

Classwork: diet optimization

Build the worksheet, minimize the cost, and read the solution.

The diet problem

Choose quantities of **Brownies**, **Ice Cream**, **Cola**, and **Cheese Cake** to obtain a **minimum-cost diet**.

The diet must contain:

- ▶ at least **500 calories**;
- ▶ at least **6 grams of chocolate**;
- ▶ at least **10 grams of sugar**;
- ▶ at least **8 grams of fat**.

The next two worksheet portions contain the nutritional values and cost **per unit** of each food.

Diet: enter the nutritional data

Create a worksheet named **Diet**. Enter the labels in **A1:E1** and **A2:A5**, then fill **B2:E5** exactly as shown.

B2:E5		<i>fx</i>	400			
	A		B	C	D	E
1	Food		Brownies	Ice Cream	Cola	Cheese Cake
2	Calories		400	200	150	500
3	Chocolate (g)		3	2	0	0
4	Sugar (g)		2	2	4	4
5	Fat (g)		2	4	1	5

Each column refers to one food. Each number is the nutritional content of **one unit** of that food.

Diet: add the unit costs

In A6, enter **Cost (\$)**. Fill B6:E6 with the four costs and display two decimal places.

B6:E6		<i>fx</i>	0.50		
	A	B	C	D	E
1	Food	Brownies	Ice Cream	Cola	Cheese Cake
2	Calories	400	200	150	500
3	Chocolate (g)	3	2	0	0
4	Sugar (g)	2	2	4	4
5	Fat (g)	2	4	1	5
6	Cost (\$)	0.50	0.20	0.30	0.80

The order remains Brownies, Ice Cream, Cola, Cheese Cake. All input data are now in A1:E6.

Diet: identify the decision cells

Enter the labels in rows 8–10. In B10:E10, enter the starting quantities **3, 0, 1, 7**.

B10:E10		<i>fx</i>	3			
	A		B	C	D	E
8	DECISION VARIABLES					
9			Brownies	Ice Cream	Cola	Cheese Cake
10	Eaten		3	0	1	7

These are initial trial values. Solver will change these four cells to choose how much of each food to eat.

Diet: calculate the total cost

Leave row 11 empty. Enter **Total Cost** in A12. Select B12 and type the formula shown in the formula bar.

B12	<i>fx</i>	=B10*B6+C10*C6+D10*D6+E10*E6			
	A	B	C	D	E
9		Brownies	Ice Cream	Cola	Cheese Cake
10	Eaten	3	0	1	7
11					
12	Total Cost	7.40			

Each term is **quantity eaten** $\times$ **unit cost**. The initial quantities give **\$7.40**. This is the objective cell.

Diet: calculate total calories

Enter **Total Calories** in A13. Select B13 and type the formula shown below.

B13	<i>fx</i>	=B10*B2+C10*C2+D10*D2+E10*E2			
	A	B	C	D	E
12	Total Cost	7.40			
13	Total Calories	4,850			

The initial quantities give **4,850 calories**. Solver must keep this total at least **500 calories**.

Diet: calculate total chocolate

Enter **Total Chocolate** in A14. Select B14 and type the formula shown below.

B14	<i>fx</i>	=B10*B3+C10*C3+D10*D3+E10*E3			
	A	B	C	D	E
12	Total Cost	7.40			
13	Total Calories	4,850			
14	Total Chocolate	9			

The initial quantities give **9 grams**. Solver must keep this total at least **6 grams of chocolate**.

Diet: calculate total sugar

Enter **Total Sugar** in A15. Select B15 and type the formula shown below.

B15	<i>fx</i>	=B10*B4+C10*C4+D10*D4+E10*E4			
	A	B	C	D	E
13	Total Calories	4,850			
14	Total Chocolate	9			
15	Total Sugar	38			

The initial quantities give **38 grams**. Solver must keep this total at least **10 grams of sugar**.

Diet: calculate total fat

Enter **Total Fat** in A16. Select B16 and type the formula shown below.

B16	<i>fx</i>	=B10*B5+C10*C5+D10*D5+E10*E5			
	A	B	C	D	E
13	Total Calories	4,850			
14	Total Chocolate	9			
15	Total Sugar	38			
16	Total Fat	42			

The initial quantities give **42 grams**. Solver must keep this total at least **8 grams of fat**.

Diet: check the worksheet before solving

With B10:E10 still equal to **3, 0, 1, 7**, check that your formula cells show these values.

B12	<i>fx</i>	=B10*B6+C10*C6+D10*D6+E10*E6			
	A	B	C	D	E
10	Eaten	3	0	1	7
11					
12	Total Cost	7.40			
13	Total Calories	4,850			
14	Total Chocolate	9			
15	Total Sugar	38			
16	Total Fat	42			

B12 is the cost to minimize. B13:B16 are the totals used in the four constraints.

Diet in Solver: minimize the cost

Schematic Excel interface

Solver Parameters

Set Objective: \$B\$12

To: ☐ Max ☒ Min

By Changing Variable Cells:

\$B\$10:\$E\$10

Subject to the Constraints:

\$B\$13 >= 500
\$B\$14 >= 6
\$B\$15 >= 10
\$B\$16 >= 8

Add
Change
Delete

☒ Make Unconstrained Variables Non-Negative

Solving Method: Simplex LP Options Solve

Open **Data** → **Solver**. Set Objective to \$B\$12 and select **Min**.

Diet in Solver: select the decision cells

Schematic Excel interface

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min

By Changing Variable Cells:

Subject to the Constraints:

\$B\$13 >= 500

\$B\$14 >= 6

\$B\$15 >= 10

\$B\$16 >= 8

Add

Change

Delete

☒ Make Unconstrained Variables Non-Negative

Solving Method:

In **By Changing Variable Cells**, enter \$B\$10:\$E\$10: the quantities of the four foods.

Diet in Solver: add the four minimum requirements

Schematic Excel interface

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min

By Changing Variable Cells:

Subject to the Constraints:

\$B\$13 >= 500

\$B\$14 >= 6

\$B\$15 >= 10

\$B\$16 >= 8

Add

Change

Delete

☒ Make Unconstrained Variables Non-Negative

Solving Method:

Use **Add** for each constraint. All four use $\geq$: calories, chocolate, sugar, and fat must reach their required minimums.

Diet in Solver: choose Simplex LP

Schematic Excel interface

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min

By Changing Variable Cells:

Subject to the Constraints:

☒ Make Unconstrained Variables Non-Negative

Solving Method:

Choose **Simplex LP** from the method list. The cost and all nutrient totals are linear in the four decision cells. If the default is GRG Nonlinear, change it here.

Diet in Solver: enforce nonnegative quantities

Schematic Excel interface

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min

By Changing Variable Cells:

Subject to the Constraints:

☒ Make Unconstrained Variables Non-Negative

Solving Method:

Tick **Make Unconstrained Variables Non-Negative**. The four quantities must be at least zero. No integer constraint is imposed.

Diet in Solver: solve the model

Schematic Excel interface

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min

By Changing Variable Cells:

Subject to the Constraints:

☒ Make Unconstrained Variables Non-Negative

Solving Method:

Check the objective, decision cells, constraints, method, and nonnegativity setting. Click **Solve** and keep the Solver solution.

Diet: read the optimal quantities and cost

Solver updates the four decision cells and recalculates the formula cells.

B10:E10		fx	0			
	A	B	C	D	E	
8	DECISION VARIABLES					
9		Brownies	Ice Cream	Cola	Cheese Cake	
10	Eaten	0	3	1	0	
11						
12	Total Cost	0.90				

The minimum-cost diet is **0 Brownies, 3 Ice Cream, 1 Cola, and 0 Cheese Cake**. Its total cost is **\$0.90**.

Diet: verify the totals and save the worksheet

Read the recalculated totals in B13:B16 and compare them with the four minimum requirements.

B13:B16		<i>fx</i>	=B10*B2+C10*C2+D10*D2+E10*E2			
	A	B	C	D	E	
12	Total Cost	0.90				
13	Total Calories	750				
14	Total Chocolate	6				
15	Total Sugar	10				
16	Total Fat	13				

750 $\geq$ 500 calories; **6** $\geq$ 6 g chocolate; **10** $\geq$ 10 g sugar;
13 $\geq$ 8 g fat.

All requirements are satisfied. **Save the workbook** with the completed Diet worksheet and the Solver solution.

Optimal configuration

Is the optimal configuration unique? Yes.

Let b, i, c, k denote brownies, ice cream, cola and cheesecake. Denote the amounts of chocolate and sugar by

$$H = 3b + 2i, \quad S = 2b + 2i + 4c + 4k.$$

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We can express the total cost as ...

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These conditions give $2i = 6$ and $2i + 4c = 10$, and therefore

$$(b, i, c, k) = (0, 3, 1, 0).$$

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$$(b, i, c, k) = (0, 3, 1, 0).$$

This configuration also satisfies the constraints! It is the **unique global minimizer**. It is integer ... no need to impose integer constraints on the solver.

Diet: increase the sugar requirement

Solve the same diet problem again, now requiring **at least 11 grams of sugar** instead of 10.

1. Open **Data** → **Solver**. Select the sugar constraint and click **Change**:

$$\text{\$B\$15} \geq 10 \quad \longrightarrow \quad \text{\$B\$15} \geq 11.$$

2. Keep the objective **\\$B\\$12** set to **Min** and the changing cells **\\$B\\$10:\\$E\\$10**. Keep all other requirements and worksheet formulas unchanged.
3. Use **Simplex LP** with **Make Unconstrained Variables Non-Negative** selected. Leave the quantities continuous: do not add integer constraints.

BEFORE REVEALING THE SOLUTION

Find the new quantities and minimum cost. Will the optimal quantities still be whole numbers?

Diet with 11 grams of sugar: Solver solution

After solving, the worksheet should show the following quantities and total cost.

B10:E10		fx	0			
	A	B	C	D	E	
8	DECISION VARIABLES					
9		Brownies	Ice Cream	Cola	Cheese Cake	
10	Eaten	0.00	3.00	1.25	0.00	
11						
12	Total Cost	0.975				

Cola is now 1.25 units.

Show two decimal places for quantities and **three** for the cost:
the exact minimum is **\$0.975**.

Why is the new optimum not integer?

Without integer constraints, Solver permits **real, nonnegative quantities**.

The same cost identity now gives

$$C = \frac{H}{40} + \frac{3S}{40} + \frac{11b}{40} + \frac{k}{2} \geq \frac{6 + 3(11)}{40} = 0.975.$$

Equality requires $b = k = 0$, $H = 6$ and $S = 11$. Hence

$$2i = 6, \quad 2i + 4c = 11 \quad \implies \quad i = 3, \quad c = \frac{5}{4} = 1.25.$$

The displayed plan is feasible and attains this bound, so it is the **unique continuous optimum**.

To require whole portions, explicitly add $\$B\$10:\$E\$10 = \mathbf{integer}$ in Solver and solve the new model.

Diet: remove the nonnegativity condition

Return to the **original diet model**: change the sugar requirement back from 11 to **10 grams**. Start from $(b, i, c, k) = (0, 3, 1, 0)$.

1. Keep **\$B\$12** as the objective to **Min**, **\$B\$10:\$E\$10** as the changing cells, and **Simplex LP** as the method.
2. Keep only the four nutritional constraints: calories ≥ 500 , chocolate ≥ 6 , sugar ≥ 10 , and fat ≥ 8 . Remove any explicit nonnegativity or integer constraints added in earlier experiments.
3. Clear **Make Unconstrained Variables Non-Negative**, then click **Solve**.



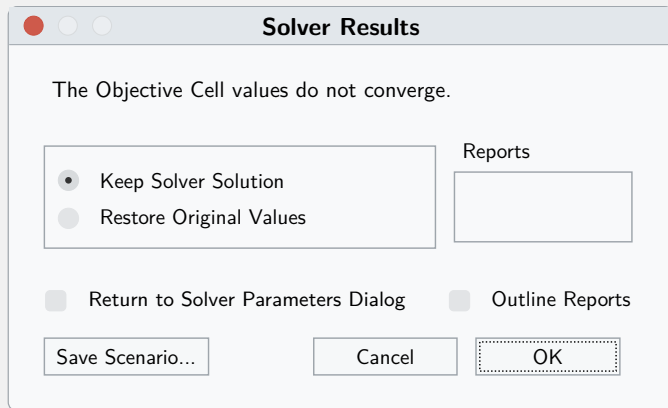
Make Unconstrained Variables Non-Negative

Unchecked

OBSERVE AND EXPLAIN

Read Solver's message. Is there a finite minimum cost when negative quantities are allowed?

Diet: read the Solver Results message



Solver reports that the objective values do not converge.

To finish the experiment, select **Restore Original Values**.

Without nonnegativity: feasible configurations

Let b, i, c, k denote brownies, ice cream, cola and cheesecake. When they may be negative, consider, for any $t \geq 0$,

$$(b, i, c, k) = (-t, 3 + 2t, 1, 0).$$

The four nutritional constraints remain satisfied:

Nutrient	Amount	Requirement
Calories	$400(-t) + 200(3 + 2t) + 150 = 750$	≥ 500
Chocolate	$3(-t) + 2(3 + 2t) = 6 + t$	≥ 6
Sugar	$2(-t) + 2(3 + 2t) + 4 = 10 + 2t$	≥ 10
Fat	$2(-t) + 4(3 + 2t) + 1 = 13 + 6t$	≥ 8

For every $t > 0$, the brownie quantity is negative. The remaining constraints permit it.

We can increase t indefinitely while satisfying every nutritional requirement.

Without nonnegativity: cost is unbounded below

Along the feasible configurations just constructed,

$$\begin{aligned}C(t) &= 0.50(-t) + 0.20(3 + 2t) + 0.30(1) + 0.80(0) \\ &= 0.90 - 0.10t.\end{aligned}$$

Therefore

$$C(t) \longrightarrow -\infty \quad \text{as } t \longrightarrow +\infty.$$

There is no finite minimum to find

The objective is **unbounded below**. Solver's message reflects a property of the model, rather than a numerical convergence difficulty.

The model treats negative brownies as a monetary credit. That credit can grow without limit while every nutritional requirement is met.

Select **Make Unconstrained Variables Non-Negative** again to restore the intended diet model. Explicit constraints $b, i, c, k \geq 0$ would have the same effect.

Amusement Park Staff Scheduling

Build a weekly staffing plan in Excel

A weekly staffing plan

An amusement park operates **seven days per week**. Each day it needs a minimum number of employees on duty.

The working pattern

Each employee works **five consecutive days**, then has **two consecutive days off**. The pattern repeats every week.

Choose how many employees follow each schedule: meet daily demand and **minimize total weekly payroll**.

1. Enter the seven schedule types

Use a new worksheet. Enter these labels in A1:D8; merge B:C separately in each row shown.

		fx		
	A	B	C	D
1	Sch.	Days off		Employees
2	A	Sunday, Monday		
3	B	Monday, Tuesday		
4	C	Tuesday, Wed.		
5	D	Wed., Thursday		
6	E	Thursday, Friday		
7	F	Friday, Saturday		
8	G	Saturday, Sunday		

The days off identify each schedule. For G, Saturday and Sunday are consecutive across the end of the displayed week.

2. Enter the days and daily demand

Enter the day names in F1:L1. Then enter the demand values in F10:L10; add the labels in column E.

	fx							
	E	F	G	H	I	J	K	L
1		Sun	Mon	Tue	Wed	Thu	Fri	Sat

F10:L10	fx	20						
	E	F	G	H	I	J	K	L
9	Scheduled Daily:							
10	Total Demand:	20	5	15	14	15	18	24

Keep this column order throughout: **Sunday to Saturday**.
Row 9 will contain the staff actually scheduled.

3. Enter the daily pay

In F12:L12, enter the pay per employee for each day. Add the labels in E12:E13. All amounts are in euros.

		<i>fx</i>							
	E		F	G	H	I	J	K	L
1			Sun	Mon	Tue	Wed	Thu	Fri	Sat
F12:L12		<i>fx</i>	40.00						
	E		F	G	H	I	J	K	L
12	Pay/Emp./Day:		40.00	30.00	30.00	30.00	30.50	30.50	35.00
13	Payroll/Week:								

Pay is due only for days actually worked. Format the rates with two decimal places; row 13 will contain the full weekly payroll.

4. A: Sunday and Monday off

In F2:L2, type **0, 0, 1, 1, 1, 1, 1**.

F2:L2		f_x	0				
	F	G	H	I	J	K	L
1	Sun	Mon	Tue	Wed	Thu	Fri	Sat
2	0	0	1	1	1	1	1
3							
4							
5							
6							
7							
8							

0 = day off; 1 = working day. An employee on schedule A works Tuesday through Saturday.

4. B: Monday and Tuesday off

In F3:L3, type 1, 0, 0, 1, 1, 1, 1.

F3:L3		f_x	1				
	F	G	H	I	J	K	L
1	Sun	Mon	Tue	Wed	Thu	Fri	Sat
2	0	0	1	1	1	1	1
3	1	0	0	1	1	1	1
4							
5							
6							
7							
8							

For schedule B, Sunday is a working day; Monday and Tuesday are days off.

4. C: Tuesday and Wednesday off

In F4:L4, type 1, 1, 0, 0, 1, 1, 1.

F4:L4		f_x	1				
	F	G	H	I	J	K	L
1	Sun	Mon	Tue	Wed	Thu	Fri	Sat
2	0	0	1	1	1	1	1
3	1	0	0	1	1	1	1
4	1	1	0	0	1	1	1
5							
6							
7							
8							

Only the Tuesday and Wednesday entries are zero in this row.

4. D: Wednesday and Thursday off

In F5:L5, type 1, 1, 1, 0, 0, 1, 1.

F5:L5		f_x	1				
	F	G	H	I	J	K	L
1	Sun	Mon	Tue	Wed	Thu	Fri	Sat
2	0	0	1	1	1	1	1
3	1	0	0	1	1	1	1
4	1	1	0	0	1	1	1
5	1	1	1	0	0	1	1
6							
7							
8							

Keep all previously entered rows. Each row describes one schedule type.

4. E: Thursday and Friday off

In F6:L6, type 1, 1, 1, 1, 0, 0, 1.

F6:L6		f_x	1				
	F	G	H	I	J	K	L
1	Sun	Mon	Tue	Wed	Thu	Fri	Sat
2	0	0	1	1	1	1	1
3	1	0	0	1	1	1	1
4	1	1	0	0	1	1	1
5	1	1	1	0	0	1	1
6	1	1	1	1	0	0	1
7							
8							

Schedule E includes Saturday and Sunday as working days.

4. F: Friday and Saturday off

In F7:L7, type 1, 1, 1, 1, 1, 0, 0.

F7:L7		f_x	1				
	F	G	H	I	J	K	L
1	Sun	Mon	Tue	Wed	Thu	Fri	Sat
2	0	0	1	1	1	1	1
3	1	0	0	1	1	1	1
4	1	1	0	0	1	1	1
5	1	1	1	0	0	1	1
6	1	1	1	1	0	0	1
7	1	1	1	1	1	0	0
8							

Schedule F works Sunday through Thursday.

4. G: Saturday and Sunday off

In F8:L8, type **0, 1, 1, 1, 1, 1, 0**.

F8:L8		f_x	0				
	F	G	H	I	J	K	L
1	Sun	Mon	Tue	Wed	Thu	Fri	Sat
2	0	0	1	1	1	1	1
3	1	0	0	1	1	1	1
4	1	1	0	0	1	1	1
5	1	1	1	0	0	1	1
6	1	1	1	1	0	0	1
7	1	1	1	1	1	0	0
8	0	1	1	1	1	1	0

The matrix is complete: each row has **five 1s and two 0s**.
Schedule G works Monday through Friday.

5. Enter a trial staffing plan

Type these trial numbers in D2:D8. These are the cells Solver will later change.

D2:D8		<i>fx</i>	4		
	A	B	C	D	
1	Sch.	Days off		Employees	
2	A	Sunday, Monday		4	
3	B	Monday, Tuesday		1	
4	C	Tuesday, Wed.		3	
5	D	Wed., Thursday		3	
6	E	Thursday, Friday		1	
7	F	Friday, Saturday		2	
8	G	Saturday, Sunday		5	

The numbers count employees assigned to each schedule. They are a starting point for checking our formulas.

6. Calculate Sunday staffing

Select F9 and enter the formula shown in the formula bar.

F9	<i>fx</i>	=SUMPRODUCT(\$D\$2:\$D\$8,F2:F8)						
	E	F	G	H	I	J	K	L
8		0	1	1	1	1	1	0
9	Scheduled Daily:	10						
10	Total Demand:	20	5	15	14	15	18	24

It multiplies each employee count by its Sunday 0/1 entry and adds the products. The result is **10**; lock \$D\$2:\$D\$8 before copying.

7. Copy the formula across the week

Fill F9 rightward through L9. The employee range stays fixed; the workday column changes.

	fx							
	E	F	G	H	I	J	K	L
1		Sun	Mon	Tue	Wed	Thu	Fri	Sat

L9	fx	=SUMPRODUCT(\$D\$2:\$D\$8,L2:L8)						
	E	F	G	H	I	J	K	L
9	Scheduled Daily:	10	14	15	13	15	16	12
10	Total Demand:	20	5	15	14	15	18	24

For example, G9 uses G2:G8. Check the seven results: 10, 14, 15, 13, 15, 16, 12.

8. Count the employees

Merge B9:C9 and enter **Scheduled Totals:**. Which formula belongs in D9?

D9		fx		
	A	B	C	D
7	F	Friday, Saturday		2
8	G	Saturday, Sunday		5
9		Scheduled Totals:		

Think first: count each employee once, regardless of how many days they work.

8. Count the employees

Merge B9:C9 and enter **Scheduled Totals:**. Which formula belongs in D9?

D9		<i>fx</i>	=SUM(D2:D8)	
	A	B	C	D
7	F	Friday, Saturday		2
8	G	Saturday, Sunday		5
9		Scheduled Totals:		19

Enter =SUM(D2:D8). The trial plan assigns **19 employees** in total.

9. Calculate the weekly payroll

Which formula belongs in F13? Row 9 contains daily staffing; row 12 contains daily pay per employee.

F13		fx						
	E	F	G	H	I	J	K	L
9	Scheduled Daily:	10	14	15	13	15	16	12
10	Total Demand:	20	5	15	14	15	18	24
11								
12	Pay/Emp./Day:	40.00	30.00	30.00	30.00	30.50	30.50	35.00
13	Payroll/Week:							

Think first: multiply the staff on each day by that day's pay, then add the seven amounts.

9. Calculate the weekly payroll

Which formula belongs in F13? Row 9 contains daily staffing; row 12 contains daily pay per employee.

F13	<i>fx</i>	=SUMPRODUCT(F9:L9,F12:L12)						
	E	F	G	H	I	J	K	L
9	Scheduled Daily:	10	14	15	13	15	16	12
10	Total Demand:	20	5	15	14	15	18	24
11								
12	Pay/Emp./Day:	40.00	30.00	30.00	30.00	30.50	30.50	35.00
13	Payroll/Week:	3,025.50						

The formula gives **EUR 3,025.50 per week**. Format F13 as a euro amount with two decimal places.

The trial plan is not feasible

Compare the calculated staffing in row 9 with the required staffing in row 10.

	fx							
	E	F	G	H	I	J	K	L
1		Sun	Mon	Tue	Wed	Thu	Fri	Sat

	fx							
	E	F	G	H	I	J	K	L
9	Scheduled Daily:	10	14	15	13	15	16	12
10	Total Demand:	20	5	15	14	15	18	24

There are too few employees on **Sunday, Wednesday, Friday and Saturday**. A low trial cost is not useful unless every daily requirement is met.

10. Minimize the weekly payroll

Schematic Excel interface

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min

By Changing Variable Cells:

Subject to the Constraints:

☐ Make Unconstrained Variables Non-Negative

Solving Method:

Open **Data** → **Solver**. Set Objective to \$F\$13, select **Min**, and use **Simplex LP**.

11. Select the decision cells

Schematic Excel interface

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min

By Changing Variable Cells:

Subject to the Constraints:

☐ Make Unconstrained Variables Non-Negative

Solving Method:

In **By Changing Variable Cells**, enter **\$D\$2:\$D\$8**: the seven employee counts.

12. Require integer employee counts

Schematic Excel interface

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min

By Changing Variable Cells:

Subject to the Constraints:

☐ Make Unconstrained Variables Non-Negative

Solving Method:

Click **Add**. Select $\$D\$2:\$D\8 , choose **int**, and confirm. Employees cannot be fractional.

13. Cover every day's demand

Schematic Excel interface

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min

By Changing Variable Cells:

Subject to the Constraints:

\$D\$2:\$D\$8 = integer

\$F\$9:\$L\$9 >= \$F\$10:\$L\$10

Add

Change

Delete

☐ Make Unconstrained Variables Non-Negative

Solving Method:

Add $\$F\$9:\$L\$9 \geq \$F\$10:\$L\10 . This single range constraint compares all seven days, column by column.

First experiment: leave one condition out

Schematic Excel interface

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min

By Changing Variable Cells:

Subject to the Constraints:

☐ **Make Unconstrained Variables Non-Negative**

Solving Method:

For this experiment, deliberately leave **Make Unconstrained Variables Non-Negative** unchecked. We will inspect what goes wrong.

Inspect the integer-solving options

Click **Options**. In this first run, Integer Optimality is **1%**; keep the integer constraints active.

Solver Options: All Methods	
Constraint Precision:	0.000001
☐ Use Automatic Scaling	
☐ Show Iteration Results	
☐ Ignore Integer Constraints	
Integer Optimality (%):	1

Confirm with **OK**, then click **Solve**. Check whether the resulting employee counts make sense.

Is this staffing plan acceptable?

Solver can return this result when non-negativity is omitted.

D8		fx	-5	
	A	B	C	D
1	Sch.	Days off		Employees
2	A	Sunday, Monday		8
3	B	Monday, Tuesday		8
4	C	Tuesday, Wed.		0
5	D	Wed., Thursday		7
6	E	Thursday, Friday		1
7	F	Friday, Saturday		4
8	G	Saturday, Sunday		-5
9		Scheduled Totals:		23

No: schedule G has -5 employees. The integer constraint permits negative integers as well as zero and positive integers.

Meeting demand is not the only requirement

All daily coverage inequalities hold for that result, and the computed payroll is EUR 3,786.50.

	fx							
	E	F	G	H	I	J	K	L
1		Sun	Mon	Tue	Wed	Thu	Fri	Sat

	fx							
	E	F	G	H	I	J	K	L
9	Scheduled Daily:	20	7	15	16	15	18	24
10	Total Demand:	20	5	15	14	15	18	24

	fx		
	E	F	
13	Payroll/Week:	3,786.50	

The plan is still impossible: a negative number of employees has no operational meaning. We must fix the model.

14. Require non-negative employee counts

Schematic Excel interface

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min

By Changing Variable Cells:

Subject to the Constraints:

\$D\$2:\$D\$8 = integer

\$F\$9:\$L\$9 >= \$F\$10:\$L\$10

Add

Change

Delete

☒ **Make Unconstrained Variables Non-Negative**

Solving Method:

Now select **Make Unconstrained Variables Non-Negative**.
Combined with the integer constraint, this means 0, 1, 2, ... employees per schedule.

15. Set Integer Optimality to zero

Reopen **Options** and change **Integer Optimality (%)** from 1 to 0.

Solver Options: All Methods

Constraint Precision:

0.000001

☐ Use Automatic Scaling

☐ Show Iteration Results

☐ Ignore Integer Constraints

Integer Optimality (%):

0

Keep **Ignore Integer Constraints** unchecked. Confirm with **OK**, then click **Solve** and keep the solution.

One optimal staffing plan

The corrected model can give the following employee assignments.

D2:D8		fx	5		
	A	B		C	D
1	Sch.	Days off			Employees
2	A	Sunday, Monday			5
3	B	Monday, Tuesday			2
4	C	Tuesday, Wed.			7
5	D	Wed., Thursday			4
6	E	Thursday, Friday			6
7	F	Friday, Saturday			1
8	G	Saturday, Sunday			0
9		Scheduled Totals:			25

There are **25 employees**. All counts are non-negative integers; schedule G is unused. Solver may return another optimal plan.

Check the coverage and the payroll

The resulting daily staffing meets or exceeds every minimum requirement.

	fx							
	E	F	G	H	I	J	K	L
1		Sun	Mon	Tue	Wed	Thu	Fri	Sat

	fx							
	E	F	G	H	I	J	K	L
9	Scheduled Daily:	20	18	16	14	15	18	24
10	Total Demand:	20	5	15	14	15	18	24

	fx		
	E	F	
13	Payroll/Week:	4,086.50	

The minimum weekly payroll is **EUR 4,086.50**. Some days have more staff than the minimum because each employee follows a five-day schedule.

What if every schedule must be used?

Is it better or worse, in terms of weekly payroll, to require **at least one employee on every schedule**?

D8		<i>fx</i>	0	
	A	B	C	D
6	E	Thursday, Friday		6
7	F	Friday, Saturday		1
8	G	Saturday, Sunday		0
9		Scheduled Totals:		25

Think first: the current plan has no employee on schedule G.

What if every schedule must be used?

Is it better or worse, in terms of weekly payroll, to require **at least one employee on every schedule**?

D8		fx	0	
	A	B	C	D
6	E	Thursday, Friday		6
7	F	Friday, Saturday		1
8	G	Saturday, Sunday		0
9		Scheduled Totals:		25

Adding a requirement reduces the feasible choices. The minimum cost can stay the same or increase; it cannot decrease.

16. Add the new lower bound

Schematic Excel interface

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min

By Changing Variable Cells:

Subject to the Constraints:

\$D\$2:\$D\$8 = integer

\$D\$2:\$D\$8 >= 1

\$F\$9:\$L\$9 >= \$F\$10:\$L\$10

Add

Change

Delete

☒ Make Unconstrained Variables Non-Negative

Solving Method:

Add $\$D\$2:\$D\$8 \geq 1$. Keep the other constraints and the 0% integer tolerance, then solve again.

A new optimal plan with every schedule used

One solution satisfying the additional requirement is shown below.

D2:D8		fx	5	
	A	B	C	D
1	Sch.	Days off		Employees
2	A	Sunday, Monday		5
3	B	Monday, Tuesday		7
4	C	Tuesday, Wed.		1
5	D	Wed., Thursday		4
6	E	Thursday, Friday		7
7	F	Friday, Saturday		1
8	G	Saturday, Sunday		1
9		Scheduled Totals:		26

Every count is at least 1, and the total rises to **26 employees**.
Alternative optimal assignments may also exist.

The cost of the additional requirement

The new plan still covers every day's demand.

	fx							
	E	F	G	H	I	J	K	L
1		Sun	Mon	Tue	Wed	Thu	Fri	Sat

	fx							
	E	F	G	H	I	J	K	L
9	Scheduled Daily:	20	14	18	21	15	18	24
10	Total Demand:	20	5	15	14	15	18	24

	fx		
	E	F	
13	Payroll/Week:	4,236.50	

The weekly cost rises from EUR 4,086.50 to **EUR 4,236.50**: an increase of **EUR 150**. Save the completed workbook.

The shipping problem

Open **The shipping problem** on Moodle.



The shipping problem