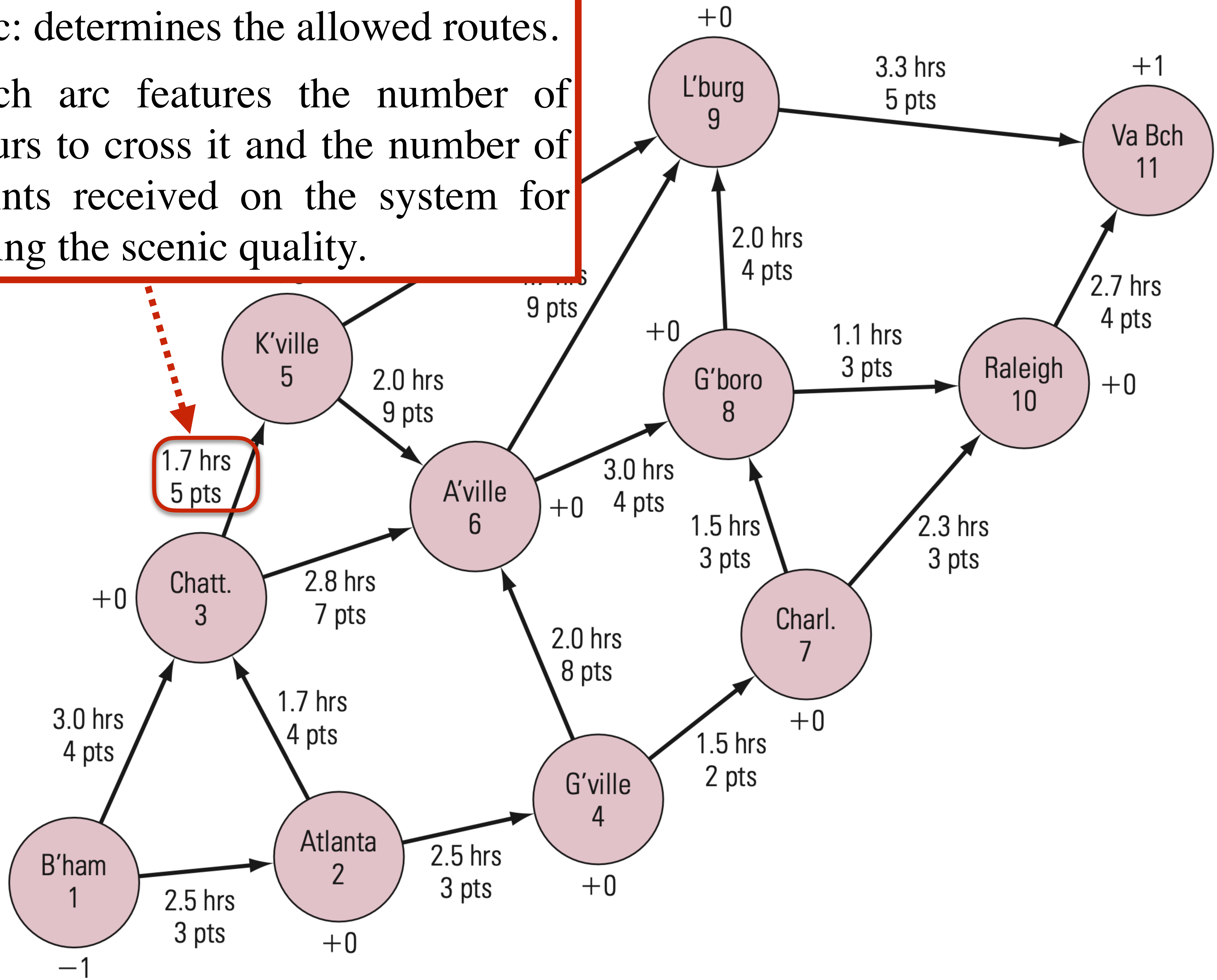
Ending point:

equivalent to a network flow with a supply of **0 units** and a demand of **1 unit**.



Arc: determines the allowed routes.

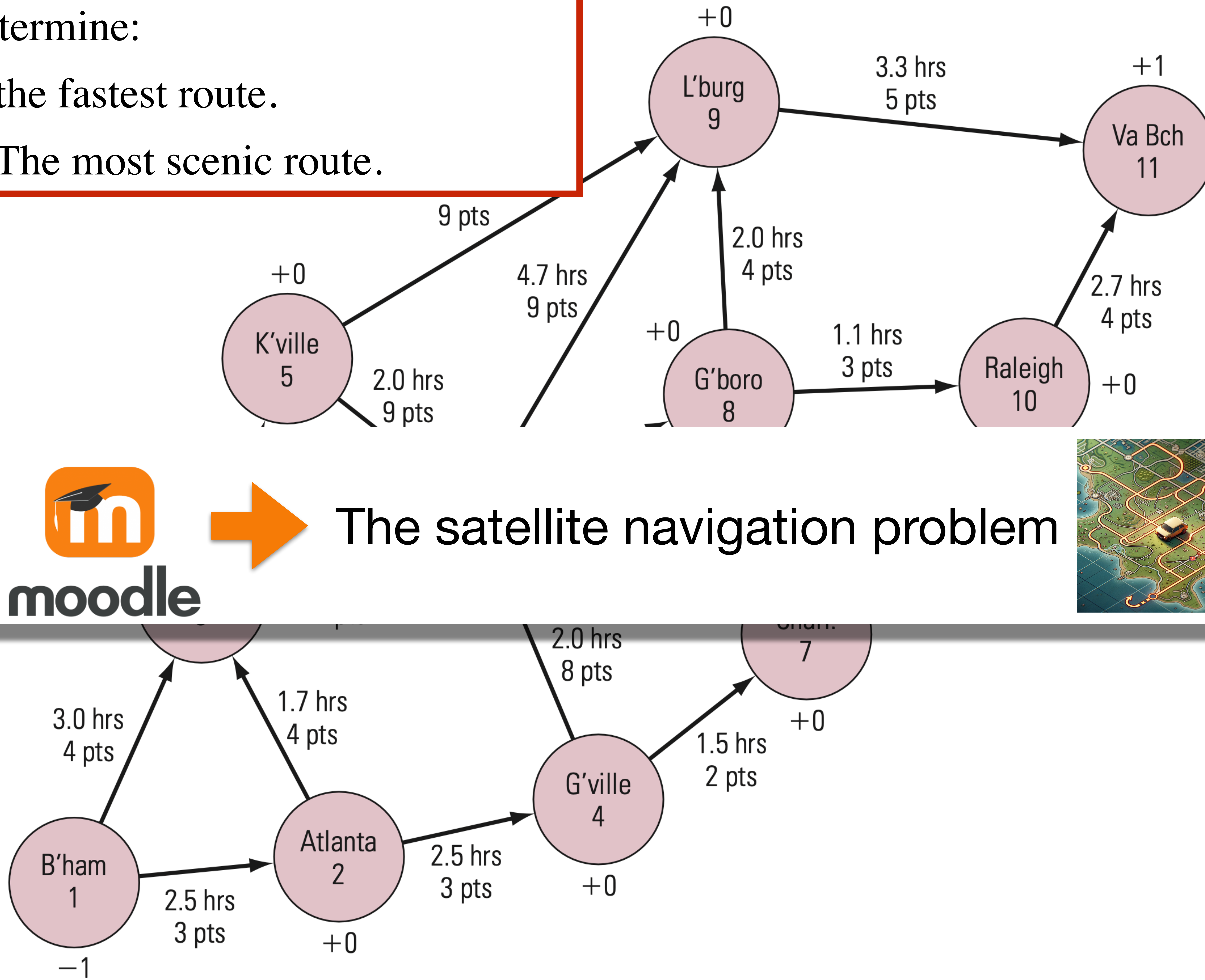
Each arc features the number of hours to cross it and the number of points received on the system for rating the scenic quality.



Determine:

1) the fastest route.

2) The most scenic route.



The satellite navigation problem