

Exercise 1: Cournot duopoly

Two firms compete in a market by simultaneously setting the quantity of a homogeneous good to produce. The two firms face a *constant marginal cost* $c = 1$ and no *fixed cost*. *Inverse market demand* is $P(Q) = 13 - 2Q$, where Q is aggregate quantity. Payoffs are given by each firm's profits.

Data of the problem

- Firms' marginal cost: $c = 1$.
- Inverse demand: $P(Q) = 13 - 2Q$ where $Q = q_1 + q_2$.
- Point 5: $F = 6$.

Answer the following questions and explain your answer in detail.

1. Compute the two firms' reaction functions. [3 points]
2. Draw the two firms' reaction functions in the (q_1, q_2) space. [3 points]
3. Find the *Nash Equilibrium* of the game. [3 points]
4. Find the payoffs that firms obtain when they play the Nash equilibrium. [3 points]
5. Suppose each firm faces the same fixed cost $F = 6$. How is the equilibrium affected? [3 points]

Exercise 2

Consider two affiliate producers of electric car battery that we denote 1 and 2. Producer 1 can engage in R&D to develop a new generation of car battery that could benefit both producers 1 and 2.

In period 1, producer 1 can decide to create a high-quality car battery (H), a low-quality one (L) or not to invest at all (N) and use the current existing technology. In period 2, for each new technology H and L, producer 2 can decide whether to buy it from producer 1 (A) or not (R). If producer 1 decides not to invest (N), the game ends in period 1 as producer 2 is forced to use the current existing technology as well.

Payoffs are as follows.

Payoffs:

- (3, 2) if producer 1 invests in high quality (H) and producer 2 buys it (A)
- (0, 1) if producer 1 invests in high quality (H) and producer 2 does not buy it (R)
- (2, 3) if producer 1 invests in low quality (L) and producer 2 buys it (A)
- (0, 1) if producer 1 invests in low quality (L) and producer 2 does not buy it (R)
- (1, 1) if producer 1 chooses not to engage in R&D (N);

Answer the following questions and explain your answer in detail.

1. Using the information above, draw the tree of the dynamic game. Carefully specify actions, payoffs and players. [3 points]
2. What is the sequentially rational action of producer 2 after H? And after L? [3 points]
3. Find the *subgame-perfect Nash equilibrium*. [3 points]

Now we want to investigate Nash equilibria of this game.

4. Write the payoff matrix of this game. Be careful when you write producer 2's strategies. [3 points]
5. Find all pure-strategy Nash equilibria. [3 points]
6. **[Bonus point]** Carefully explain why (H, AR) is a Nash equilibrium of the game but not a subgame-perfect Nash equilibrium? [3 points]