Ex. 3

Assume $n$ (odd) individual ($i = 1, .., n$) can vote for a level of public good under a majority voting rule. Individuals have quasi-linear preferences over the public and private good described by the following utility function:

$$U_i = \beta_i \ln(G) + Z_i$$

Assume that $\beta_1 < \beta_2 < ... < \beta_n$. The income for each individual is $w_i$. The cost of the public good, $G$, is $c = 1$ and it is equally shared among individual, then each individual pays $\frac{1}{n}$ of the total provision. The price of the private good $Z_i$ is normalized to 1.

a) Show wether the median voter theorem (MVT) applies

the number $n$ is odd an the preferences are singles peaked ( the utility function is concave )

$$\frac{d^2 U_i}{dG^2} = -\frac{1}{G^2}\beta_i < 0$$

therefore the MDV applies and the result of the voting is the level of public good preferred by the median voter. We can simply solve the maximization problem of the median voter, that by using the budget constraint to obtain $Z_i$, gives:

$$\max_G U_i = \beta_i \ln(G) + w_i - \frac{G}{n}$$

the first order condition gives:

$$\frac{\partial U_i}{\partial G} = \beta_i \frac{1}{G} - \frac{1}{n} = 0 \tag{1}$$

that, after denoting $G_m$ and $\beta_m$ the respective level solving (1), after rearranging we have:

$$G_m = n\beta_m \tag{2}$$

b) Show whether the result of the majority voting is Pareto efficient,

The Pareto efficient level of public good is obtained from the following condition:

$$\sum_{i=1}^{n} MRS_i = \sum_{i=1}^{n} \frac{\frac{\partial U_i}{\partial G}}{\frac{\partial U_i}{\partial Z_i}} = \sum_{i=1}^{n} \frac{\beta_i}{G} = 1$$

that after solving gives:

$$G^e = \sum_{i=1}^{n} \beta_i = n\left(\frac{1}{n}\sum_{i=1}^{n} \beta_i\right)$$

the majority voting rule is efficient if

$$\beta_m = \frac{1}{n}\sum_{i=1}^{n}\beta_i \tag{3}$$

in fact if we plug (3) into (2) we obtain $G^e$.