

**Integer linear programming
with
EXCEL SOLVER**



The Jacuzzi problem

The Jacuzzi company sells two models of hot tubs, the Aqua-Spa and the Hydro-Lux, which it produces by purchasing prefabricated fiberglass hot tub shells and installing a common water pump and an appropriate amount of tubing.

Each Aqua-Spa produced requires 1 pump, 9 hours of labor, and 12 feet of tubing, and contributes \$350 to profits. Each Hydro-Lux produced requires 1 pump, 6 hours of labor, and 16 feet of tubing, and contributes \$300 to profits. Assuming the company has 200 pumps, 1,566 labor hours, and 2,880 feet of tubing available.

	A	B	C	D	E
1	The Jacuzzi Problem				
2					
3		Aqua-Spa	Hydro-Luxes		
4	Number to make	0	0	Total Profit	
5	Unit profit	\$350.00	\$300.00	\$0.00	
6					
7					
8	Constraints			Used	Available
9	Pumps required	1	1	0	200
10	Labor required	9	6	0	1566
11	Tubing required	12	16	0	2880

	A	B	C	D	E
1	The Jacuzzi Problem				
2					
3		Aqua-Spa	Hydro-Luxes		
4	Number to make	0	0	Total Profit	
5	Unit profit	\$350.00	\$300.00	\$0.00	
6					
7					
8	Constraints			Used	Available
9	Pumps required	1	1	0	200
10	Labor required	9	6	0	1566
11	Tubing required	12	16	0	2880

=B4*B5+C4*C5

	A	B	C	D	E
1	The Jacuzzi Problem				
2					=B9*\$B\$4+C9*\$C\$4
3		Aqua-Spa	Hydro-Luxes		
4	Number to make	0	0	Total Profit	
5	Unit profit	\$350.00	\$300.00	\$0.00	
6					
7					
8	Constraints			Used	Available
9	Pumps required	1	1	0	200
10	Labor required	9	6	0	1566
11	Tubing required	12	16	0	2880

	A	B	C	D	E
1	The Jacuzzi Problem				
2					
3		Aqua-Spa	Hydro-Luxes		
4	Number to make	0	0	Total Profit	
5	Unit profit	\$350.00	\$300.00	\$0.00	
6					
7					
8	Constraints			Used	Available
9	Pumps required	1	1	0	200
10	Labor required	9	6	0	1566
11	Tubing required	12	16	0	2880

Hand fill

Solver Parameters

Set Objective:

\$D\$5

To:

☒ Max

☐ Min

☐ Value Of:

0

By Changing Variable Cells:

\$B\$4:\$C\$4

Subject to the Constraints:

\$B\$4:\$C\$4 >= 0

\$D\$9:\$D\$11 <= \$E\$9:\$E\$11

Add

Change

Delete

Reset All

Load/Save

☐ Make Unconstrained Variables Non-Negative

Select a Solving Method:

Simplex LP

Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Close

Solve

[illegible]

	A	B	C	
1	The Jacuzzi Problem			
2				
3		Aqua-Spa	Hydro-Luxes	
4	Number to make	0	0	Total
5	Unit profit	\$350.00	\$300.00	\$0
6				
7				
8	Constraints			U
9	Pumps required	1	1	
10	Labor required	9	6	
11	Tubing required	12	16	

Solver Parameters

Set Objective:

\$D\$5

To:

☒ Max
 ☐ Min
 ☐ Value Of:

0

By Changing Variable Cells:

\$B\$4:\$C\$4

Subject to the Constraints:

\$B\$4:\$C\$4 >= 0

\$D\$9:\$D\$11 <= \$E\$9:\$E\$11

Add

Change

Delete

Reset All

Load/Save

☐ Make Unconstrained Variables Non-Negative

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Solving Method

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Close

Solve

	A	B	C	
1	The Jacuzzi Problem			
2				
3		Aqua-Spa	Hydro-Luxes	
4	Number to make	0	0	Total
5	Unit profit	\$350.00	\$300.00	\$0
6				
7				
8	Constraints			U
9	Pumps required	1	1	
10	Labor required	9	6	
11	Tubing required	12	16	

Solver Parameters

Set Objective:

\$D\$5

To:

☒ Max
 ☐ Min
 ☐ Value Of:

0

By Changing Variable Cells:

\$B\$4:\$C\$4

Subject to the Constraints:

\$B\$4:\$C\$4 >= 0

\$D\$9:\$D\$11 <= \$E\$9:\$E\$11

Add

Change

Delete

Reset All

Load/Save

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Solving Method

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Close

Solve

	A	B	C	
1	The Jacuzzi Problem			
2				
3		Aqua-Spa	Hydro-Luxes	
4	Number to make	0	0	Total
5	Unit profit	\$350.00	\$300.00	\$0
6	We do not impose that they are integers! This is called the “Relaxed” solution.			
7				
8	Constraints			U
9	Pumps required	1	1	
10	Labor required	9	6	
11	Tubing required	12	16	

Solver Parameters

Set Objective:

\$D\$5

To:

☒ Max
 ☐ Min
 ☐ Value Of:

0

By Changing Variable Cells:

\$B\$4:\$C\$4

Subject to the Constraints:

\$B\$4:\$C\$4 >= 0

\$D\$9:\$D\$11 <= \$E\$9:\$E\$11

Add

Change

Delete

Reset All

Load/Save

Make Unconstrained Variables Non-Negative

Solving Method:

Simplex LP

Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Close

Solve

Solver Parameters

Set Objective:

\$D\$5

To:

Max

Min

Value Of:

0

By Changing Variable Cells:

\$B\$4:\$C\$4

Subject to the Constraints:

\$B\$4:\$C\$4 >= 0

\$D\$9:\$D\$11 <= \$E\$9:\$E\$11

Add

Change

Delete

Reset All

Load/Save

Make Unconstrained Variables Non-Negative

Select a Solving Method:

Simplex LP

Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Close

Solve

B	C	D	E
Aqua-Spa	Hydro-Luxes		
0	0	Total Profit	
\$350.00	\$300.00	\$0.00	
Constraints		Used	Available
1	1	0	200
9	6	0	1566
12	16	0	2880

Solver Parameters

Set Objective:

\$D\$5

To:

☒ Max

☐ Min

☐ Value Of:

0

By Changing Variable Cells:

\$B\$4:\$C\$4

Subject to the Constraints:

\$B\$4:\$C\$4 >= 0

\$D\$9:\$D\$11 <= \$E\$9:\$E\$11

Add

Change

Delete

Reset All

Load/Save

☐ Make Unconstrained Variables Non-Negative

Select a Solving Method:

Simplex LP

Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Close

Solve

B	C	D	E
Aqua-Spa	Hydro-Luxes		
0	0	Total Profit	
\$350.00	\$300.00	\$0.00	
Constraints		Used	Available
1	1	0	200
9	6	0	1566
12	16	0	2880

The “Relaxed” solution happens to have, in this case, integer solutions! Hence, it is a viable solution for the company.

	A	happens to have, In this case, integer solutions! Hence, it is a viable solution for the company.			E
1	The Jacuzzi Problem				
2					
3		Aqua-Spa	Hydro-Luxes		
4	Number to make	122	78	Total Profit	
5	Unit profit	\$350.00	\$300.00	\$66,100.00	
6					
7					
8	Constraints			Used	Available
9	Pumps required	1	1	200	200
10	Labor required	9	6	1566	1566
11	Tubing required	12	16	2712	2880

	A				E
1	The Jacuzzi Problem	happens to have, in this case, integer solutions! Hence, it is a viable solution for the company.			This is, however, not always the case ...
2					
3		Aqua-Spa	Hydro-Luxes		
4	Number to make	122	78	Total Profit	
5	Unit profit	\$350.00	\$300.00	\$66,100.00	
6					
7					
8	Constraints			Used	Available
9	Pumps required	1	1	200	200
10	Labor required	9	6	1566	1566
11	Tubing required	12	16	2712	2880

The “Relaxed” solution happens to have, in this case, integer solutions! Hence, it is a viable solution for the company.

This is, however, not always the case ...

	A	Consider a different set of constraints			E
1	The Jacuzzi Problem				
2					
3		Aqua-Spa	Hydro-Luxes		
4	Number to make	0	0	Total Profit	
5	Unit profit	\$350.00	\$300.00	\$0.00	
6					
7					
8	Constraints			Used	Available
9	Pumps required	1	1	0	200
10	Labor required	9	6	0	1520
11	Tubing required	12	16	0	2650

Solver Parameters

Set Objective:

To:

☒ Max ☐ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

\$B\$4:\$C\$4 >= 0

\$D\$9:\$D\$11 <= \$E\$9:\$E\$11

Add

Change

Delete

Reset All

Load/Save

☐ Make Unconstrained Variables Non-Negative

Select a Solving Method:

Simplex LP

Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Close

Solve

Solver settings again without integer constraints.

Solver settings again without integer constraints.		E	
Aqua-Spa	Hydro-Luxes		
0	0	Total Profit	
\$350.00	\$300.00	\$0.00	
Constraints		Used	Available
1	1	0	200
9	6	0	1520
12	16	0	2650

We have an unfeasible solution. It is not acceptable!

	A	B	C	D	E
1	The Jacuzzi Problem				
2					
3		Aqua-Spa	Hydro-Luxes		
4	Number to make	116.94444	77.916667	Total Profit	
5	Unit profit	\$350.00	\$300.00	\$64,305.56	
6					
7					
8	Constraints			Used	Available
9	Pumps required	1	1	195	200
10	Labor required	9	6	1520	1520
11	Tubing required	12	16	2650	2650

Rounding is not a good patch.

- 1) We are not guaranteed to find the optimum.
- 2) We are not guaranteed to satisfy the constraints imposed by the available resources.

	A				
1	The Jacuzzi Problem				
2					
3		Aqua-Spa	Hydro-Luxes		
4	Number to make	116.94444	77.916667	Total Profit	
5	Unit profit	\$350.00	\$300.00	\$64,305.56	
6					
7					
8	Constraints			Used	Available
9	Pumps required	1	1	195	200
10	Labor required	9	6	1520	1520
11	Tubing required	12	16	2650	2650

	A	
1	The Jacuzzi Problem	
2		
3		
4	Number to make	
5	Unit profit	
6		
7		
8	Const	
9	Pumps required	
10	Labor required	
11	Tubing required	

Add the integer constraint.

Set Objective:

To: ☒ Max ☐ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

-
-
-

☐ Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Close Solve

E
Available
200
1520
2650

Solver found an integer solution: is this the optimal?

	A	B	C	D	E
1	The Jacuzzi Problem				
2					
3		Aqua-Spa	Hydro-Luxes		
4	Number to make	117	77	Total Profit	
5	Unit profit	\$350.00	\$300.00	\$64,050.00	
6					
7					
8	Constraints			Used	Available
9	Pumps required	1	1	194	200
10	Labor required	9	6	1515	1520
11	Tubing required	12	16	2636	2650

Try with this.
Update all the cells...

	A	B	C	D	E
1	The Jacuzzi Problem				
2					
3		Aqua-Spa	Hydro-Luxes		
4	Number to make	118	76	Total Profit	
5	Unit profit	\$350.00	\$300.00	\$64,100.00	
6					
7					
8	Constraints			Used	Available
9	Pumps required	1	1	194	200
10	Labor required	9	6	1518	1520
11	Tubing required	12	16	2632	2650

This is a better solution!
Why didn't the SOLVER find this solution?

	A	B	C	D	E
1	The Jacuzzi Problem				
2					
3		Aqua-Spa	Hydro-Luxes		
4	Number to make	118	76	Total Profit	
5	Unit profit	\$350.00	\$300.00	\$64,100.00	
6					
7					
8	Constraints			Used	Available
9	Pumps required	1	1	194	200
10	Labor required	9	6	1518	1520
11	Tubing required	12	16	2632	2650

This is a better solution!
Why didn't the SOLVER find this solution?

	A	B	C	D	E
1	The Jacuzzi Problem				
2					
3		Aqua-Spa	Hydro-Luxes		
4	Number to make	118	76	Total Profit	
5	Units available				
6	By default, Solver uses a suboptimality tolerance factor. So, when Solver found the integer solution with the objective function value of \$64,050, it determined that this solution was within the tolerance of the optimal integer solution and abandoned its search.				
7					
8					Available
9	Pumps required	1	1	134	200
10	Labor required	9	6	1518	1520
11	Tubing required	12	16	2632	2650

	A
1	<u>The Jacuzzi Pro</u>
2	
3	
4	Number to ma
5	Unit profit
6	
7	
8	
9	Pumps require
10	Labor required
11	Tubing require

Solver Parameters

Set Objective:

\$D\$5

To:

☒ Max

☐ Min

☐ Value Of:

0

By Changing Variable Cells:

\$B\$4:\$C\$4

Subject to the Constraints:

\$B\$4:\$C\$4 = integer

\$B\$4:\$C\$4 >= 0

\$D\$9:\$D\$11 <= \$E\$9:\$E\$11

Add

Change

Delete

Reset All

Load/Save

☐ Make Unconstrained Variables Non-Negative

Select a Solving Method:

Simplex LP

Options

Solving Method

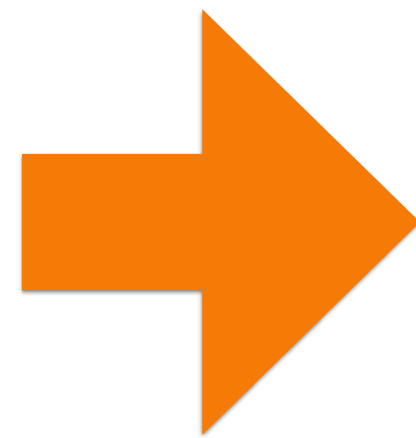
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Close

Solve

	E
Profit	
0.00	
d	Available
4	200
8	1520
2	2650

	A	B	C	D	E
1	The Jacuzzi Problem	The problem is fixed: SOLVER found the optimal solution.			
2					
3		Aqua-Spa	Hydro-Luxes		
4	Number to make	118	76	Total Profit	
5	Unit profit	\$350.00	\$300.00	\$64,100.00	
6					
7					
8	Constraints			Used	Available
9	Pumps required	1	1	194	200
10	Labor required	9	6	1518	1520
11	Tubing required	12	16	2632	2650



The project financing problem

