

Practice session 5

Game Theory - MSc EEBL

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Exercise 1. A simple static game of incomplete information

In this exercise, the goal is to determine whether some strategy profiles are Bayesian Nash equilibria or not. See Exercise 2 for the method to find *all* Bayesian Nash equilibria in a similar game.

Consider the following *static game of incomplete information*.

| | L | R |
|---|-------|-------|
| U | 2, 2 | 1, -3 |
| D | -3, 1 | 0, 0 |

prob. 1/2

| | L | R |
|---|-------|-------|
| U | 0, 0 | 1, -3 |
| D | -3, 1 | 2, 2 |

prob. 1/2

We further assume that player 1 is fully informed about which of these two games is played whereas player 2 only knows that each game has the same probability of occurring.

1. Write the normal-form representation of this game.
2. For this question only, assume that player 1 **always** play U, that is, player 1's strategy is UU.
 - (a) What is player 2's expected payoff when playing L? When playing R? Deduce player 2's best response to UU.
 - (b) Show that if player 2 plays L, then UU is actually a best response for player 1.
 - (c) Conclude whether (UU, L) is a *Bayesian Nash equilibrium*.
3. Show that (UD, R) is not a Bayesian Nash equilibrium of this game.
4. Assume now that player 2 believes that the game on the left is played with probability $\alpha \in [0, 1]$.

- (a) Find a threshold value $\hat{\alpha}$, such that for all $\alpha \leq \hat{\alpha}$, R is a best response to UD for player 2.
- (b) Is (UD, R) is a Bayesian Nash equilibrium for $\alpha \leq \hat{\alpha}$?

Exercise 2. Hiring Decisions

Consider a Firm (F) and a Worker (W). If W has a *high ability* ($T_w = \text{high}$), they would like to *Work* when they are hired; if instead W has a *low ability* ($T_w = \text{low}$), they would rather *Shirk*.

F wants to *Hire* W if the latter is willing to *Work* and *Not Hire* them otherwise. W knows their ability level while F does not. However, F *believes* that W's ability is high with probability $\mathbb{P}(T_w = \text{high}) = \frac{2}{3}$ and low with probability $\mathbb{P}(T_w = \text{low}) = \frac{1}{3}$. The payoffs (different for each type of W) are as follows:

| | Work | Shirk |
|-------|------|-------|
| Hire | 1, 2 | 0, 1 |
| Don't | 0, 0 | 0, 0 |

$T_w = \text{High}$

| | Work | Shirk |
|-------|------|-------|
| Hire | 1, 1 | -1, 2 |
| Don't | 0, 0 | 0, 0 |

$T_w = \text{Low}$

1. Describe the game as a normal-form game.
2. Write the strategy profile for each player.
3. Write the game as an extensive-form game.
4. Find all (pure-strategy) Bayesian Nash equilibria of the game.

Exercise 3. Matching technologies

Consider two firms: Firm 1 is producing a video game console and firm 2 is producing a video game. Each firm can choose to *cooperate* (C) or *not cooperate* (N). For instance, firm 1 can cooperate by providing better development tools and firm 2 can cooperate by better optimizing their game to run on firm 1's console.

Each firm's product can be either of type *High* (H) or *Low* (L). Payoffs are as follows.

| | C | N |
|---|-------|-------|
| C | 3, 3 | -2, 0 |
| N | 0, -2 | 0, 0 |

$G_1 : \{H, H\}$

| | C | N |
|---|--------|-------|
| C | -1, -1 | -2, 0 |
| N | 0, -2 | 0, 0 |

$G_2 : \{L, L\} \text{ or } \{H, L\} \text{ or } \{L, H\}$

In words, firms enjoy cooperation only when they both have high types. Each firm only knows their own type. They both think that the probability that the other firm is of type H is $1/2$. Let $p_1(H) = p_2(H) = 1/2$ denote this belief.

1. (a) What is the probability that firms are in configuration G_1 ? Same question for G_2 .
(b) Now assume firm 1 knows it is of type H. From its point of view, what is the probability that it faces configuration G_1 ?
2. What is a strategy for firm $i = 1, 2$ in this game?
3. Assume firm 2 decides to cooperate for each of its type, that is, firm 2 plays CN.
 - (a) What is the best response of firm 1 when it is of type H?
 - (b) What is the best response of firm 1 when it is of type L?
 - (c) Deduce a Bayesian Nash equilibrium of this game.