Exercise 1: Stackelberg duopoly

Two firms compete in a market by *sequentially* setting the quantity of a homogeneous good to produce. The game has two *stages*: in stage 1, Firm 1 sets quantity q_1; in stage 2. Firm 2 observes q_1, and then sets q_2. Each firm has a *constant marginal cost* $c = 2$. Firms face no *fixed cost. Inverse market demand* is $P(Q) = 14 - Q$, where Q is the aggregate quantity produced. Payoffs are given by each firm's profits.

Data of the problem

- Marginal cost: $c = 2$.
- Inverse demand: $P(Q) = 14 - Q$ where $Q = q_1 + q_2$.

Answer the following questions and explain your answer in detail.

1. Find the *subgame-perfect Nash equilibrium* of the game. *[4 points]*
2. Find the market price and the profit that firms obtain in equilibrium. *[4 points]*
3. Use the concept of *first-mover advantage* to compare the two firms' profits. *[3 points]*
4. Use the data of the problem to compare:

   i. industry quantity; and *[2 points]*

   ii. industry profits *[2 points]*,

   in the Stackelberg and Cournot equilibria (*hint: you are allowed to use the general formula for Cournot quantity and profits*).

Exercise 2: A static game of incomplete information

Consider the following *static game of complete information*.

	L	R
U	$2, 2$	$1, -3$
D	$-3, 1$	$0, 0$

Let us call player 1 (resp. player 2) the row (resp. column) player. That is, player 1's actions are U and D and player 2's actions are L and R.

Answer the following questions and explain your answer in detail.

1. Find all pure-strategy Nash equilibria of this game. *[2 points]*
2. Show that, in fact, each player has a dominant strategy. *[2 points]*
3. Given question 2, what can you say about the existence of mixed-strategy equilibria in this game (*Hint:* No computation required)? *[3 points]*

From now on, assume that there is some *uncertainty* about payoffs. The game of *incomplete information* can be represented as follows.

	L	R
U	$2, 2$	$1, -3$
D	$-3, 1$	$0, 0$

prob. 1/2

	L	R
U	$0, 0$	$1, -3$
D	$-3, 1$	$2, 2$

prob. 1/2

Notice that the game on the left is the same as the previous one. Assume that player 2 believes that each matrix occurs with probability one half. Player 1, however, knows which one is played.

4. Assume that player 1 **always** play U, that is, player 1's strategy is UU.
   (a) Compute player 2's *expected payoffs* when playing L. Same when playing R. Show that player 2 prefers to play L to R. *[3 points]*
   (b) Show that if player 2 plays L, then UU is actually a best response for player 1. *[3 points]*
   (c) Conclude whether (UU, L) is a *Bayesian Nash equilibrium*. *[2 points]*