Practice session 2

Game Theory - MSc EEBL

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September 22, 2022

Exercise 1. *Cournot Duopoly*

Two firms compete in a market by simultaneously setting the quantities of a (homogeneous) good to produce (q_i, q_j). Each firm faces a *constant marginal cost* $c_k \in \mathbb{R}_+$. The two firms face the *inverse demand function* $P(Q) = a - bQ$, where Q is the aggregate quantity produced and $(a, b) \in \mathbb{R}_+^2$ are demand parameters. Payoffs are given by each firm's profits.

1. Describe the game as a normal-form game (Players, Strategies, Payoffs).
2. Write the maximization problem for each firm.
3. Solve the maximization problem for each firm, obtaining its *reaction function*. (*Hint: Take the first derivative of profits with respect to each firm's quantity, taking the quantity produced by the other as given).*
4. Find the *NE* of the game, i.e., equilibrium quantities for both firms (*Hint: Use the two reaction functions and solve for* $q_i^\star$).
5. Comment on the equilibrium quantities when a, c_1 and c_2 vary.
6. Find the payoffs $\pi_i(q_i^\star, q_j^\star)$ for $i = 1, 2$ obtained by firms when they play this *NE* (*Hint: Replace equilibrium quantities in profits*).
7. Now suppose that both firms also face a *fixed cost* $F = 1$. Does this affect the equilibrium quantity? Does this change equilibrium payoffs? Explain.

Exercise 2. *Rock paper scissors*

Pat and Carl meet to play the famous game *Rock paper scissors*. According to this game, both players simultaneously choose between rock, paper, or scissors. Not surprisingly, rock beats scissors, scissors beat paper, and paper beats rock. If a player wins, they get 1 Euro from the other player. If they loose, they pay 1 Euro to the other player. If both players choose the same action, then they both get nothing.

1. Fill the *payoff matrix* using the available information and write the game as a *normal-form* game.
2. Is there any strictly *dominated strategy* for players?
3. Find the *pure-strategy Nash equilibria (NE)* of this game, if any.
4. Find the *mixed-strategy* NE of this game, if any.

Exercise 3. *Setting a Standard*

A new type of consumer product is about to be introduced in a market in which two firms are active (for example, a video game). The two firms own *competing technologies* (for example, two game consoles) that can be used to run this product, and would like their technology to be the *standard* in the market.

Each firm would prefer its technology to be used exclusively to run the product, as this would increase its sales. In particular, each firm has a payoff of zero if no standard is set (both firms use their own technology). If only one firm's technology is adopted as a standard, that firm gets a payoff of 2, and the other gets 1. Finally, if both firms employ the other firm's technology, they both get a payoff of 0.

1. Fill the payoff matrix using the available information and write the game as a normal-form game.
2. Is there any strictly *dominated strategy* for the two players?
3. Find the *pure-strategy Nash equilibria (NE)* of this game, if any.
4. Find the *mixed-strategy NE* of this game, if any.
5. Now, suppose that firm 1 has a *superior technology*, that is, the latter gets a payoff of 3 when it manages to set the standard. Does this affect the *pure and mixed-strategy NE*? Explain (*Hint: payoffs do not change for firm* 2).

Exercise 4. *Bertrand duopoly with differentiated products* (*Optional*)

Two firms compete in a market by simultaneously setting the prices, (p_i, p_j), of a differentiated good. Each firm faces a *constant marginal cost* $c = 2$. The demand for each firm's good is $q_i(p_i, p_j) = 6 - p_i + bp_j$ and $q_j(p_i, p_j) = 6 - p_j + bp_i$, where b is a parameter capturing product differentiation. Assume that $b \in [0, 1]$. Payoffs are given by each firm's profits.

1. Describe the game as a normal-form game (Players, Strategies, Payoffs).
2. Write the maximization problem for each firm.
3. Solve the maximization problem for each firm, obtaining its *reaction function*. (*Hint: Take the first derivative of profits with respect to each firm's price, taking the price set by the other as given).*
4. Find the *NE* of the game, i.e., equilibrium quantities for both firms (*Hint: Use the two reaction functions and solve for* $p_i^\star$).
5. How do prices vary with b? Explain.
6. Find the payoffs ($\pi_i(p_i^\star, p_j^\star)$) obtained by firms when they play this *NE* (*Hint: Replace equilibrium prices in profits*).