From Osborne Introduction to game theory

? EXERCISE 32.2 (Voter participation) Two candidates, A and B, compete in an election. Of the n citizens, k support candidate A and m $(= n - k)$ support candidate B. Each citizen decides whether to vote, at a cost, for the candidate she supports, or to abstain. A citizen who abstains receives the payoff of 2 if the candidate she supports wins, 1 if this candidate ties for first place, and 0 if this candidate loses. A citizen who votes receives the payoffs $2 - c$, $1 - c$, and $-c$ in these three cases, where $0 < c < 1$.

a. For $k = m = 1$, is the game the same (except for the names of the actions) as any considered so far in this chapter?

b. For $k = m$, find the set of Nash equilibria. (Is the action profile in which everyone votes a Nash equilibrium? Is there any Nash equilibrium in which the candidates tie and not everyone votes? Is there any Nash equilibrium in which one of the candidates wins by one vote? Is there any Nash equilibrium in which one of the candidates wins by two or more votes?)

c. What is the set of Nash equilibria for $k < m$?

If, when sitting in a traffic jam, you have ever thought about the time you might save if another road were built, the next exercise may lead you to think again.

34.2 Voter participation

a. For $k = m = 1$ the game is shown in Figure 8.1. It is the same, except for the names of the actions, as the *Prisoner's Dilemma*.

		B supporter	
		abstain	*vote*
A supporter	*abstain*	$1, 1$	$0, 2 - c$
	vote	$2 - c, 0$	$1 - c, 1 - c$

Figure 8.1 The game of voter participation in Exercise 34.2.

b. For $k = m$, denote the number of citizens voting for A by n_A and the number voting for B by n_B. The cases in which $n_A \leq n_B$ are symmetric with those in which $n_A \geq n_B$; I restrict attention to the latter.

$n_A = n_B = k$ (all citizens vote): A citizen who switches from voting to abstaining causes the candidate she supports to lose rather than tie, reducing her payoff from $1 - c$ to 0. Since $c < 1$, this situation is a Nash equilibrium.

$n_A = n_B < k$ (not all citizens vote; the candidates tie): A citizen who switches from abstaining to voting causes the candidate she supports to win rather than tie, increasing her payoff from 1 to $2 - c$. Thus this situation is not a Nash equilibrium.

$n_A = n_B + 1$ or $n_B = n_A + 1$ (a candidate wins by one vote): A supporter of the losing candidate who switches from abstaining to voting causes the candidate she supports to tie rather than lose, increasing her payoff from 0 to $1 - c$. Thus this situation is not a Nash equilibrium.

$n_A \geq n_B + 2$ or $n_B \geq n_A + 2$ (a candidate wins by two or more votes): A supporter of the winning candidate who switches from voting to ab-